Solutions to CSEC Maths P2 June 2019

Question 1(a)(i)

Required to evaluate $\frac{2\frac{1}{4}-1\frac{3}{5}}{3}$.

$$\frac{2\frac{1}{4}-1\frac{3}{5}}{3}=\left(\frac{9}{4}-\frac{8}{5}\right)\div 3$$

$$=\left(\frac{5(9)-4(8)}{20}\right)\div\frac{3}{1}$$

$$=\left(\frac{45-32}{20}\right)\div\frac{3}{1}$$

$$=\frac{13}{20}\times\frac{1}{3}$$

$$=\frac{13}{60}$$

Question 1(a)(ii)

Required to evaluate $2.14\sin 75°$, giving your answer to 2 decimal places.

$2.14\sin 75° = 2.07$ (to 2 decimal places)

Question 1(b)(i)

The data given as follows.

Item	**Amounts Allocated**
Rent	$\$x$
Food	$629
Other living expenses	$\$2x$
Savings	$1,750
Total	**$4,320**

Required to calculate the annual take-home pay.

Rate of pay = \$4, 320 per fortnight (every 2 weeks)

One year = 52 weeks

$\therefore$ There are $\frac{52}{2} = 26$ fortnight per year.

So, $26 \times \$4,320 = \$112,320$ per year

Question 1(b)(ii)

Required to determine the amount of money that Irma allocated for rent each month.

$$x + 629 + 2x + 1750 = 4320$$

$$x + 2x + 2379 = 4320$$

$$3x = 4320 - 2379$$

$$3x = 1941$$

$$x = \frac{1941}{3}$$

$$x = 647$$

Allocation for rent per fortnight = \$647

There are two fortnights per month.

So, Irma's rent per month = $\$647 \times 2$

$= \$1294$

Question 1(b)(iii)

Required to calculate how long it will take to pay off tuition.

$$\text{Tuition cost} = \$150,000$$

$$\text{Savings per fortnight} = \$1,750$$

$$\text{Total savings per year} = \$1,750 \times 26$$

$$= \$45,500$$

$$\text{Number of years to pay off} = \frac{\$150,000}{\$45,500}$$

$$= 3.30 \quad \text{(to 2 decimal places)}$$

OR

The question says that she saves the same amount of money each MONTH. There are two fortnights every month and 12 months per year. Therefore, there are 24 fortnights to consider.

$$\text{Savings per year} = \$1,750 \times 24$$

$$= \$42,000$$

$$\text{Number of years to pay off} = \frac{\$150,000}{\$42,000}$$

$$= 3.57 \quad \text{(to 2 decimal places)}$$

Either way she must work at least 4 years.

Question 2(a)(i)

Required to simplify completely $3p^2 \times 4p^2$.

$3p^2 \times 4p^2$

$= 12p^{2+5}$

$= 12p^7$

Question 2(a)(ii)

Required to simplify completely $\frac{3x}{4y^3} \div \frac{21x^2}{20y^2}$.

$\frac{3x}{4y^3} \div \frac{21x^2}{20y^2}$

$= \frac{3x}{4y^3} \times \frac{20y^2}{21x^2}$

$= \frac{60xy^2}{84x^2y^3}$

$= \frac{5xy^2}{7x^2y^3}$

$= \frac{5}{7xy}$

Question 2(b)

Required to solve the equation $\frac{3}{7x-1} + \frac{1}{x} = 0$.

$\frac{3}{7x-1} + \frac{1}{x} = 0$

$\frac{3x+1\,(7x-1)}{x(7x-1\,)} = 0$

If a fraction is equal to 0, then the numerator is 0.

We have,

$3x + 7x - 1 = 0$

$10x - 1 = 0$

$10x = 1$

$x = \frac{1}{10}$

Question 2(c)(i)

Required to express y in terms of x.

We are given that when a number, x, is multiplied by 2, the result is squared to give a new number, y.

This can be expressed as:

$(x \times 2)^2 = y$

$(2x)^2 = y$

$y = 4x^2$

Question 2(c)(ii)

Required to determine the two values of x that satisfy the equation $y = x$ and the equation derived in (c)(i).

$y = x$ → Equation 1

$y = 4x^2$ → Equation 2

Equating both equations gives:

$x = 4x^2$

$4x^2 - x = 0$

$x(4x - 1) = 0$

Either $x = 0$ or $4x - 1 = 0$

$$4x = 1$$

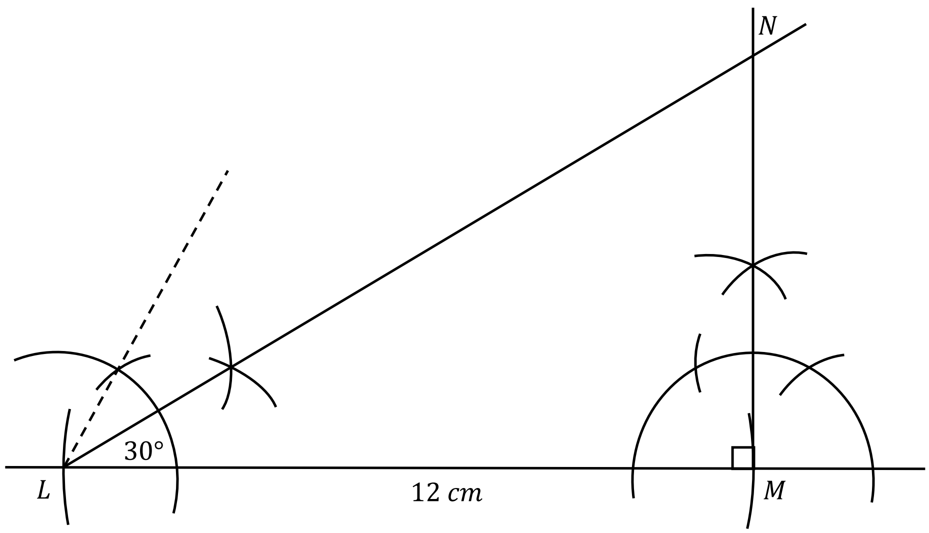
$$x = \frac{1}{4}$$

$\therefore x = 0$ or $x = \frac{1}{4}$

Question 3(a)

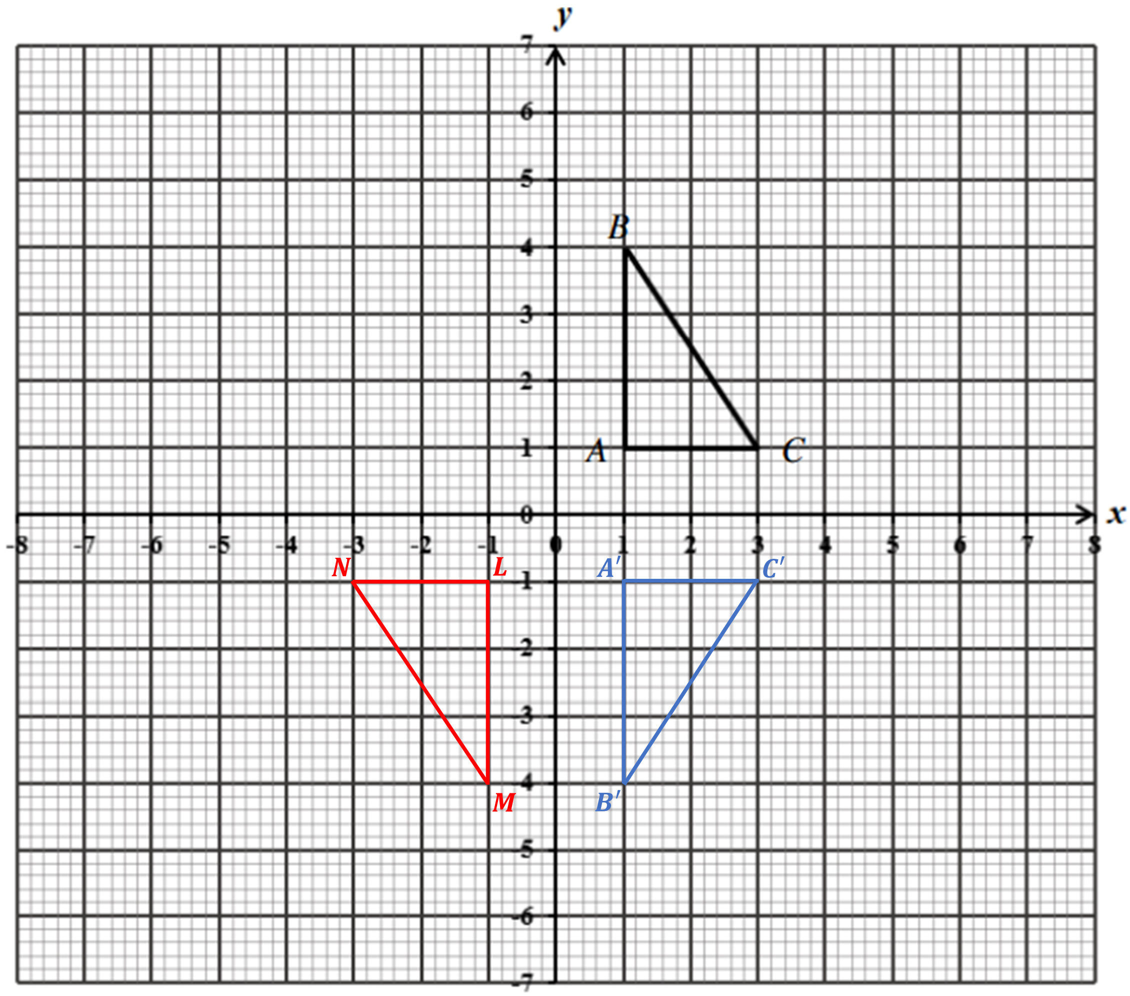
Required to construct triangle NLM.

$LM = 12\ cm$, $\angle MLN = 30°$ and $\angle LMN = 90°$



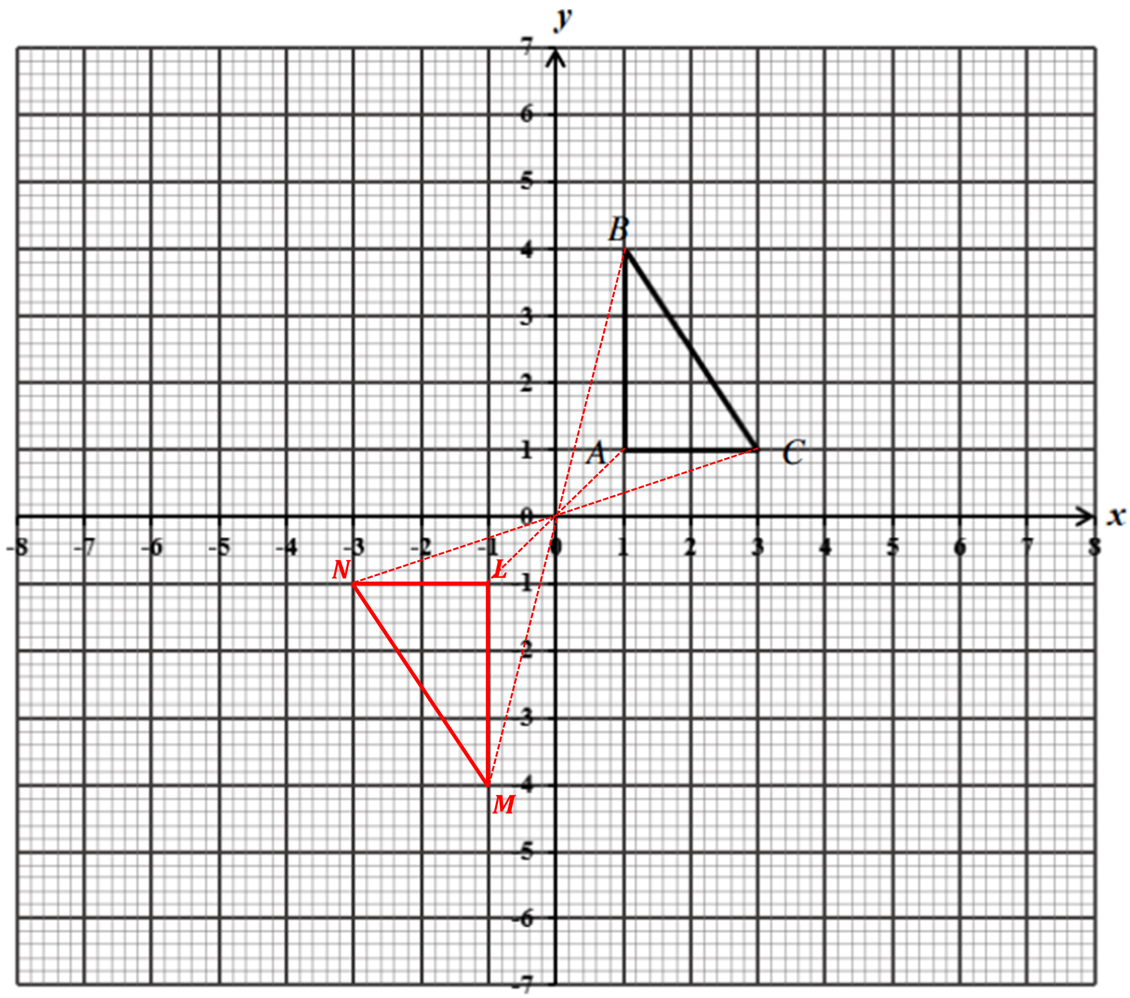
Question 3(b)(i)

Required to draw and label ΔLMN.



Question 3(b)(ii)

Required to describe fully the transformation that maps ΔABC onto ΔLMN.



The single transformation that maps ΔABC onto ΔLMN is a 180^{o} clockwise or anti-clockwise rotation about the origin OR a reflection in the origin.

Question 3(b)(iii)

Required to state the 2×2 matrix for the transformation that maps ΔABC onto ΔLMN.

The 2×2 matrix which represents this transformation is $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

Question 4(a)(i)

Required to write an equation connecting P and V.

P varies inversely as the square of V.

$P \propto \frac{1}{V^2}$

$\therefore P = \frac{k}{V^2}$

Question 4(a)(ii)

Required to calculate the value of V when $P = 1$, given that $V = 3$ when $P = 4$.

$4 = \frac{k}{3^2}$

$4 = \frac{k}{9}$

$4 \times 9 = k$

$\therefore k = 36$

If $P = 1$,

Then, we have,

$1 = \frac{36}{V^2}$

$V^2 = 36$

$V = \sqrt{36}$

$V = 6$

Question 4(b)(i)

Required to calculate the value of x for $-7 < 3x + 5 \leq 7$.

Consider $-7 < 3x + 5$

$-7 - 5 < 3x$

$-12 < 3x$

$-4 < x$

Consider $3x + 5 \leq 7$

$3x \leq 7 - 5$

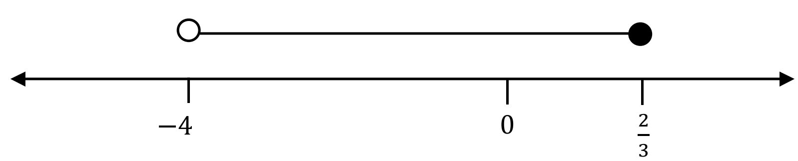
$3x \leq 2$

$x \leq \frac{2}{3}$

Hence, $-4 < x \leq \frac{2}{3}$.

Question 4(b)(ii)

Required to draw the graph of $-4 < x \leq \frac{2}{3}$.



Question 4(c)(i)

Required to determine the coordinates of Q.

$\frac{x}{3} + \frac{y}{7} = 1$

The line crosses the y-axis at $x = 0$.

When $x = 0$,

$\frac{0}{3} + \frac{y}{7} = 1$

$0 + \frac{y}{7} = 1$

$\frac{y}{7} = 1$

$y = 1 \times 7$

$y = 7$

$\therefore$ Coordinates of $Q = (0,7)$

Question 4(c)(ii)

Required to find the gradient of the line $\frac{x}{3} + \frac{y}{7} = 1$.

$\frac{x}{3} + \frac{y}{7} = 1$

Multiplying by the LCM of the denominators gives:

$7x + 3y = 21$

$3y = -7x + 21$

$y = -\frac{7}{3}x + \frac{21}{3}$

$y = -\frac{7}{3}x + 7$

The gradient of the line is given by "m".

Gradient $= -\frac{7}{3}$

Question 5(a)(i)

Required to determine the lower class boundary for the class 21-30.

The lower class boundary of 21-30 is 20.5 litres.

Question 5(a)(ii)

Required to find the class width for the class 21-30.

Class width = Upper Class Boundary – Lower Class Boundary

$= 30.5 - 20.5$

$= 10$ vehicles

Question 5(b)

Required to determine how many vehicles were recorded in the class 31-40.

Number of vehicles recorded $= 101 - 59$

$= 42$ vehicles

Question 5(c)

Required to determine the probability that the volume of petrol needed to fill the tank is more than 50.5 litres.

$$P(need > 50.5\ litres\ to\ fill\ tank) = \frac{Number\ of\ Desired\ Outcomes}{Total\ Number\ of\ Outcomes}$$

$$= \frac{150-129}{150}$$

$$= \frac{21}{150}$$

$$= \frac{7}{50}$$

Question 5(d)

Required to determine why Byron's estimate is incorrect.

Byron estimates the median amount of petrol to be 43.5 litres.

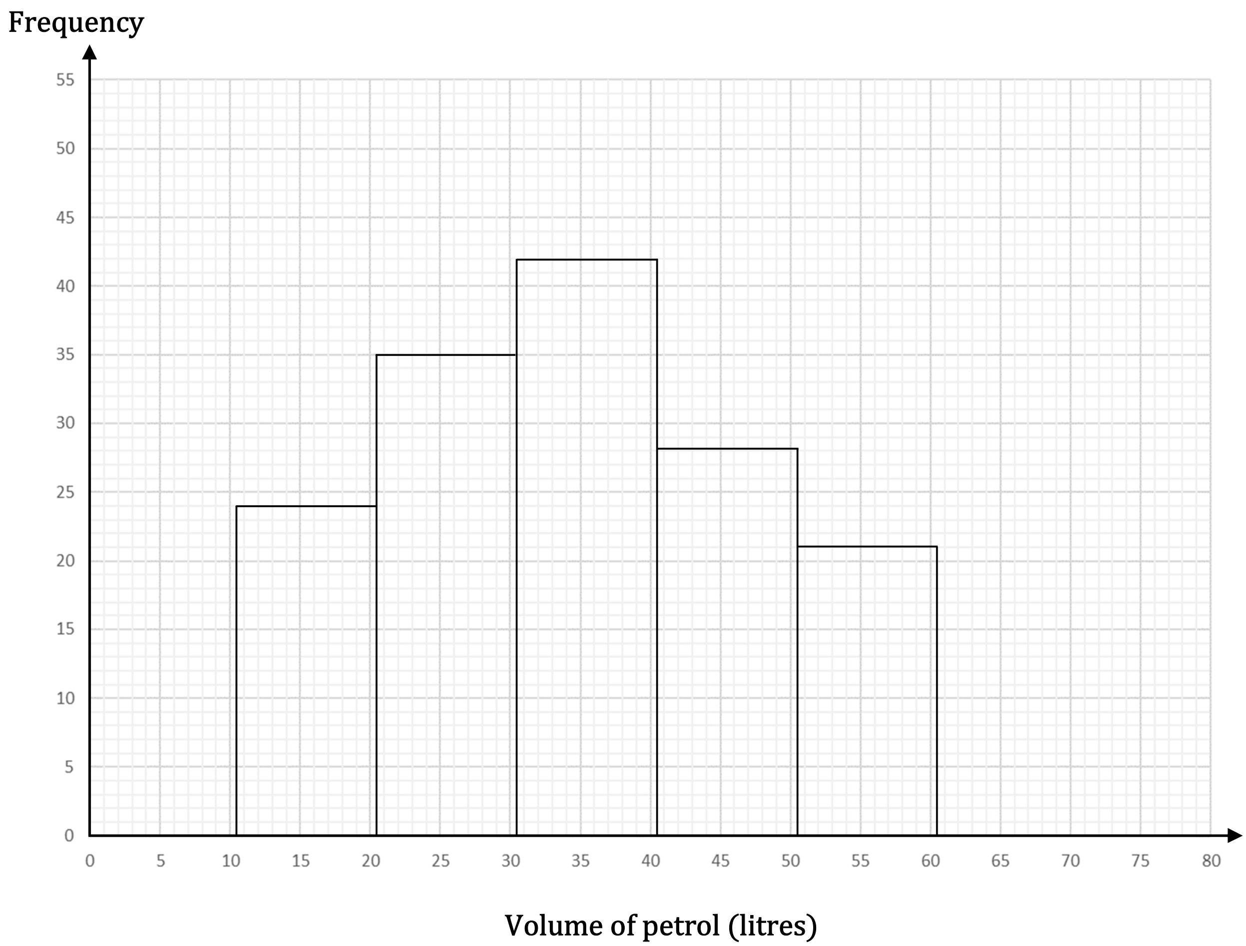
Bryon's estimate is wrong because the median would be given at the 75th vehicle (half the cumulative frequency) which would be found in the interval 31-40.

Question 5(e)

Required to construct a histogram to represent the distribution of the volume of petrol needed to fill the tanks of the 150 vehicles.

Volume (litres)	Lower Class Boundary	Upper Class Boundary	Frequency	Cumulative Frequency
$11 - 20$	10.5	20.5	24	24
$21 - 30$	20.5	30.5	35	59
$31 - 40$	30.5	40.5	42	101
$41 - 50$	40.5	50.5	28	129
$51 - 60$	50.5	60.5	21	150

Title: Graph showing a histogram representing the distribution of the volume of petrol needed to fill the tanks of the 150 vehicles.



Question 6(a)(i)

Scale is $1:25\ 000$.

$\therefore 1\ cm = 25\ 000\ cm$

$0.5\ cm = 0.5 \times 25\ 000$

$= 12\ 500\ cm$

Now,

$100\ 000\ cm = 1\ km$

$1\ cm = \frac{1}{100\ 000}\ km$

$12\ 500\ cm = \frac{1}{100\ 000} \times 12\ 500$

$12\ 500\ cm = 0.125\ km$

Question 6(a)(ii)

An area of $1cm^2$ on the map is represented by a $1\ cm \times 1\ cm$ square on the map. The actual area represented in real like by this would be

$25{,}000 \times\ 25{,}000 = 625{,}000{,}000\ cm^2$

Alternatively, since 25,000cm is one quarter of a km, then we can express this area as such:

$0.25 \times\ 0.25 = 0.0625\ km^2$

An area of $2.25\ cm^2$ is 2.25 times the area of $1\ cm^2$.

$\therefore$ The area in real life will be 2.25 times the $0.0625\ km^2$.

$2.25 \times\ 0.0625 = 0.140625\ km^2$

Question 6(b)(i)

Volume of water in $X = \pi r^2 h$

$$= \pi \times \left(\frac{3d}{2}\right)^2 \times 4$$

$$= \pi \times \frac{9d^2}{4} \times 4$$

$$= 9\pi d^2 \ cm^3$$

Question 6(b)(ii)

Required to calculate the height Y

Volume of water in $Y = \pi \left(\frac{d}{2}\right)^2 h$

$$= \frac{\pi d^2 h}{4}$$

Hence,

$$9\pi d^2 = \frac{\pi d^2 h}{4}$$

$$9 = \frac{h}{4}$$

$$h = 36 \ cm$$

So, the height of the water in Jay Y is 36 cm.

Question 7(a)(i)

Required to calculate the value of T_1

$T_n = 3n^2 - 2$

If $\quad n = 1,$

Then $\quad T_1 = 3(1)^2 - 2$

$= 3 - 2$

$= 1$

Question 7(a)(ii)

$T_n = 3n^2 - 2$

If $\quad n = 3,$

Then $\quad T_3 = 3(3)^2 - 2$

$= 3(9) - 2$

$= 27 - 2$

$= 25$

Question 7(a)(iii)

$T_n = 145$

So, we have

$3n^2 - 2 = 145$

$3n^2 = 145 + 2$

$3n^2 = 147$

$n^2 = \frac{147}{3}$

$n^2 = 49$

$n = 7 \quad$ since n is a positive integer

Question 7(b)(i)

Required to write down the next two terms in the sequence

The first 8 terms of the sequence are 1, 1, 2, 3, 5, 8, 13, 21.

$U(1) = 1$

$U(2) = 1$

$U(3) = 2$

$U(4) = 3$

$U(5) = 5$

$U(6) = 8$

$U(7) = 13$

$U(8) = 21$

Each term is the sum of the two terms that came before it (from the 3rd term onwards).

This is Fibonacci's sequence.

$U(9) = U(8) + U(7)$

$= 21 + 13$

$= 34$

$U(10) = U(9) + U(8)$

$= 34 + 21$

$= 55$

Question 7(b)(ii)

The term in the sequence which is the sum of $U(18)$ and $U(19)$ is $U(20)$.

Any term is the sum of the two terms that came immediately before it.

Question 7(b)(iii)

RTS that $U(20) - U(19) = U(19) - U(17)$.

$U(n) = U(n-1) + U(n-2)$

So,

$U(20) - U(19) = U(18)$ and $U(19) - U(17) = U(18)$

Hence, $U(20) - U(19) = U(19) - U(17)$.

Question 8(a)(i)

Required to state the value of x that cannot be in the domain of f.

This is the value of x when the denominator is equal to zero.

So,

$2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

Question 8(a)(ii)(a)

$f(x) = \frac{9}{2x+1}$ and $g(x) = x - 3$

$fg(x) = f[g(x)]$

$= \frac{9}{2(x-3)+1}$

$= \frac{9}{2x-6+1}$

$= \frac{9}{2x-5}$

Question 8(a)(ii)(b)

$f(x) = \frac{9}{2x+1}$

Let $y = f(x)$.

$y = \frac{9}{2x+1}$

Interchange the variables x and y.

$x = \frac{9}{2y+1}$

Make y the subject.

$x(2y+1)=9$

$2xy+x=9$

$2xy=9-x$

$y=\frac{9-x}{2x}$

$\therefore f^{-1}(x)=\frac{9-x}{2x}$

Question 8(b)(i)

RTS that the area of $ABCD$ is $x^2+2x+4=0$.

Area of rectangle $= l \times b$

Length of DC, $l=4+3x$

Length of AD, $b=2+3x$

Area of $ABCD=(4+3x)(2+3x)$

$=4(2+3x)+3x(2+3x)$

$=8+12x+6x+9x^2$

$=9x^2+18x+8$

Given that area is 44 cm^2, then

$9x^2+18x+8=44$

$9x^2+18x-36=0$

$x^2+2x-4=0$

Question 8(b)(ii)

$x^2 + 2x - 4 = 0$

where $a = 1, b = 2, c = -4$.

Using the quadratic formula to calculate x,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4+16}}{2}$$

$$= \frac{-2 \pm \sqrt{20}}{2}$$

Either $x = \frac{-2-\sqrt{20}}{2}$ or $x = \frac{-2-\sqrt{20}}{2}$

$x = -3.236$ $x = 1.236$

$\therefore x = -3.236$ and $x = 1.236$ (to 3 decimal places)

Question 8(b)(ii)

Perimeter of the unshaded region

$= (3x) + (3x + 2) + (3x + 4) + (3x) + (4) + (2)$

$= 12x + 12$

$= 12(1.23605) + 12$ (since x is positive)

$= 26.8326$

$= 26.833\ cm$ (correct to 3 decimal places)

Question 9(a)(i)

RTF $\angle DBA$

The angle in a semi-circle is right-angle. So, $A\hat{B}C = 90°$.

$A\hat{B}D = A\hat{B}C - D\hat{B}C$

$= 90° - 46°$

$= 44°$

Question 9(a)(ii)

The angles subtended by a chord (DC) at the circumference of a circle ($D\hat{B}C$ and $D\hat{A}C$) and standing on the same arc are equal.

$\therefore D\hat{A}C = 46°$

Question 9(a)(iii)

The angles subtended by a chord (BC) at the circumference of a circle ($B\hat{A}C$ and $B\hat{D}C$) and standing on the same arc are equal.

$\therefore B\hat{D}C = 28°$

Exterior angle of a triangle is equal to the sum of the interior opposite angles.

$B\hat{C}E = 28° + 46°$

$= 74°$

Question 9(b)(i)

Required to find the length of PS

Note that $\angle PQS = \angle RSQ$ because of alternating or Z-angles.

$\sin PQS = \frac{PS}{QS}$

$\sin 30° = \frac{PS}{8}$

$\therefore PS = \sin 30° \times 8$

$= 4\ cm$

Question 9(b)(ii)

Required to find the length of PQ

Using Pythagoras' Theorem,

$QS^2 = PQ^2 + PS^2$

$\therefore PQ^2 = QS^2 - PS^2$

$\therefore PQ^2 = 8^2 - 4^2$

$PQ = \sqrt{64 - 16}$

$PQ = \sqrt{48}$

$PQ = 6.93\ cm$ (to 2 decimal places)

Question 9(b)(iii)

Required to find the area of $PQRS$

Area of a trapezium, $PQRS = \frac{1}{2}(PQ + RS^*) \times PS$

$= \frac{1}{2}(6.93 + 8.54) \times 4$

$= 30.94\ cm$ (to 2 decimal places)

*Using the sine rule to find RS:

$$\frac{RS}{sinRQS} = \frac{QS}{sinQRS^{**}}$$

$$RS = \frac{8}{\sin 68^o} \times \sin 82^o$$

$$RS = 8.54\ cm \qquad \text{(to 2 decimal places)}$$

**To find $\angle QRS$:

$$\angle QRS = 180° - (82° + 30°)$$

$$= 180° - 112°$$

$$= 68°$$

Question 10(a)(i)(a)

$$\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix}\begin{pmatrix} k \\ 5 \end{pmatrix} = \begin{pmatrix} (-1 \times k) + (3 \times 5) \\ (4 \times k) + (h \times 5) \end{pmatrix}$$

$$= \begin{pmatrix} -k + 15 \\ 4k + 5h \end{pmatrix}$$

Question 10(a)(i)(b)

Required to find the values of h and k

$$\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix}\begin{pmatrix} k \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -k + 15 \\ 4k + 5h \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Equating corresponding entries:

$-k + 15 = 0$

$k = 15$

$4k + 5h = 0$

$4(15) + 5h = 0$

$60 + 5h = 0$

$5h = -60$

$h = \frac{-60}{5}$

$h = -12$

$\therefore h = -12$ and $k = 15$

Question 10(a)(ii)

$2x + 3y = 5$

$-5x + y = 13$

Expressing the given equations in a matrix form:

$$\begin{pmatrix} 2 & 3 \\ -5 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 13 \end{pmatrix}$$

$$\text{Inverse} = \frac{1}{(2\times1)-(3\times-5)} \times \begin{pmatrix} 1 & -3 \\ 5 & 2 \end{pmatrix}$$

$$= \frac{1}{17}\begin{pmatrix} 1 & -3 \\ 5 & 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{17}\begin{pmatrix} 1 & -3 \\ 5 & 2 \end{pmatrix} \times \begin{pmatrix} 5 \\ 13 \end{pmatrix}$$

$$= \frac{1}{17}\begin{pmatrix} (1\times5) & + & (-3\times13) \\ (5\times5) & + & (2\times13) \end{pmatrix}$$

$$= \frac{1}{17}\begin{pmatrix} 5 & - & 39 \\ 25 & + & 26 \end{pmatrix}$$

$$= \frac{1}{17}\begin{pmatrix} -34 \\ 51 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{17}\times -34 \\ \frac{1}{17}\times 51 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

So, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

Therefore, $x = -2$ and $y = 3$

Question 10(b)(i)

$\overrightarrow{OA} = \begin{pmatrix} 9 \\ 0 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}, \quad \overrightarrow{AD} = \frac{1}{3}AB, \quad \overrightarrow{OE} = \frac{1}{3}OA$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= \begin{pmatrix} -9 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} -6 \\ 6 \end{pmatrix}$

Question 10(b)(ii)

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$

$= \begin{pmatrix} 9 \\ 0 \end{pmatrix} + \frac{1}{3}\overrightarrow{AB}$

$= \begin{pmatrix} 9 \\ 0 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -6 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 9 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ 2 \end{pmatrix}$

Question 10(b)(iii)

$\overrightarrow{BE} = \overrightarrow{BO} + \overrightarrow{OE}$

$= \begin{pmatrix} -3 \\ -6 \end{pmatrix} + \frac{1}{3}(\overrightarrow{OA})$

$= \begin{pmatrix} -3 \\ -6 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 9 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} -3 \\ -6 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ -6 \end{pmatrix}$