

Solutions to CSEC Maths P2 July 2021

SECTION I

Answer ALL questions.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the EXACT value of [2]

$$1\frac{4}{7} \div \frac{2}{3} - 1\frac{5}{7}$$

With the use of a calculator, $1\frac{4}{7} \div \frac{2}{3} - 1\frac{5}{7} = \frac{9}{14}$

- (b) When Meghan started working, she was paid \$85 each week. After a six-month probationary period, her pay was increased by 20%. How much was she paid each week **after** the increase? [1]

Her pay increased to 120%.

She was paid $\frac{120}{100} \times \$85 = \102

- (c) In 1965, the population of Country A was 2 714 000. In 2015, the population was 3 663 900.

- (i) (a) Write the population in 2015 correct to 3 significant figures. [1]

$3\,663\,900 = 3\,660\,000$ (to 3 significant figures)

- (b) Write the population in 1965 in standard form. [1]

$2\,714\,000 = 2.714 \times 10^6$

(ii) Determine the percentage increase in the population from 1965 to 2015. [2]

$$\begin{aligned}\text{Percentage increase} &= \frac{3\,663\,900 - 2\,714\,000}{2\,714\,000} \times 100 \\ &= 35\%\end{aligned}$$

(d) The ratio of teachers to male students to female students in a school is 3:17:18.

If the TOTAL number of students in the school is 630, determine the number of teachers in the school. [2]

The ratio of teachers : male students : female students is 3:17:18.

630 means 17 + 18 parts

Therefore,

35 parts = 630

$$1 \text{ part} = \frac{630}{35}$$

$$\begin{aligned}\therefore \text{Number of teachers in the school} &= \frac{630}{35} \times 3 \\ &= 54\end{aligned}$$

Total 9 marks

2. (a) Two quantities, n and T , are related as follows:

$$n = \sqrt{T}.$$

(i) Find the value of n when $T = 49$. [1]

$$n = \sqrt{T}$$

When $T = 49$,

$$n = \sqrt{49}$$

$$\therefore n = 7 \text{ or } n = -7$$

(ii) Make T the subject of the formula. [1]

$$n = \sqrt{T}$$

Square both sides,

$$n^2 = (\sqrt{T})^2$$

$$T = n^2$$

(b) Ally is x years. Jim is 5 years older than Ally and Chris is twice as old as Ally.

(i) Write expressions in terms of x for Jim's age and Chris' age. [2]

Jim's age..... $x + 5$

Chris' age..... $2x$

(ii) In **two years' time**, the product of Ally's age and Chris' age will be the same as the square of Jim's **present** age.

Show that the equation $x^2 - 4x - 21 = 0$ represents the information given above. [3]

$$\text{Ally} = x$$

$$\text{Jim} = x + 5$$

$$\text{Chris} = 2x$$

In two year's time,

$$\text{Ally} = x + 2$$

$$\text{Jim} = x + 5 + 2$$

$$\text{Chris} = 2x + 2$$

Therefore,

We have,

$$(x + 2)(2x + 2) = (x + 5)^2$$

$$2x^2 + 2x + 4x + 4 = x^2 + 10x + 25$$

$$2x^2 + 6x + 4 = x^2 + 10x + 25$$

$$2x^2 - x^2 + 6x - 10x + 4 - 25 = 0$$

$$x^2 - 4x - 21 = 0$$

(iii) Calculate Ally's present age. [2]

$$x^2 - 4x - 21 = 0$$

$$x^2 - 7x + 3x - 21 = 0$$

$$x(x - 7) + 3(x - 7) = 0$$

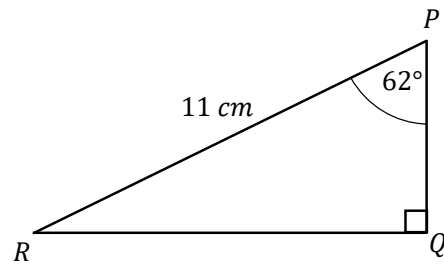
$$(x + 3)(x - 7) = 0$$

$$\begin{array}{l} \text{Either} \quad x + 3 = 0 \quad \text{OR} \quad x - 7 = 0 \\ \quad \quad \quad x = -3 \quad \quad \quad \quad \quad x = 7 \end{array}$$

Since x cannot be negative, Ally's present age is 7 years.

Total 9 marks

3. (a) The diagram below shows the triangle PQR in which angle $QPR = 62^\circ$, angle $PQR = 90^\circ$ and $PR = 11$ cm.



Calculate

- (i) the size of angle PRQ [1]

$$\begin{aligned} PRQ &= 180 - (90 + 62) \\ &= 28 \end{aligned}$$

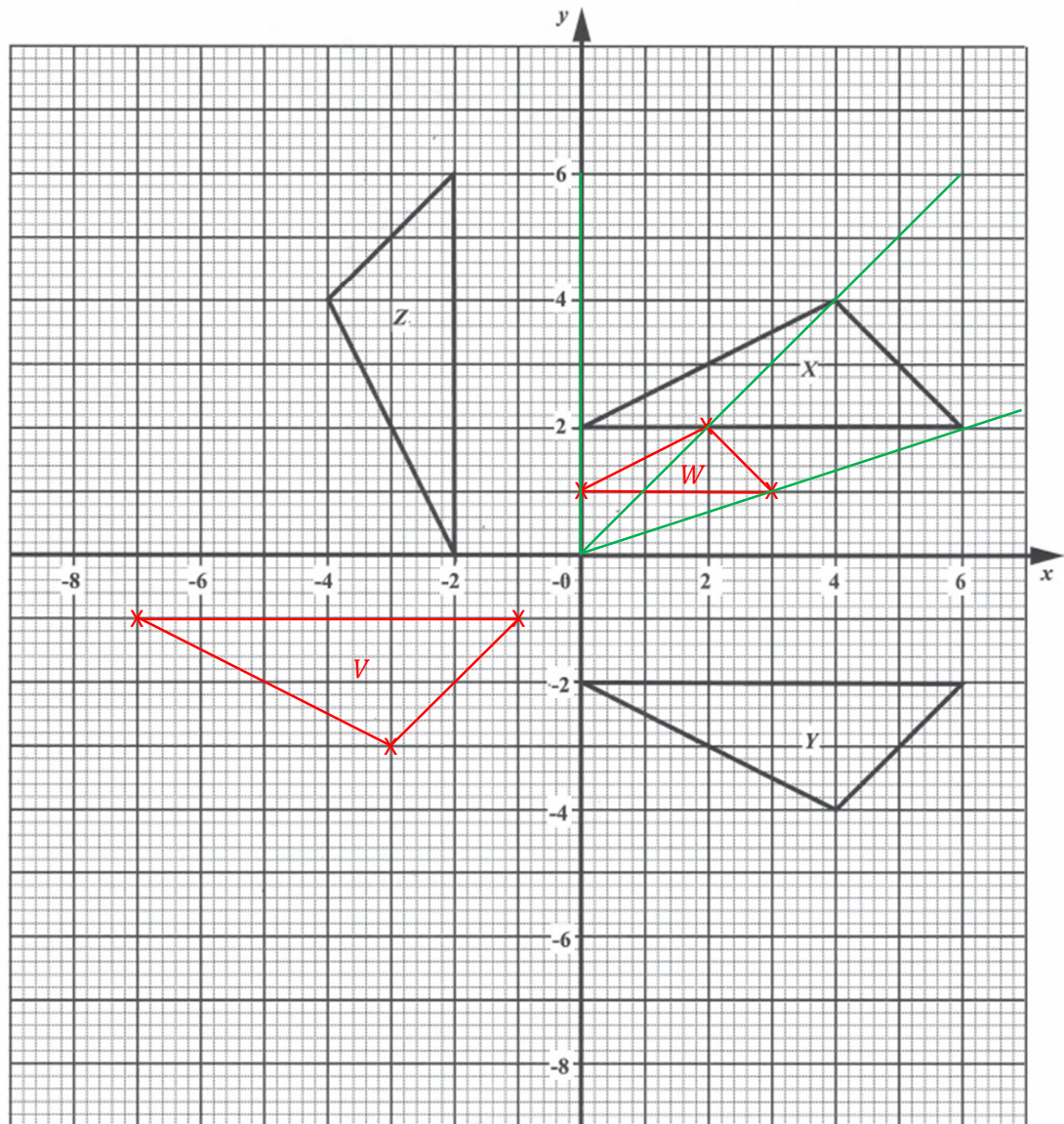
- (ii) the length of the side RQ [2]

$$\sin 62^\circ = \frac{\text{opp}}{\text{hyp}}$$

$$\sin 62 = \frac{RQ}{11}$$

$$\begin{aligned} RQ &= 11 \times \sin 62 \\ &= 9.71 \text{ cm} \end{aligned}$$

(b) The diagram below shows three triangles, X , Y and Z , on a square grid.



- (i) Triangle X is mapped onto Triangle Y by a reflection. State the equation of the mirror line. [1]

The equation of the mirror line is $y = 0$.

- (ii) Describe fully the transformation which maps Triangle X onto Triangle Z . [2]

The transformation which maps Triangle X onto Triangle Z is a rotation about the origin, clockwise 270° .

- (i) On the diagram on page 10, translate Triangle Y using the vector $\begin{pmatrix} -7 \\ 1 \end{pmatrix}$. Label the image V . [1]

The translation vector is $\begin{pmatrix} -7 \\ 1 \end{pmatrix}$.

(See diagram above).

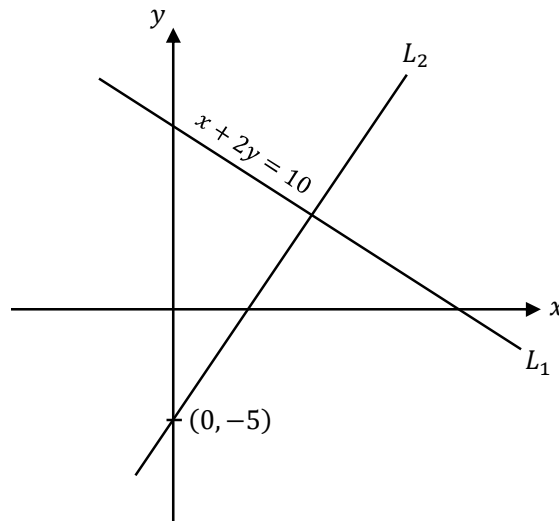
- (ii) On the diagram on page 10, enlarge Triangle X about the centre, $C(0,0)$, and scale factor $\frac{1}{2}$. Label this image W . [2]

Note that the green lines represent guidelines.

(See diagram above).

Total 9 marks

4. (a) The diagram below shows two lines L_1 and L_2 . The equation of the line L_1 is $x + 2y = 10$. The line L_2 passes through the point $(0, -5)$ and is **perpendicular** to L_1 .



- (i) Express the equation of the line L_1 in the form $y = mx + c$. [1]

$$x + 2y = 10$$

$$2y = -x + 10$$

$$y = -\frac{1}{2}x + 5$$

which is in the form $y = mx + c$

where $m = -\frac{1}{2}$ and $c = 5$

- (ii) State the gradient of the line L_1 . [1]

The gradient of the line L_1 is $m = -\frac{1}{2}$.

- (iii) Hence, determine the equation of the line L_2 . [2]

The gradient of L_2 is the negative reciprocal of L_1 .

The gradient of the line L_1 is $-\frac{1}{2}$.

\therefore The gradient of the line L_2 is 2.

The line L_2 passes through the point $(0, -5)$. Therefore, $c = -5$.

Substituting $m = 2$ and $c = -5$ into the equation of a line gives

$$y = mx + c$$

$$y = 2x - 5$$

\therefore The equation of line L_2 is $y = 2x - 5$.

(b) Given that $f(x) = \frac{1}{3}x + 4$ and $g(x) = \frac{3x}{x+1}$,

(i) determine the value of $f(9)$ [1]

$$\begin{aligned} f(9) &= \frac{1}{3}(9) + 4 \\ &= 3 + 4 \\ &= 7 \end{aligned}$$

(ii) calculate the value of $fg(-3)$ [2]

$$\begin{aligned} g(-3) &= \frac{3(-3)}{(-3)+1} \\ &= \frac{-9}{-2} \\ &= \frac{9}{2} \end{aligned}$$

$$\begin{aligned}\therefore f\left(\frac{9}{2}\right) &= \frac{1}{3}\left(\frac{9}{2}\right) + 4 \\ &= \frac{3}{2} + 4 \\ &= 5\frac{1}{2}\end{aligned}$$

(iii) determine the value of x , for which $g(x) = \frac{5}{2}$. [2]

$$\frac{5}{2} = \frac{3x}{x+1}$$

$$5(x+1) = 2(3x)$$

$$5x + 5 = 6x$$

$$x = 5$$

Total 9 marks

5. (a) One hundred students were surveyed on the amount of money they spent on data for their cellphones during a week. The table below shows the results as well as the midpoint for each class interval.

Amount Spent (\$)	Number of Students (f)	Midpoint (\$)
$50 < x \leq 60$	7	55
$60 < x \leq 70$	11	65
$70 < x \leq 80$	31	75
$80 < x \leq 90$	29	85
$90 < x \leq 100$	22	95

Using the table,

- (i) (a) determine the modal class of the amount of money spent [1]

The modal class is $70 < x \leq 80$.

- (b) calculate an estimate of the mean amount of money spent, giving your answer correct to 2 decimal places. [2]

Amount Spent (\$)	Number of Students (f)	Midpoint (\$)	fx
$50 < x \leq 60$	7	55	385
$60 < x \leq 70$	11	65	715
$70 < x \leq 80$	31	75	2325
$80 < x \leq 90$	29	85	2465
$90 < x \leq 100$	22	95	2090
	$\Sigma f = 100$		$\Sigma fx = 7980$

$$\begin{aligned}\text{Mean} &= \frac{\sum fx}{\sum f} \\ &= \frac{7980}{100} \\ &= 79.8\end{aligned}$$

- (ii) Damion reports that the median amount spent is \$84. Briefly explain why Damion's report could be correct. [1]

The middle value falls in the $80 < x \leq 90$ interval.

(There are 50 students before and 50 after.)

The $\left(\frac{100+1}{2}\right)$ th student lies in the $80 < x \leq 90$ interval.

Therefore, Damion's report is correct.

- (b) The two-way/contingency table below gives information on the mode of transportation to school for 100 students.

	Walk	Cycle	Drive	Total
Boy	15	19	14	48
Girl	8	18	26	52
Total	23	37	40	100

- (i) Complete the table by inserting the missing values. [2]

$$\begin{aligned}\text{The number of boys that cycle} &= 48 - (15 + 14) \\ &= 19\end{aligned}$$

$$\begin{aligned}\text{The number of girls that walk} &= 23 - 15 \\ &= 8\end{aligned}$$

$$\begin{aligned}\text{Number of girls} &= 8 + 18 + 26 \\ &= 52\end{aligned}$$

$$\begin{aligned}\text{Number of students that cycle} &= 19 + 18 \\ &= 37\end{aligned}$$

- (ii) A student is selected at random. What is the probability that he/she was being driven to school on that day? [1]

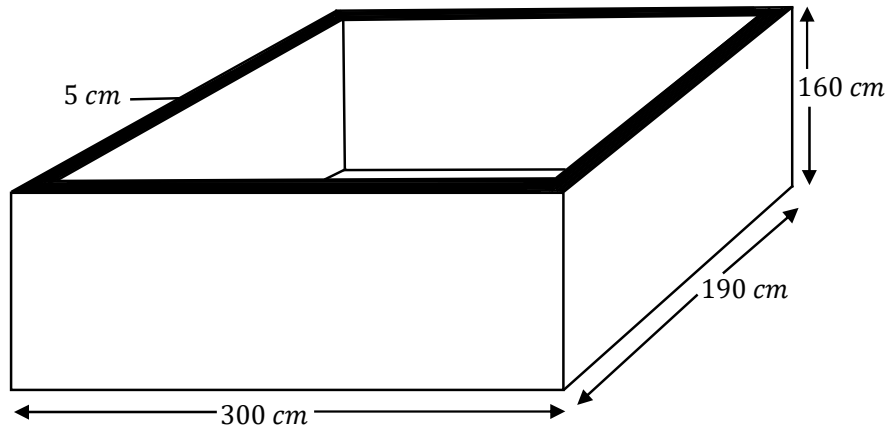
$$\begin{aligned}P(\text{driven}) &= \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}} \\ &= \frac{40}{100} \\ &= \frac{2}{5} \text{ or } 40\%\end{aligned}$$

- (iii) One of the girls is selected at random. What is the probability that she did NOT cycle to school? [2]

$$\begin{aligned}P(\text{did not cycle}) &= \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}} \\ &= \frac{34}{52} \\ &= \frac{17}{26} \text{ or } 65.4\%\end{aligned}$$

Total 9 marks

6. Farmer Brown makes troughs to feed his farm animals, using wood that is 5 cm thick. As shown in the diagram below, the troughs are rectangular-based, open at the top and have external dimensions of 300 cm by 190 cm by 160 cm.



- (a) Show, by calculation, that the internal capacity (volume) of the trough is

$$8\,091\,000\text{ cm}^3.$$

[3]

$$\text{Internal height} = 160 - 5$$

$$= 155$$

$$\text{Internal width} = 190 - 10$$

$$= 180$$

$$\text{Internal length} = 300 - 10$$

$$= 290$$

$$\text{Internal capacity} = 290 \times 180 \times 155$$

$$= 8\,091\,000\text{ cm}^3$$

(b) Calculate the volume of wood needed to make a trough. [3]

$$\begin{aligned}\text{Volume of wood} &= \text{Full volume} - \text{Internal capacity} \\ &= (160 \times 190 \times 300) - 8\,091\,000 \\ &= 1\,029\,000 \text{ cm}^3\end{aligned}$$

(c) Farmer Brown must paint the INTERNAL surface of the trough. Given that 1 gallon of paint covers approximately $280\,000 \text{ cm}^2$ of surface, determine the TOTAL amount of paint, in litres, that is needed to paint the internal surface of the trough. [3]

(1 gallon \approx 3.79 litres)

$$\begin{aligned}\text{Surface Area} &= 2(290 \times 155) + 2(180 \times 155) + (290 \times 180) \\ &= 197\,900 \text{ cm}^2\end{aligned}$$

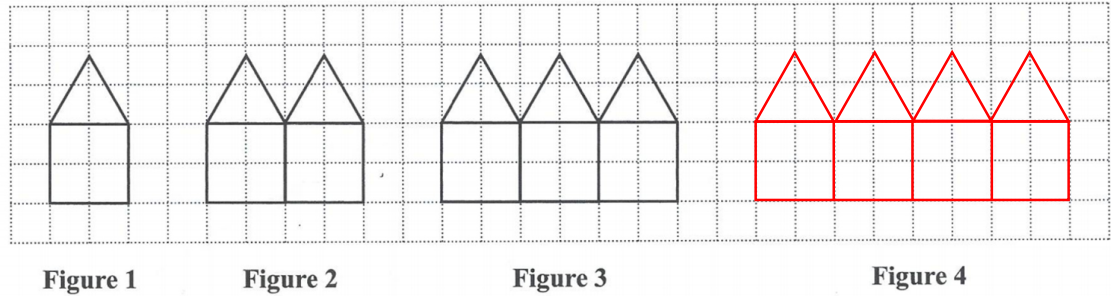
$$280\,000 \text{ cm}^2 = 1 \text{ gallon}$$

$$\begin{aligned}197\,900 \text{ cm}^2 &= \frac{1}{280\,000} \times 197\,900 \\ &= \frac{1979}{2800} \text{ gallons}\end{aligned}$$

$$\begin{aligned}\text{Number of litres} &= \frac{1979}{2800} \times 3.79 \\ &= 2.68 \text{ litres (to 3 s.f.)}\end{aligned}$$

Total 9 marks

7. The first 3 figures in a sequence of shapes, formed by connecting lines of unit length, are shown below.



- (a) Draw Figure 4 of the pattern in the space provided above. [2]
- (b) The number of lines, L , in each shape and the perimeter, P , of the shape follow a pattern. Study the pattern of numbers in each row of the table below and answer the questions that follow.

Complete the table below showing the number of lines and the perimeter of each figure.

	Figure	Number of Lines (L)	Perimeter (P)	
	1	6	5	
	2	11	8	
	3	16	11	
	⋮	⋮	⋮	
(i)	5	<u>26</u>	<u>17</u>	[2]
	⋮	⋮	⋮	
(ii)	<u> </u>	66	<u> </u>	[2]
	⋮	⋮	⋮	
(iii)	n	<u>$5n + 1$</u>	<u>$3n + 2$</u>	[2]

$$(i) \quad L = 5(5) + 1$$

$$= 26$$

$$P = 3(5) + 2$$

$$= 17$$

(ii) To find the Figure,

$$66 = 5n + 1$$

$$5n = 65$$

$$n = \frac{65}{5}$$

$$n = 13$$

To find the Perimeter,

$$66 = 3n + 2$$

$$3n = 64$$

$$n = \frac{64}{3}$$

$$n = 41$$

- (c) Write a simplified expression, in terms of n , for the **difference**, d , between the number of lines and the perimeter of any figure, n . [2]

$$d = (5n + 1) - (3n + 2)$$

$$= 5n + 1 - 3n - 2$$

$$= 2n - 1$$

Total 10 marks

SECTION II

Answer ALL questions.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. Marla buys 2 types of mobile phones, B-Flo and C-Flex, from a company to retail. One B-Flo mobile phone costs \$60 while one C-Flex costs \$80. She buys x number of B-Flo phones and y number of C-Flex phones.

(a) (i) Marla must not spend more than \$1 200. Write an inequality to represent this information. [1]

Inequality: $60x + 80y \leq 1200$

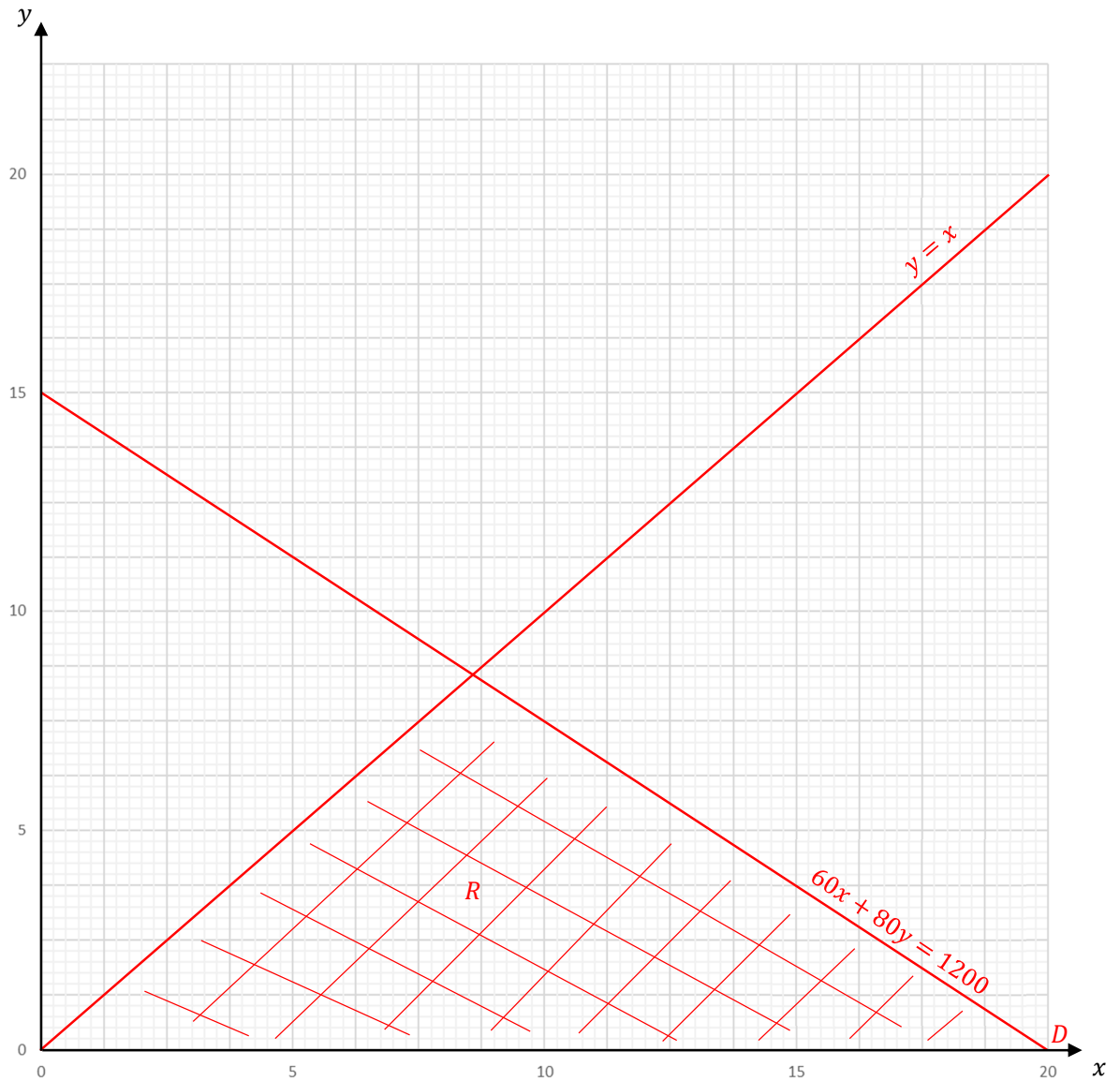
(ii) The number of B-Flo phones must be greater than or equal to the number of C-Flex phones. Write down an inequality in x and y to show this information. [1]

Inequality: $x \geq y$

(iii) Represent the two inequalities on page 22 on the grid shown below.

Label as R the region which satisfies both inequalities.

[4]



Note: The scaling of the graph on the question paper contained an error.

The inequalities are:

$$60x + 80y \leq 1200$$

$$x \geq y$$

Rewriting the inequalities as equations:

$$60x + 80y \leq 1200$$

$$60x + 80y = 1200$$

$$3x + 4y = 60$$

and

$$x \geq y$$

$$y = x$$

- (iv) The total number of mobile phones is represented by $x + y$. According to the graph on page 23, what is the largest possible value of $x + y$? [1]

The highest number of $x + y$ occur at point D where $x = 20$ and $y = 0$.

\therefore highest number = 20

(b) The table below shows pairs of values for the function $y = x^2 + x - 4$.

x	-4	-3	-2	-1	0	1	2	3
y	8	2	-2	-4	-4	-2	2	8

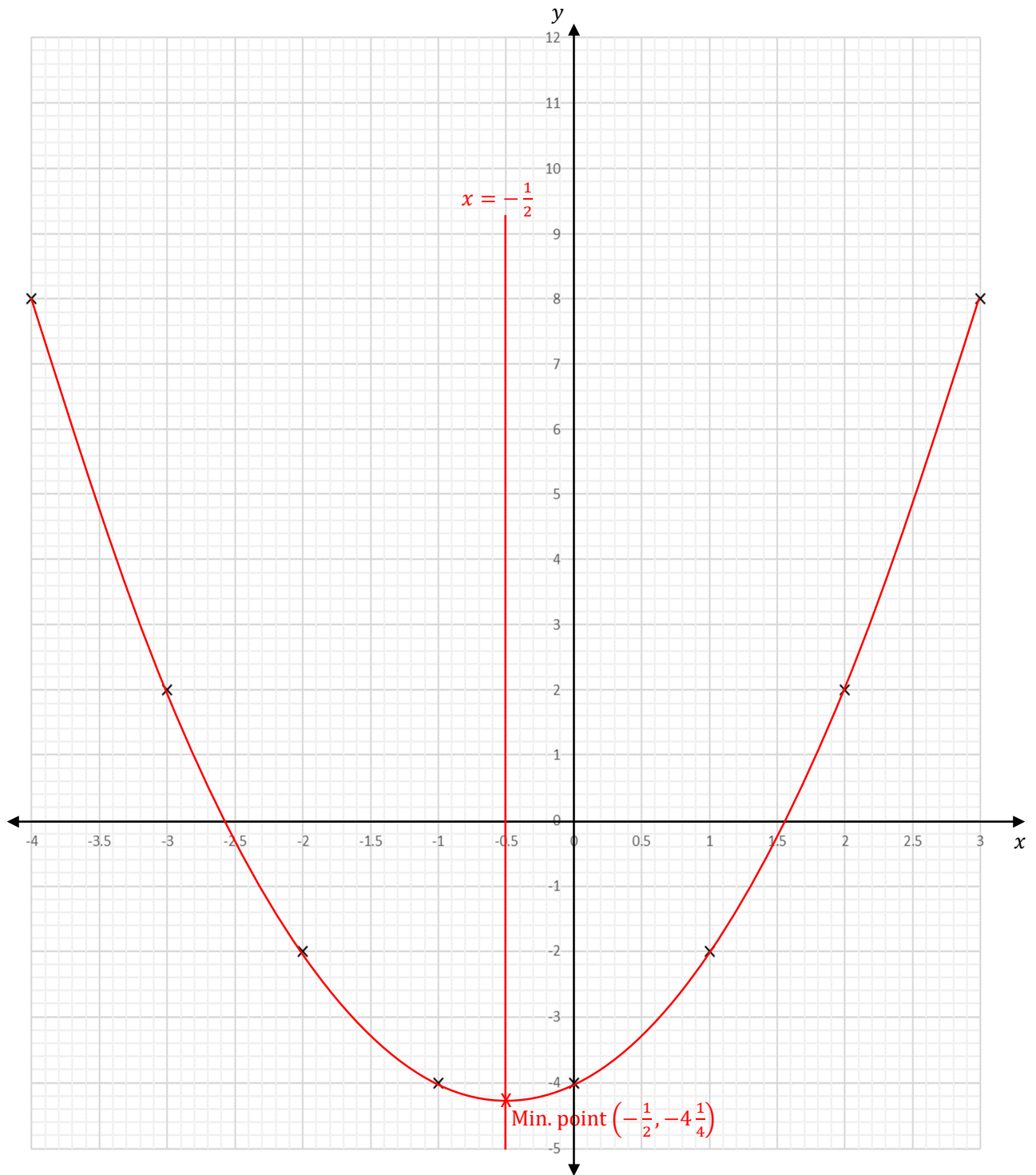
- (i) On the grid provided **on page 25**, plot the remaining 4 points and draw the graph of the function $y = x^2 + x - 4$ for $-4 \leq x \leq 3$. [3]

(See graph below)

- (ii) Write down the maximum or minimum value of the function. [1]

The minimum point is $\left(-\frac{1}{2}, -4\frac{1}{4}\right)$.

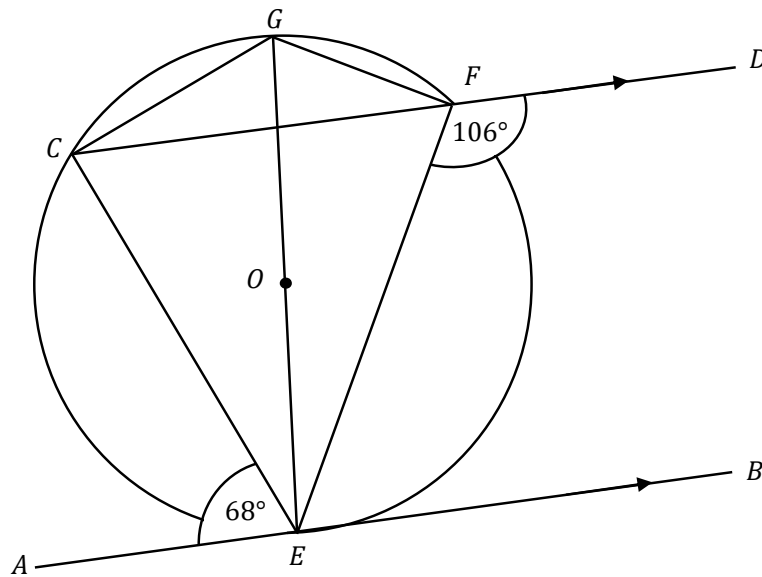
- (iii) Using a ruler, draw the axis of symmetry on the graph **on page 25**. [1]



Total 12 marks

GEOMETRY AND TRIGONOMETRY

9. (a) In the diagram below, E, C, G and F are points on the circumference of a circle. EG is a diameter of the circle. The tangent AEB is parallel to CD . Angle $AEC = 68^\circ$ and angle $EFD = 106^\circ$.



Determine the value of EACH of the following angles. Show detailed working where necessary and give a reason to support your answer.

- (i) ECD [2]

$\hat{AEC} = \hat{ECD}$ since they are corresponding angles.

Therefore, $\hat{ECD} = 68^\circ$.

(ii) CEG [2]

The line AB is a tangent to the circle.

The tangent meets radius at 90° .

$$\begin{aligned} \widehat{CEG} &= 90 - 68 \\ &= 22^\circ \end{aligned}$$

(iii) CGF [2]

$\widehat{AEF} = \widehat{EFD}$ since they are corresponding angles.

Therefore, $\widehat{AEF} = 106^\circ$.

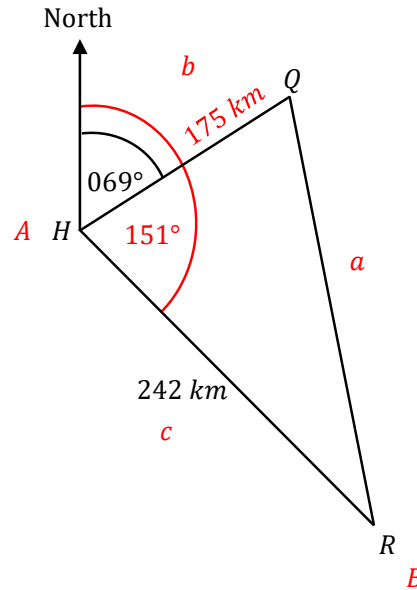
The line AB is a tangent to the circle. The tangent meets radius at 90° .

Therefore, $\widehat{AEG} = 90^\circ$.

$$\begin{aligned} \text{Now, } \widehat{GEF} &= 106 - 90 \\ &= 16^\circ \end{aligned}$$

$$\begin{aligned} \text{Hence, } \widehat{CGF} &= 180 - (22 + 16) \\ &= 180 - 38 \\ &= 142^\circ \end{aligned}$$

(b) From a harbour, H , the bearing of two ships, Q and R , are 069° and 151° respectively. Q is 175 km from H while R is 242 km from H .



(i) Complete the diagram above to show the information given. [1]

(See diagram above)

(ii) Calculate QR , the distance between the two ships, to the nearest km. [3]

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$(QR)^2 = 175^2 + 242^2 - 2(175)(242) \cos 82^\circ$$

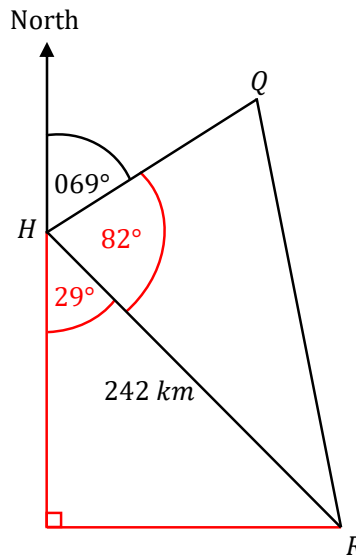
$$(QR)^2 = 77401.03835$$

$$QR = \sqrt{77401.03835}$$

$$QR = 278.2 \text{ km}$$

$$QR = 278 \text{ km (to the nearest km)}$$

- (iii) Calculate how far due south is Ship R of the harbour, H . [2]



Now, $180 - (82 + 69) = 29^\circ$ (as shown on the diagram above)

So,

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos(29) = \frac{\text{adj}}{242}$$

$$\begin{aligned} \text{adj} &= 242 \cos 29 \\ &= 211.7 \text{ km} \end{aligned}$$

Hence, Ship R is 211.7 km due south of the harbour, H .

Total 12 marks

VECTORS AND MATRICES

10. (a) (i) Calculate the matrix product $\begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix} \begin{pmatrix} 2 & 1 & -4 \\ 0 & 3 & 6 \end{pmatrix}$. [2]

$$\begin{aligned} \begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix} \begin{pmatrix} 2 & 1 & -4 \\ 0 & 3 & 6 \end{pmatrix} &= \begin{pmatrix} 10+0 & 5+12 & -20+24 \\ -6+0 & -3-6 & 12-12 \end{pmatrix} \\ &= \begin{pmatrix} 10 & 17 & 4 \\ -6 & -9 & 0 \end{pmatrix} \end{aligned}$$

(ii) State why the two matrices in (a)(i) are conformable for multiplication. [1]

The two matrices are conformable for multiplication because the number of columns in the first matrix is equal to the number of rows in the second matrix.

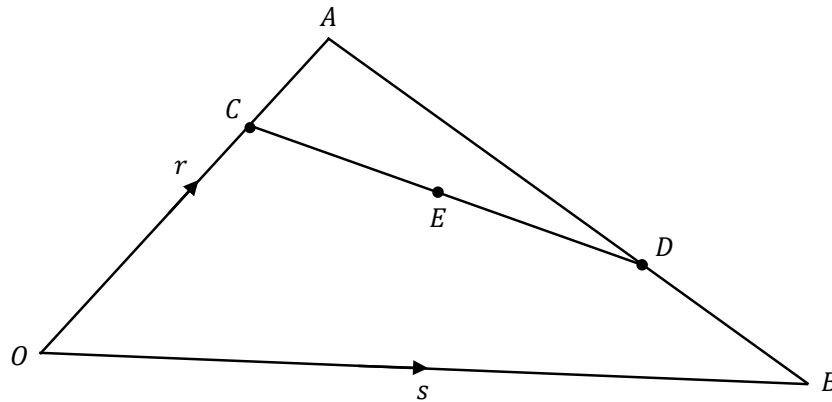
(b) Determine the inverse of $\begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}$. [2]

Let $A = \begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}$ which is in the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$$\begin{aligned} \det(A) &= ad - bc & \text{adj}(A) &= \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \\ &= -10 - (-12) & &= \begin{pmatrix} -2 & -4 \\ 3 & 5 \end{pmatrix} \\ &= 2 & & \end{aligned}$$

$$\begin{aligned}\therefore A^{-1} &= \frac{1}{\det} \times \text{adj} \\ &= \frac{1}{2} \begin{pmatrix} -2 & -4 \\ 3 & 5 \end{pmatrix} \\ &= \begin{pmatrix} -1 & -2 \\ -\frac{3}{2} & \frac{5}{2} \end{pmatrix}\end{aligned}$$

(c) The diagram below shows triangle OAB in which $\overrightarrow{OA} = r$ and $\overrightarrow{OB} = s$. In addition, E is the midpoint of CD , $OC = \frac{3}{4}OA$ and $AD = \frac{2}{3}AB$.



Write in terms of r and s , in the simplest form, an expression for

(i) \overrightarrow{CD} [2]

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB} \\ &= -r + s\end{aligned}$$

$$\begin{aligned}\overrightarrow{AD} &= \frac{2}{3}\overrightarrow{AB} \\ &= \frac{2}{3}(-r + s)\end{aligned}$$

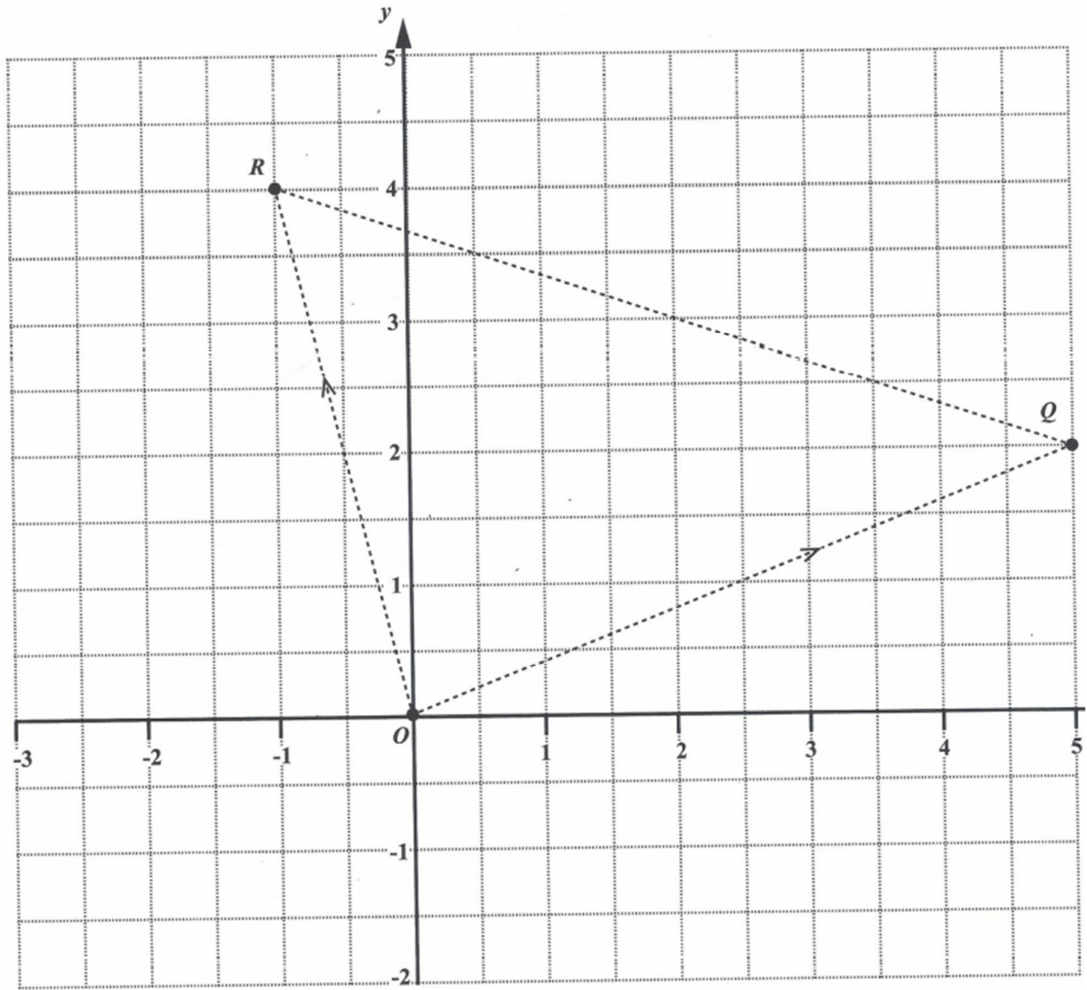
$$\begin{aligned}\overrightarrow{CD} &= \overrightarrow{CA} + \overrightarrow{AD} \\ &= \frac{1}{4}r + \frac{2}{3}(-r + s) \\ &= \frac{1}{4}r - \frac{2}{3}r + \frac{2}{3}s \\ &= -\frac{5}{12}r + \frac{2}{3}s\end{aligned}$$

(ii) \overrightarrow{OE} [2]

$$\begin{aligned}\overrightarrow{CE} &= \frac{1}{2}\overrightarrow{CD} \\ &= \frac{1}{2}\left(-\frac{5}{12}r + \frac{2}{3}s\right)\end{aligned}$$

$$\begin{aligned}\overrightarrow{OE} &= \overrightarrow{OC} + \overrightarrow{CE} \\ &= \frac{3}{4}r + \frac{1}{2}\left(-\frac{5}{12}r + \frac{2}{3}s\right) \\ &= \frac{3}{4}r - \frac{5}{24}r + \frac{2}{6}s \\ &= \frac{13}{24}r + \frac{1}{3}s\end{aligned}$$

(d) The points O , Q and R have coordinates $(0, 0)$, $(5, 2)$ and $(-1, 4)$ respectively.



(i) Write \overrightarrow{OR} as a column vector. [1]

The coordinate of R is $(-1, 4)$.

So, $\overrightarrow{OR} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ as a column vector.

(ii) Determine $|\overrightarrow{QR}|$. [2]

$$\begin{aligned}\overrightarrow{QR} &= \overrightarrow{OR} - \overrightarrow{OQ} \\ &= \begin{pmatrix} -1 \\ 4 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} -6 \\ 2 \end{pmatrix}\end{aligned}$$

$$\begin{aligned}|\overrightarrow{QR}| &= \sqrt{(-6)^2 + (2)^2} \\ &= \sqrt{40} \\ &= 2\sqrt{10}\end{aligned}$$

Total 12 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.