Solutions to CSEC Maths P2 June 2010

Question 1(a)(i)

Required to calculate: $\frac{1\frac{1}{2}-\frac{2}{5}}{4\frac{2}{5}\times\frac{3}{4}}$

$$\text{Numerator} = 1\frac{1}{2}-\frac{2}{5}$$

$$= \frac{3}{2}-\frac{2}{5}$$

$$= \frac{15-4}{10}$$

$$= \frac{11}{10}$$

$$\text{Denominator} = 4\frac{2}{5}\times\frac{3}{4}$$

$$= \frac{22}{5}\times\frac{3}{4}$$

$$= \frac{66}{20}$$

$$= \frac{33}{10}$$

$$\therefore \text{Numerator} \div \text{Denominator} = \frac{11}{10}\div\frac{33}{10}$$

$$= \frac{11}{10}\times\frac{10}{33}$$

$$= \frac{11}{33}$$

$$= \frac{1}{3}$$

Question 1(a)(ii)

Required to calculate: $2.5^2 - \frac{2.89}{17}$

Using a calculator,

$$2.5^2 - \frac{2.89}{17} = 6.25 - 0.17$$

$$= 6.08$$

$$= 6.1 \qquad \text{(to 2 significant figures)}$$

Question 1(b)(i)

Required to calculate the cost of 1 T-shirt.

150 T-shirts $= \$1920$

1 T-shirt $= \frac{\$1920}{150}$

$= \$12.80$

Question 1(b)(ii)

Required to calculate the amount for 150 T-shirts at \$19.99 each.

1 T-shirt $= \$19.99$

150 T-shirts $= \$19.99 \times 150$

$= \$2998.50$

Question 1(b)(iii)

Required to calculate the profit.

$$\text{Profit} = \text{Selling Price} - \text{Cost Price}$$

$$= \$2998.50 - \$1920$$

$$= \$1078.50$$

Question 1(b)(iv)

Required to calculate the percentage profit.

$$\text{Percentage profit} = \frac{\text{Profit}}{\text{Cost Price}} \times 100\%$$

$$= \frac{1078.50}{1920} \times 100\%$$

$$= 56.2\%$$

$$= 56\% \qquad \text{(to the nearest whole number)}$$

Question 2(a)(i)

Required to find the value of $a + b + c$.

$$a + b + c = (-1) + 2 + (-3)$$
$$= -1 + 2 - 3$$
$$= 2 - 4$$
$$= -2$$

Question 2(a)(ii)

Required to find the value of $b^2 - c^2$.

$$b^2 - c^2 = (2)^2 - (-3)^2$$
$$= 4 - 9$$
$$= -5$$

Question 2(b)(i)

Required to express the statement given as an algebraic expression.

Phrase: "Seven times the sum of x and y."

Algebraic expression: $7(x + y)$

Question 2(b)(ii)

Required to express the statement given as an algebraic expression.

Phrase: "The product of TWO consecutive numbers when the smaller is y."

If the smaller number is y, then the next larger, consecutive number is $y + 1$.

Algebraic expression: $y(y + 1)$

Question 2(c)

Required to solve the given pair of simultaneous equations.

$2x + y = 7 \quad \rightarrow$ Equation 1

$x - 2y = 1 \quad \rightarrow$ Equation 2

Multiplying Equation 2 by 2 gives:

$2x - 4y = 2 \quad \rightarrow$ Equation 3

Equation 1 – Equation 3 gives:

$5y = 5$

$y = \frac{5}{5}$

$y = 1$

Substituting $y = 1$ into Equation 2 gives:

$x - 2(1) = 1$

$x - 2 = 1$

$x = 1 + 2$

$x = 3$

$\therefore x = 3$ and $y = 1$

Question 2(d)(i)

Required to factorise completely $4y^2 - z^2$.

$4y^2 - z^2 = (2y + z)(2y - z)$ → difference of two squares

Question 2(d)(ii)

Required to factorise completely $2ax - 2ay - bx + by$.

$$2ax - 2ay - bx + by$$
$$= 2a(x - y) - b(x - y)$$
$$= (2a - b)(x - y)$$

Question 2(d)(iii)

Required to factorise completely $3x^2 + 10x - 8$.

$$3x^2 + 10x - 8$$
$$= 3x^2 + 12x - 2x - 8$$
$$= 3x(x + 4) - 2(x + 4)$$
$$= (3x - 2)(x + 4)$$

Question 3(a)(i)

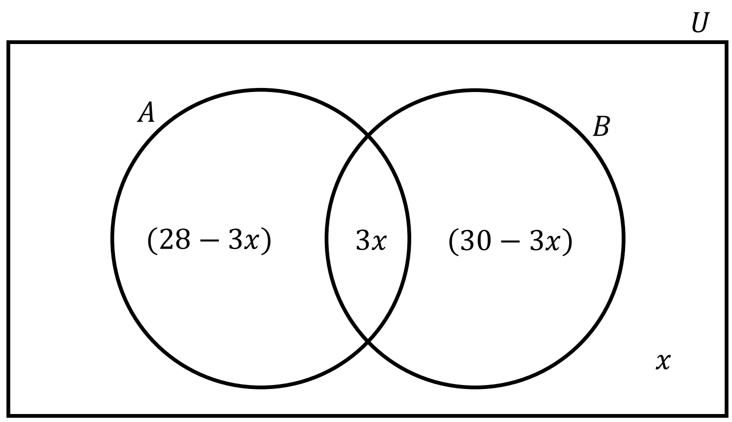
Required to copy and complete the Venn diagram to represent the given information.

28 visited Antigua

30 visited Barbados

$3x$ visited both Antigua and Barbados

x visited neither Antigua nor Barbados



Question 3(a)(ii)

Required to find an expression for the total number of tourists.

$$\text{Total number of tourists} = (28 - 3x) + 3x + (30 - 3x) + x$$

$$= 28 - 3x + 3x + 30 - 3x + x$$

$$= 58 - 2x$$

Question 3(a)(iii)

Required to calculate the value of x.

The survey was conducted among 40 tourists.

$58 - 2x = 40$

$2x = 58 - 40$

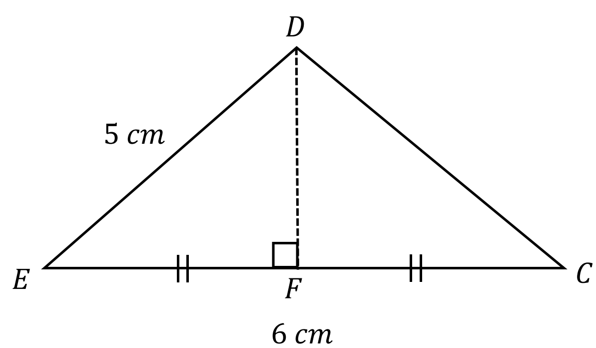
$2x = 18$

$x = \frac{18}{2}$

$x = 9$

Question 3(b)(i)

Required to calculate the length of EF.



F is the midpoint of EC.

$\therefore EF = \frac{6}{2}$

$= 3\ cm$

Question 3(b)(ii)

Required to calculate the length of DF.

By Pythagoras' Theorem,

$$(EF)^2 + (DF)^2 = (DE)^2$$

$$(3)^2 + (DF)^2 = (5)^2$$

$$9 + (DF)^2 = 25$$

$$(DF)^2 = 25 - 9$$

$$(DF)^2 = 16$$

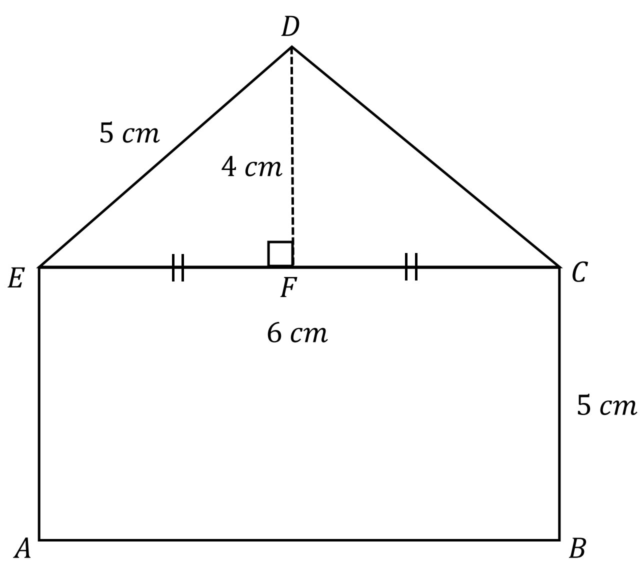
$$DF = \sqrt{16}$$

$$DF = 4\ cm$$

$\therefore$ The length of DF is $4\ cm$.

Question 3(b)(iii)

Required to calculate the area of the face $ABCDE$.



Area of $\Delta DEC = \frac{b \times h}{2}$

$= \frac{6 \times 4}{2}$

$= \frac{24}{2}$

$= 12\ cm^2$

Area of rectangle $ABCE = l \times b$

$= 6 \times 5$

$= 30\ cm^2$

$\therefore$ Area of the entire face $ABCDE = 12 + 30$

$= 42\ cm^2$

Question 4(a)(i)

Required to find the value of k.

Substituting $y = 50$ and $x = 10$ into $y = kx^2$ gives:

$50 = k(10)^2$

$50 = 100k$

$k = \frac{50}{100}$

$k = \frac{1}{2}$

Question 4(a)(ii)

Required to calculate the value of y when $x = 30$.

$y = \frac{1}{2}x^2$

When $x = 30$,

$y = \frac{1}{2}(30)^2$

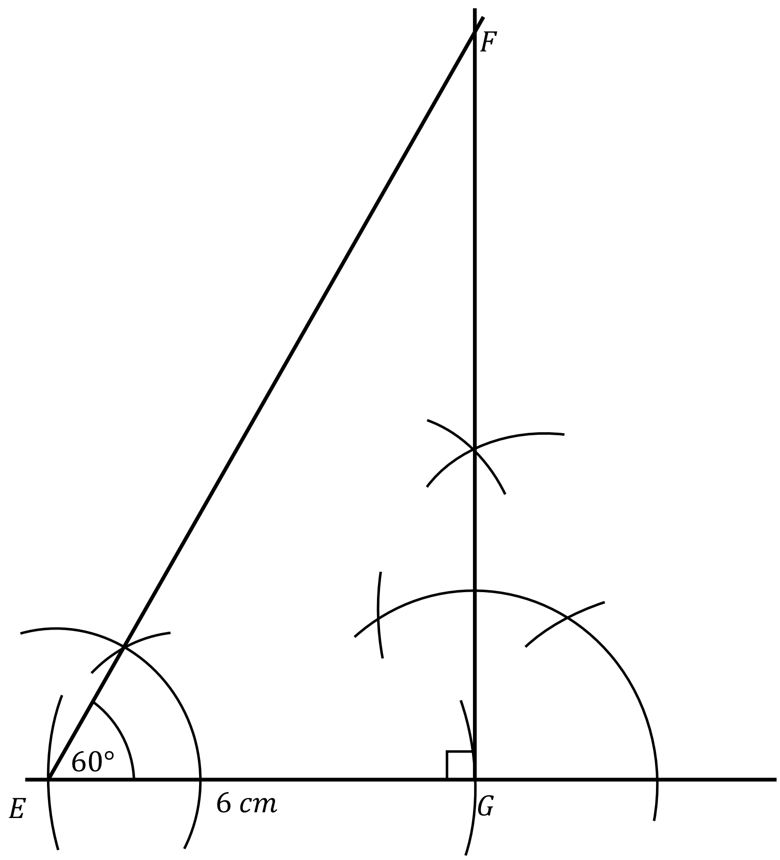
$= \frac{1}{2}(900)$

$= 450$

Question 4(b)(i)

Required to construct triangle EFG.

$EG = 6\ cm$, $\angle FEG = 60°$ and $\angle EGF = 90°$.



Question 4(b)(ii)(a)

Required to find the length of EF.

By measurement, $EF = 12.0\ cm$.

Question 4(b)(ii)(b)

Required to find the size of $\angle EFG$.

By measurement, $E\hat{F}G = 30°$.

Question 5(a)(i)(a)

Required to find the calculate the value of $f(4)$.

$f(x) = 2x - 5$

$f(4) = 2(4) - 5$

$= 8 - 5$

$= 3$

Question 5(a)(i)(b)

Required to find the calculate the value of $gf(4)$.

$gf(4) = g[f(4)]$

$= g(3)$

$= (3)^2 + 3$

$= 9 + 3$

$= 12$

$\therefore gf(4) = 12$

Question 5(a)(ii)

Required to find $f^{-1}(x)$.

$f(x) = 2x - 5$

Let $y = f(x)$.

$y = 2x - 5$

Interchanging variables x and y gives:

$x = 2y - 5$

Making y the subject of the formula gives:

$x + 5 = 2y$

$\frac{x+5}{2} = y$

$\therefore f^{-1}(x) = \frac{x+5}{2}$

Question 5(b)(i)

Required to use the graph to determine the scale used on the x-axis.

On the x-axis, the scale used is $2\ cm = 1\ unit$ or $1\ cm = 0.5\ unit$.

Question 5(b)(ii)

Required to find the value of y for which $x = -1.5$.

When $x = -1.5, y = -3.8$. (by read-off)

Question 5(b)(iii)

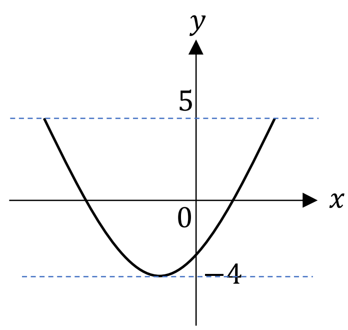
Required to find the values of x for which $y = 0$.

When $y = 0, x = -3$ and $x = 1$. (by read-off)

Question 5(b)(iv)

Required to determine the range of values of y, giving your answer in the form

$a \leq y \leq b$, where a and b are real numbers.



The range of values of y is: $-4 \leq y \leq 5$ which is of the form $a \leq y \leq b$.

Question 6(a)(i)

Required to determine the value of x.

Since alternate angles

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Question 6(a)(ii)

Required to calculate an expression for $gf(x)$.

$$gf(x) = g[f(x)]$$
$$= g(6x + 8)$$
$$= \frac{(6x+8)-2}{3}$$
$$= \frac{6x+8-2}{3}$$
$$= \frac{6x+6}{3}$$
$$= \frac{3(2x+2)}{3}$$
$$= 2x + 2$$

$\therefore gf(x) = 2x + 2$

Question 6(a)(iii)

Required to calculate $f^{-1}(x)$.

$f(x) = 6x + 8$

Let $y = f(x)$.

$y = 6x + 8$

Interchange variables x and y.

$x = 6y + 8$

Make y the subject.

$x - 8 = 6y$

$\frac{x-8}{6} = y$

$y = \frac{x-8}{6}$

$\therefore f^{-1}(x) = \frac{x-8}{6}$

Question 6(b)(i)

Required to find the coordinates of A and B.

From the graph,

The coordinate of A is $(-2, 3)$.

The coordinate of B is $(4, 6)$.

Question 6(b)(ii)

Required to find the gradient of AB.

Points are $A(-2, 3)$ and $B(4, 6)$.

Gradient of $AB = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{6-3}{4-(-2)}$

$= \frac{3}{6}$

$= \frac{1}{2}$

Question 6(b)(iii)

Required to find the equation of the line passing through A and B.

Substituting $m = \frac{1}{2}$ and point $A(-2, 3)$ into $y - y_1 = m(x - x_1)$ gives,

$y - 3 = \frac{1}{2}\big(x - (-2)\big)$

$y - 3 = \frac{1}{2}(x + 2)$

$y - 3 = \frac{1}{2}x + 1$

$y = \frac{1}{2}x + 1 + 3$

$y = \frac{1}{2}x + 4$

$\therefore$ The equation of the line passing through A and B is $y = \frac{1}{2}x + 4$.

Question 7(a)

Required to copy and complete the table to show cumulative frequency for the distribution.

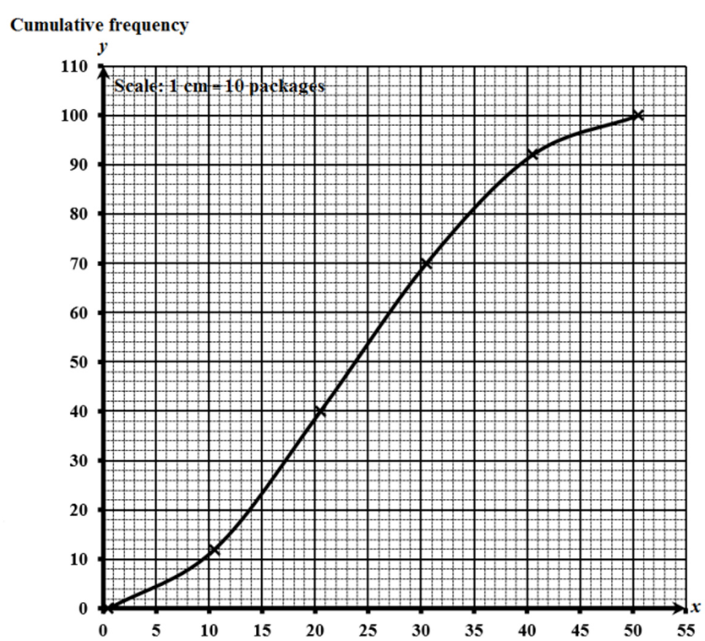
The completed table is shown below.

Mass, m (kg)	Upper Class Boundary (UCB)	Number of Packages (Frequency)	Cumulative Frequency
1-10	10.5	12	12
11-20	20.5	28	40
21-30	30.5	30	70
31-40	40.5	22	92
41-50	50.5	8	100

Question 7(b)

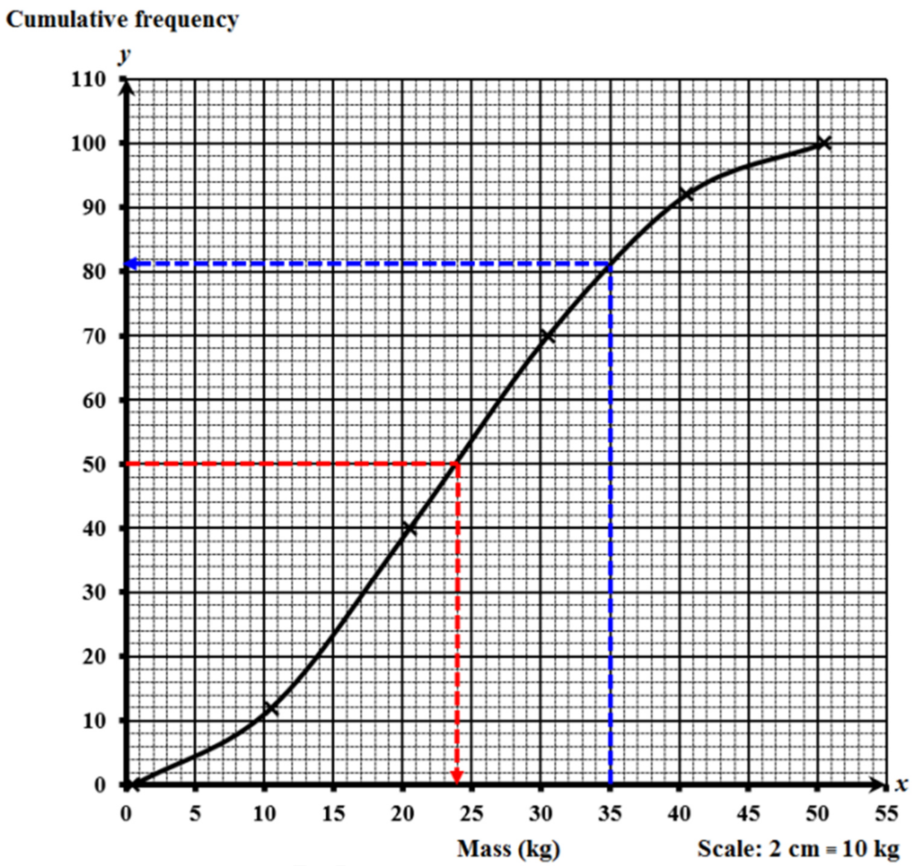
Required to draw the cumulative frequency curve for the data.

The cumulative frequency graph for the data is shown below.



Question 7(c)(i)

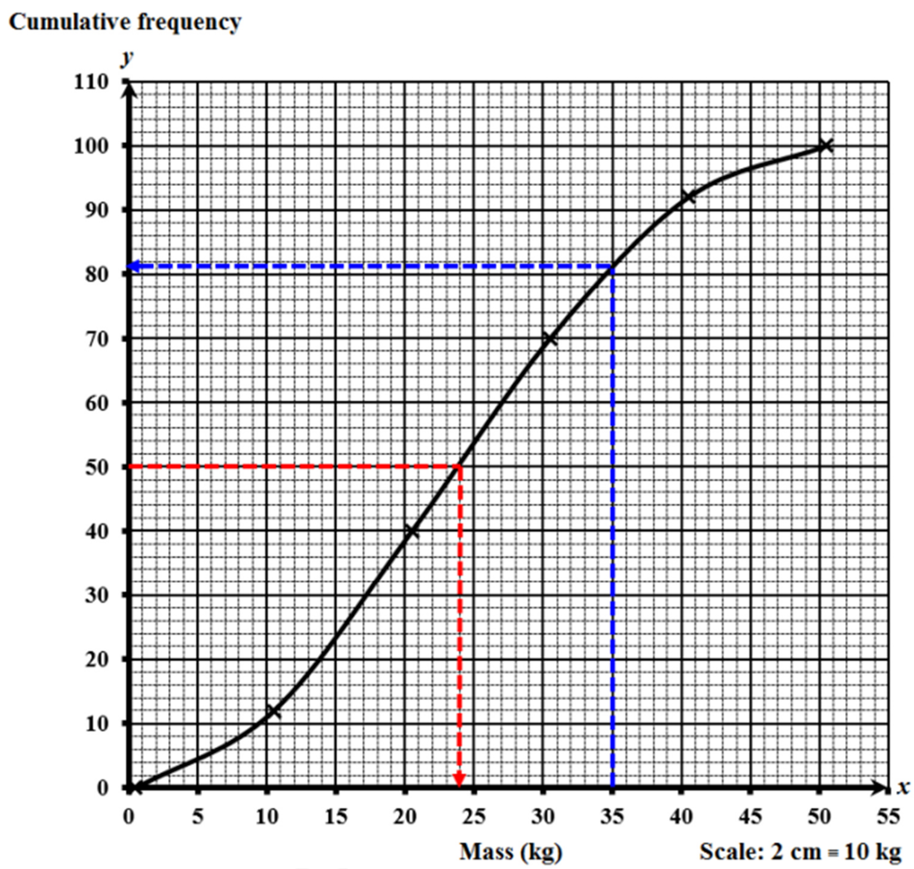
Required to find the median mass.



The median mass for the data is 24 kg.

Question 7(c)(ii)

Required to find the probability that the mass of a package is less than 35 kg.



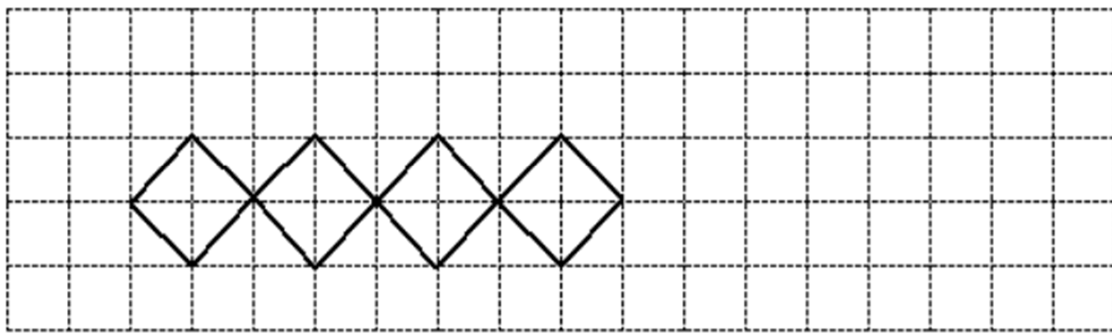
$$P(mass\ is\ less\ than\ 35\ kg) = \frac{Number\ of\ packages\ less\ than\ 35\ kg}{Total\ number\ of\ packages}$$

$$= \frac{81}{100} \quad \text{or} \quad 0.81 \quad \text{or} \quad 81\%$$

Question 8(a)

Required to draw the fourth diagram in the sequence.

The fourth diagram in the sequence is shown below:



Question 8(b)(i)

Required to find how many sticks are in the sixth diagram.

Number of sticks $= 6 \times 4$

$= 24$ sticks

Question 8(b)(ii)

Required to find how many thumb tacks are in the seventh diagram.

Number of sticks $= 7(4)$

The rule connecting t and s gives

$1 + \left(\frac{3}{4} \times 7(4)\right) = 22$

Hence, the number of thumb tacks are in the seventh diagram is 22.

Question 8(c)

Required to copy and complete the missing values in the table.

The completed table is shown below.

No. of Sticks $\boldsymbol{s}$	**Rule Connecting $\boldsymbol{t}$ and $\boldsymbol{s}$**	**No. of Thumb Tacks** $\boldsymbol{t}$
4	$1+\left(\frac{3}{4}\times 4\right)$	4
8	$1+\left(\frac{3}{4}\times 8\right)$	7
12	$1+\left(\frac{3}{4}\times 12\right)$	10
52	$\underline{1+\left(\frac{3}{4}\times 52\right)}$	$\underline{40}$
$\underline{72}$	$\underline{1+\left(\frac{3}{4}\times 72\right)}$	55

When $s=52$,

$$t=1+\left(\frac{3}{4}\times 52\right)$$

$$=1+39$$

$$=40$$

When $t=55$,

$$1+\left(\frac{3}{4}\times s\right)=55$$

$$\frac{3}{4}s=55-1$$

$$\frac{3}{4}s=54$$

$$s=54\times\frac{4}{3}$$

$$s=72$$

Question 8(d)

Required write in terms of s and t, to show how t is related to s.

The required equation is: $t = 1 + \left(\frac{3}{4} \times s\right)$.

Question 9(a)

Required to solve $y = x^2 - x + 3$ and $y = 6 - 3x$ simultaneously.

$y = x^2 - x + 3$ $\quad\rightarrow$ Equation 1

$y = 6 - 3x$ $\quad\rightarrow$ Equation 2

Equating Equation 1 and Equation 2 gives,

$$x^2 - x + 3 = 6 - 3x$$

$$x^2 - x + 3x + 3 - 6 = 0$$

$$x^2 + 2x - 3 = 0$$

$$x^2 + 3x - x - 3 = 0$$

$$x(x + 3) - 1(x + 3) = 0$$

$$(x - 1)(x + 3) = 0$$

Either $\quad x - 1 = 0 \quad$ or $\quad x + 3 = 0$

$\qquad x = 1 \qquad\qquad x = -3$

When $x = 1$,

$$y = 6 - 3(1)$$
$$= 6 - 3$$
$$= 3$$

When $x = -3$,

$$y = 6 - 3(-3)$$
$$= 6 + 9$$
$$= 15$$

$\therefore x = 1, y - 3 \quad$ or $\quad x = -3, y = 15$

Question 9(b)(i)

Required to express $4x^2 - 8x - 2$ in the form $a(x+h)^2 + k$.

$4x^2 - 8x - 2$

$= 4(x^2 - 2x) - 2$

$= 4(x^2 - 2x + 1) - 2 - 4(1)$

$= 4(x-1)^2 - 2 - 4$

$= 4(x-1)^2 - 6$

which is of the form $a(x+h)^2 + k$ where $a = 4, h = -1$ and $k = -6$.

Question 9(b)(ii)

Required to find the value of x for which $f(x)$ is a minimum.

Value of x for which $f(x)$ is a minimum $= -h$

$= -(-1)$

$= 1$

Question 9(c)(i)

Required to find the value of x.

Gradient $= \frac{12-0}{x-0}$

Since the gradient is equal to $0.6\ ms^{-2}$, then we have,

$0.6 = \frac{12}{x}$

$x = \frac{12}{0.6}$

$x = 20\ s$

Question 9(c)(ii)

Required to find the gradient of the graph during the second stage and to explain, in one sentence, what the car is doing in this stage.

In the second stage of the journey, the velocity is constant as the gradient is 0. Hence, the car is moving at a constant speed of 12 ms^{-1}.

Question 9(c)(iii)

Required to find the distance travelled during the third stage.

Distance travelled = Area under the graph

$$= \frac{1}{2}(60 - 25) \times 12$$

$$= 210\ m$$

Question 10(a)(i)

Required to calculate $X\hat{Y}Z$.

The opposite angles of the cyclic quadrilateral $WXYZ$ are supplementary.

$X\hat{Y}Z = 180° - 64°$

$= 116°$

Question 10(a)(ii)

Required to calculate $Y\hat{X}Z$.

The angle made by the tangent to a circle and a chord, angle VYZ at the point of contact is equal to the angle in the alternate segment, angle YXZ.

So, $Y\hat{X}Z = 23°$.

Question 10(a)(iii)

Required to calculate $O\hat{X}Z$.

Since OX and OZ are radii of the same circle, then $OX = OZ$.

Triangle OXZ is an isosceles triangle since the two sides are equal and the base angles are also equal.

The sum of angles in a triangle add up to 180°.

$O\hat{X}Z = \frac{(180° - 128°)}{2}$

$= \frac{52°}{2}$

$= 26°$

Question 10(b)(i)

Required to calculate the value of x.

The angles at a point in a straight line add up to 180°.

$x = 180° - (48° + 56°)$

$= 76°$

Question 10(b)(ii)

Required to calculate the length of RP.

Consider the ΔPQR. Using the cosine rule,

$RP^2 = PQ^2 + QR^2 - 2(PQ)(QR)\cos R\hat{Q}P$

$RP^2 = (220)^2 + (360)^2 - 2(220)(360)\cos 56°$

$RP^2 = 48\,400 + 129\,600 - 158\,400\cos 56°$

$RP^2 = 89\,423.84409$

$RP = \sqrt{89\,423.84409}$

$RP = 299\ km$ (to the nearest whole number)

Question 10(b)(iii)

Required to calculate the bearing of R from P.

Consider the ΔPQR. Using the sine rule,

$$\frac{RQ}{\sin R\hat{P}Q} = \frac{RP}{\sin 56°}$$

$$\frac{360}{\sin R\hat{P}Q} = \frac{299}{\sin 56°}$$

$\sin R\hat{P}Q = \frac{360 \times \sin 56°}{299}$

$R\hat{P}Q = \sin^{-1}\left(\frac{360 \times \sin 56°}{299}\right)$

$R\hat{P}Q = 86.5°$

$\therefore$ Bearing of R from $P = 132° + 86.5°$

$= 218.5°$

Question 11(a)

Required to calculate the inverse of the matrix, $M = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$.

$$\det(M) = ad - bc$$
$$= (3)(4) - (5)(2)$$
$$= 12 - 10$$
$$= 2$$

$$adj(M) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$
$$= \begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix}$$

$$\therefore M^{-1} = \frac{1}{det} \times adj(A)$$
$$= \frac{1}{2} \times \begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} \frac{4}{2} & \frac{-5}{2} \\ \frac{-2}{2} & \frac{3}{2} \end{pmatrix}$$
$$= \begin{pmatrix} 2 & -\frac{5}{2} \\ -1 & \frac{3}{2} \end{pmatrix}$$

Question 11(b)(i)

Required to calculate the value of a and b.

So, we have,

$$\begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix}\begin{pmatrix} 7 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

$$\begin{pmatrix} (0 \times 7) + (a \times 2) \\ (b \times 7) + (0 \times 2) \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

$$\begin{pmatrix} 0 + 2a \\ 7b + 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

$$\begin{pmatrix} 2a \\ 7b \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

Comparing the equivalent matrices and equating the corresponding entries gives:

$2a = 2$ and $7b = -7$

$a = \frac{2}{2}$ $\qquad$ $b = \frac{-7}{7}$

$a = 1$ $\qquad$ $b = -1$

$\therefore a = 1$ and $b = -1$.

Question 11(b)(ii)

Required to describe the transformation that M represents.

$$M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

The matrix M represents a $90°$ clockwise rotation about the origin.

Question 11(c)(i)(a)

Required to write an expression in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ for $\overrightarrow{WY}$.

$$\overrightarrow{WY} = \overrightarrow{WV} + \overrightarrow{VY}$$

$$= \boldsymbol{a} + (-\boldsymbol{b})$$

$$= \boldsymbol{a} - \boldsymbol{b}$$

Question 11(c)(i)(b)

Required to write an expression in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ for $\overrightarrow{WS}$.

$$\overrightarrow{WS} = \frac{1}{3}\overrightarrow{WY}$$

$$= \frac{1}{3}(\boldsymbol{a} - \boldsymbol{b})$$

$$= \frac{1}{3}\boldsymbol{a} - \frac{1}{3}\boldsymbol{b}$$

Question 11(c)(i)(c)

Required to write an expression in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ for $\overrightarrow{SX}$.

$$\overrightarrow{SX} = \overrightarrow{SW} + \overrightarrow{WX}$$

$$= -\frac{1}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b} + \boldsymbol{a}$$

$$= \frac{2}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$$

Question 11(c)(ii)

Required to prove that R, S and X are collinear.

$$\overrightarrow{RX} = \overrightarrow{RW} + \overrightarrow{WX}$$

$$\overrightarrow{RX} = \frac{1}{2}b + a$$

$$\overrightarrow{RX} = a + \frac{1}{2}b$$

Now,

$$\overrightarrow{SX} = \frac{2}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$$

$$= \frac{2}{3}\left(\boldsymbol{a} + \frac{\boldsymbol{1}}{\boldsymbol{2}}\boldsymbol{b}\right)$$

$$= \frac{2}{3}\overrightarrow{RX}$$

Since $\overrightarrow{SX}$ is related to $\overrightarrow{RX}$ by the scalar factor of $\frac{2}{3}$, they are parallel.

The vectors share a common point R, therefore, they are collinear.