Solutions to CSEC Maths P2 January 2011

Question 1a part (i)

Calculate the exact value of

$$(5.8^2 + 1.02) \times 2.5$$

$$= 34.66 \times 2.5$$

$$= \mathit{86.65\ to\ 2\ decimal\ places}$$

Question 1a part (ii)

Calculate the exact value of

$$\frac{\mathbf{2\frac{4}{9}}}{\mathbf{4\frac{2}{3}}} - \frac{\mathbf{3}}{\mathbf{7}}$$

$$= \frac{\frac{22}{9}}{\frac{14}{3}} - \frac{3}{7}$$

$$= \left(\frac{22}{9} \times \frac{3}{14}\right) - \frac{3}{7}$$

$$= \left(\frac{66}{126}\right) - \frac{3}{7}$$

$$= \frac{11}{21} - \frac{3}{7}$$

$$= \frac{11 - 3(3)}{21}$$

$$= \frac{\mathbf{2}}{\mathbf{21}}$$

Question 1b part (i)

Data Given

Employee is paid a basic wage of $9.50 per hour for a 40 hour week and one and a half times this rate for overtime.

Calculate the basic weekly wage of one employee for a 40-hour work week

$$Basic\ weekly\ wage\ = hourly\ rate\ \times$$
$$number\ of\ hours\ in\ a\ basic\ work\ week$$

$$= \$9.50\ \times\ 40$$

$$= \$380\ per\ week$$

Question 1b part (ii)

Calculate the overtime wage for an employee who worked 6 hours overtime

$$Overtime\ Rate = 1.5\ \times \$9.50$$

$$= \$14.25\ per\ overtime\ hour$$

$$Overtime\ wage\ for\ 6\ hours\ = \$14.25\ \times 6$$

$$= \$85.50$$

Question 1b part (iii)

Data Given

A total of $12 084.00 in basic and overtime wages were paid to 30 employees by the company in a certain week.

Calculate the total paid in overtime wages

$$Basic\ wage\ of\ 30\ employees\ for\ a\ 40\ hour\ work\ week$$

$$= Number\ of\ employees\ \times basic\ weekly\ wage$$

$$= 30\ \times \$380$$

$$= \$11,400$$

$$Total\ wages\ paid = \$12{,}084 \qquad (Given\ Data)$$

$$Total\ paid\ in\ overtime\ wages$$

$$= Total\ wages\ paid - \ Basic\ wage\ of\ 30\ employees\ for\ a\ 40\ hour\ work\ week$$

$$= \$12{,}084 - \$11,400$$

$$= \$12{,}084 - \$11,400$$

$$= \$684.00$$

Question 1b part (iv)

Calculate the total number of overtime hours worked by employees

$$Total\ number\ of\ overtime\ hours\ worked\ by\ employees = \frac{Total\ paid\ in\ overtime\ wages}{Overtime\ Rate}$$

$$= \frac{\$684.00}{\$14.25\ per\ hour}$$

$$= \quad 48\ hours$$

Question 2a

Simplify $\frac{2x}{5} - \frac{x}{3}$

$$\frac{2x}{5} - \frac{x}{3}$$

$$= \frac{3(2x) - 5(x)}{15}$$

$$= \frac{6x - 5x}{15}$$

$$= \frac{x}{15}$$

Question 2b

Factorise $a^2b + 2ab$

$$a^2b + 2ab$$

$$= ab(a + 2)$$

Question 2c

Express p as the subject of the formula $q = \frac{p^2 - r}{t}$

$$\frac{q}{1} = \frac{p^2 - r}{t}$$

$$qt = (p^2 - r)\,(1)$$

$$p^2 = qt + r$$

$$p = \sqrt{qt + r}$$

Question 2d part (i)

Data Given

Type of Box	Number of Donuts per Box
Small box	x
Large Box	2x + 3

In total 8 small boxes and 5 large boxes were sold.

Write an expression in terms of x to represent the Total number of donuts sold.

$$Total\ number\ of\ donuts\ sold = Number\ of\ small\ boxes\ sold + Number\ of\ big\ boxes\ sold$$

$$= 8(x) + 5(2x + 3)$$

$$= 8x + 10x + 15$$

$$= 18x + 15$$

Question 2d part (ii)

Data Given

The total number of donuts sold was 195.

(a) Calculate the number of donuts in a small box

$$\text{Total number of donuts sold} = 18x + 15$$

$$195 = 18x + 15$$

$$18x = 195 - 15$$

$$18x = 180$$

$$x = 10$$

$\text{Based on given data}$: $\text{Number of donuts in small box} = x$

$$\therefore \text{Number of donuts in small box} = 10$$

(b) Calculate the number of donuts in a large box

$\text{Based on given data}$: $\text{Number of donuts in large box} = 2x + 3$

$$\therefore \text{Number of donuts in large box} = 2(10) + 3$$

$$= 23$$

Question 3a

Simplify $7\,p^5q^3 \times 2\,p^2q$

$$= 7 \times 2 \times p^{5+2} \times q^{3+1}$$

$$= 14\,p^7q^4$$

Question 3b part (i)

Data Given

One carton of milk measures 6 cm by 4 cm by 10 cm.

Calculate, in cm^3, the volume of milk in a carton

$$Volume\ of\ milk\ in\ a\ carton = L \times W \times H$$

$$= 6 \times 4 \times 10$$

$$= 240\ cm^3$$

Question 3b part (ii)

Data Given

For a recipe to make ice-cream 3 litres of milk are required.

Calculate the number of cartons of milk that should be bought to make the ice-cream

$$1\ litre = 1000\ cm^3$$

$$3\ litres\ of\ milk = 3\ \times 1000$$

$$= 3000\ cm^3\ of\ milk$$

$$Number\ of\ cartons\ of\ milk\ required = \frac{Volume\ of\ milk\ required}{Volume\ of\ a\ carton}$$

$$= \frac{3000\ cm^3}{240\ cm^3}$$

$$= 12.5$$

Therefore 13 cartons of milk should be purchased make the ice − cream according to its recipe.

Question 3b part (iii)

Data Given

One carton of milk is poured into a cylindrical cup of internal diameter 5 cm.

$\pi = 3.14$

Calculate the height of milk in the cup

$$Volume\ of\ milk\ in\ one\ carton = Volume\ of\ milk\ in\ the\ cup$$

$$Volume\ of\ milk\ in\ the\ cup = \pi r^2 h$$

Where $r = radius\ of\ the\ cup$

$h = height\ of\ the\ milk\ in\ the\ cup$

$$r = \frac{internal\ diameter\ of\ the\ cup}{2}$$

$$= \frac{5\ cm}{2}$$

$$= 2.5\ cm$$

$$Volume\ of\ milk\ in\ the\ cup = \pi r^2 h$$

$$240\ cm^3 = (3.14) \times (2.5\ cm)^2 \times h$$

$$h = \frac{240}{(3.14) \times (2.5\ )^2}$$

$$h = 12.23\ cm$$

$$\boldsymbol{h = 12.2\ cm\ to\ 3\ significant\ figures}$$

Question 4a part (i)

Data Given

$U = \{Whole\ numbers\ from\ 1\ to\ 12\}$

$H = \{Odd\ numbers\ between\ 4\ and\ 12\}$

$J = \{Prime\ Numbers\ from\ 1\ to\ 12\}$

List the members of H

$$H = \{Odd\ numbers\ between\ 4\ and\ 12\}$$

$$= \{5, 7, 9, 11\}$$

Question 4a part (ii)

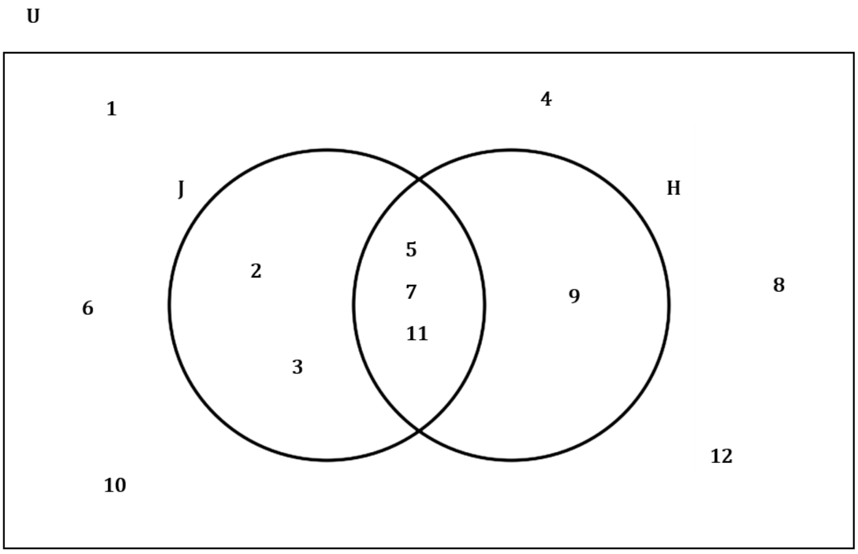
List the members of J

$$J = \{\ Prime\ Numbers\ from\ 1\ to\ 12\}$$

$$= \{2, 3, 5, 7, 11\}$$

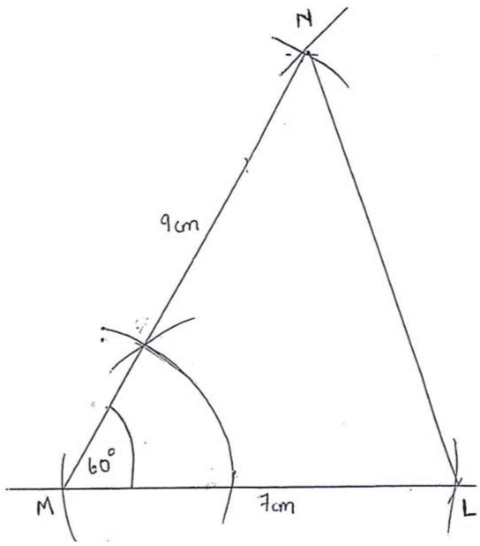
Question 4a part (iii)

Draw a Venn diagram to represent U, H and J



Question 4b part (i)

Construct a triangle LMN with angle $LMN = 60°$, $MN = 9\ cm\ and\ LM = 7\ cm$



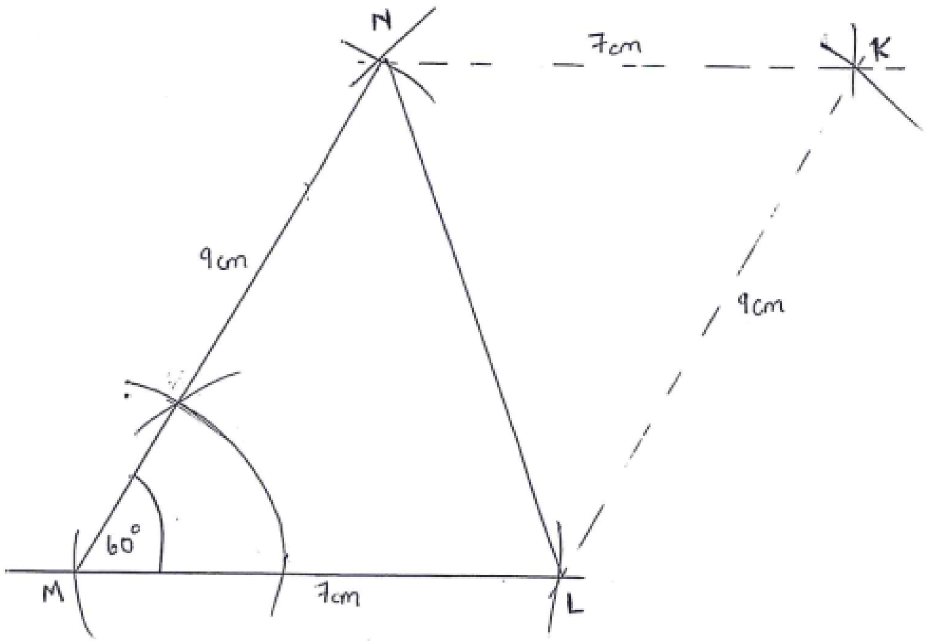
Question 4b part (ii)

Measure and state the size of $M\hat{N}L$

Using a protractor $M\hat{N}L$ was found to be 48°.

Question 4b part (iii)

Show the point, K, such that KLMN is a parallelogram



Since the opposite sides of a parallelogram are equivalent in length;

$ML = NK = 7\ cm$

$MN = LK = 9\ cm$

Question 5a (i)

Data Given

Equation of a straight line is $3y = 2x - 6$

Determine the gradient of the line

$$3y = 2x - 6$$

In the form $y = mx + c$, *where the gradient of the line is given by* "m".

$$y = \frac{2}{3}x - 2$$

$$Gradient\ (m_1) = \frac{2}{3}$$

Question 5a (ii)

Determine the equation of the line perpendicular to $3y = 2x - 6$ that passes through the point $(4, 7)$

$$Gradient\ of\ Line = m_2$$

$$m_1 \times m_2 = -1\ ; \qquad since\ the\ lines\ are\ perpendicular$$

$$m_2 = \frac{-1}{m_1}$$

$$= \frac{-1}{\frac{2}{3}}$$

$$= -\frac{3}{2}$$

$$Equation\ of\ Line:\ (y - y_1) = m\ (x - x_1)$$

$$(y - 7) = -\frac{3}{2}\ (x - 4)$$

$$(y-7) = -\frac{3}{2}x + \frac{12}{2}$$

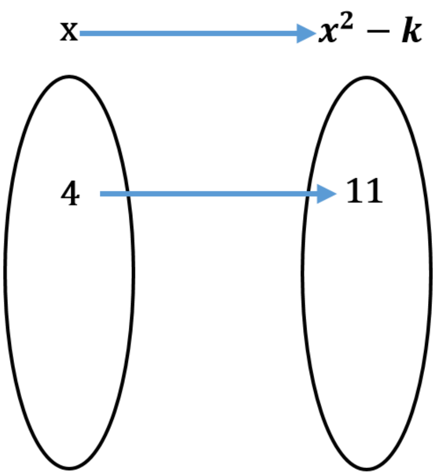
$$2y - 14 = -3x + 12$$

$$2y = -3x + 26$$

Question 5b part (i)

Data Given: Diagram showing $f: x \rightarrow x^2 - k$ where $x \in \{3, 4, 5, 6, 7, 8, 9, 10\}$

Calculate the value of k



Using the data given where 4 is mapped onto 11.

$$x^2 - k = 11$$

$$k = x^2 - 11$$

$$k = (4)^2 - 11$$

$$k = 16 - 11$$

$$k = 5$$

Question 5b part (ii)

Calculate the value of $f(3)$

$$f(x) = x^2 - k$$

Substituting $x = 3$ *and* $k = 5$ *gives*

$$f(3) = (3)^2 - 5$$

$$f(3) = 9 - 5$$

$$= 4$$

Question 5b part (iii)

Calculate the value of x when $f(x) = 95$

$$f(x) = x^2 - k$$

$$x^2 - k = 95$$

$$x^2 = 95 + k$$

$$x^2 = 95 + 5$$

$$x^2 = 100$$

$$x = \sqrt{100}$$

$$x = \pm 10$$

Based on given data the value of x would be 10 when f(x) = 95.

Question 6 part (i)

Data Given: Line graph showing the monthly sales, in thousands of dollars, at a school cafeteria for the period January to May 2010.

Copy and complete the table

Month	**Jan**	**Feb**	**Mar**	**Apr**	**May**
Sales in $Thousands	38	35	27	15	10

Question 6 (ii)

Find the two months in between which there was the greatest decrease in sales

$$Decrease\ between\ Jan\ and\ Feb = \$38{,}000 - \$35{,}000$$

$$= \$3{,}000$$

$$Decrease\ between\ Feb\ and\ Mar = \$35{,}000 - \$27{,}000$$

$$= \$8{,}000$$

$$Decrease\ between\ Mar\ and\ Apr = \$27{,}000 - \$15{,}000$$

$$= \$12{,}000$$

$$Decrease\ between\ Apr\ and\ May = \$15{,}000 - \$10{,}000$$

$$= \$5{,}000$$

$$\therefore The\ largest\ decrease\ occured\ between\ the\ months\ of\ March\ and\ April.$$

Question 6 part (iii)

Calculate the mean monthly sales for the period January to May 2011

$$Total\ monthly\ sales\ from\ \text{Jan to May} = \$\ (38{,}000 + 35{,}000 + 27{,}000 + 15{,}000 + 10{,}000)$$

$$= \$125{,}000$$

$$Mean\ monthly\ sales\ from\ \text{Jan to May} = \frac{Total\ monthly\ sales\ from\ \text{Jan to May}}{Number\ of\ months}$$

$$= \frac{\$125{,}000}{5}$$

$$= \$25{,}000$$

Question 6 part(iv)

Data Given: Total sales for the period January to June was $150,000.

Calculate the sales for the month of June

$$Sales\ for\ the\ month\ of\ June = Total\ sales\ for\ Jan\ to\ June - Total\ sales\ from\ Jan\ to\ May$$

$$= \$\ (150{,}000 - 125{,}000)$$

$$= \$25{,}000$$

Question 6 part(v)

Comment on the sales in June compared to the sales for the months January to May

The sales showed an increase when compared to the two previous months; April and May.

The sales were equivalent to the mean monthly sales for the period January to May 2010.

The month of June sales was the only month that displayed an increase since January.

Question 7 (i)

Data Given: Diagram showing the triangle RST and its transformation R'S'T' after undergoing a transformation.

Write the coordinates of R and R'

$$Coordinates\ of\ R\ = (2, 4)$$
$$Coordinates\ of\ R' = (2, 0)$$

Question 7 (ii)

Describe the transformation which maps triangle RST onto triangle R'S'T'

If the line y = 2 is drawn, it can be seen the vertices of RST and R'S'T' are located the same distance away from the line on opposite sides.

RST and R'S'T' are also identical and R'S'T' is a lateral inversion of RST.

Therefore the triangle RST was reflected along the line y = 2 to give the triangle R'S'T'.

Question 7 (iii) part (a)

Data Given: RST undergoes an enlargement, centre (0, 4), scale factor, 3.

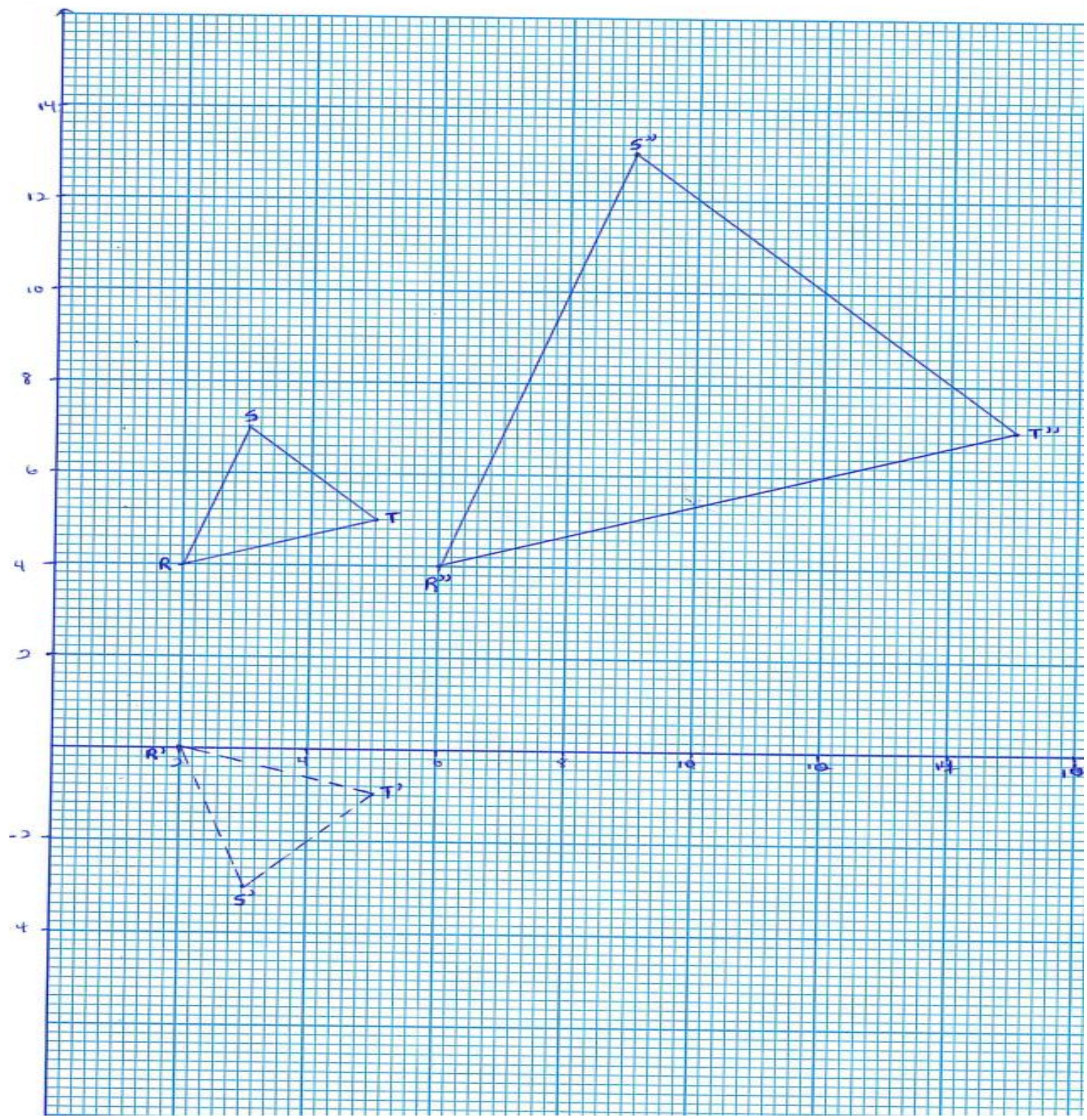
Draw triangle R''S''T'', the image of triangle RST under the enlargement

For the enlargement; centre C = (0, 4)

$$Coordinates\ of\ R'' = 3 \times CR$$
$$= [(3 \times 2), (3 \times 0 + 4)]$$
$$= (6, 4)$$

$$Coordinates\ of\ S'' = 3 \times CS$$
$$= [(3 \times 3), (3(7 - 4) + 4)]$$
$$= (9, 13)$$

$$\begin{aligned} Coordinates\ of\ T'' &= 3 \times CT \\ &= [(3 \times 5), (3(1) + 4)] \\ &= (15, 7) \end{aligned}$$



Question 7 (iii) part b

Data Given: Area of triangle RST = 4 square units

Calculate the area of triangle R''S''T''

$$Area\ of\ triangle\ R''S''T'' = (scale\ factor\ )^2 \times area\ of\ triangle\ RST$$

$$= (3\ )^2 \times 4$$

$$= 36\ square\ units$$

Question 7 (iii) part c

State two geometrical relationships between triangles RST and R''S''T''

Triangles RST and R''S''T'' similar where RST is the object and R''S''T'' is an enlargement of the object.

Regarding the sides of RST and R''S''T'':

$$\frac{R''S''}{RS} = \frac{S''T''}{ST} = \frac{R''T''}{RT} = 3 \quad \text{which is the scale factor}$$

Question 8a part (i)

Data Given: An answer sheet showing a rectangle, A, of area 20 square units and perimeter 24 units.

Draw and label

(a) rectangle B of area 27 square units and perimeter 24 units

$Area\ of\ rectangle = l \times w = 27\ square\ units$

$Perimeter\ of\ rectangle = 2l + 2w = 24\ units$

$l \times w = 27$ ……………… Equation 1

$2l + 2w = 24$ ………….. Equation 2

From Equation 2

$l = 12 - w$………………Equation 3

Substituting Equation 3 into Equation 1

$(12 - w)w = 27$

$w^2 - 12w + 27 = 0$

$(w - 9)(w - 3) = 0$

$\therefore w = 9\ or\ w = 3$

Taking w=3;

$l = 12 - w$

$l = 12 - 3 = 9$

Therefore for Rectangle B; length = 9 units and width = 3 units

(b) rectangle C of area 32 square units and perimeter 24 units

$Area\ of\ rectangle = l \times w = 32\ square\ units$

$Perimeter\ of\ rectangle = 2l + 2w = 24\ units$

$l \times w = 32$ ……………….. Equation 1

$2l + 2w = 24$ ………….. Equation 2

From Equation 2

$l = 12 - w$………………Equation 3

Substituting Equation 3 into Equation 1

$(12 - w)w = 32$

$w^2 - 12w + 32 = 0$

$(w - 8)(w - 4) = 0$

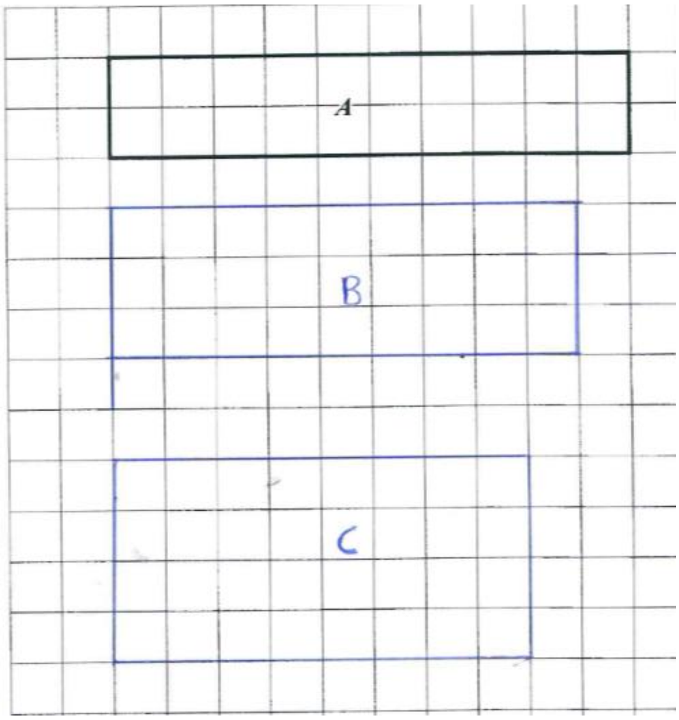
$\therefore w = 8\ or\ w = 4$

Taking w=4;

$l = 12 - w$

$l = 12 - 4 = 8$

Therefore for Rectangle B; length = 8 units and width = 4 units



Question 8a part (ii)

Complete the table to show the dimensions of Rectangles B and C

Rectangle	Length	Width	Area (square units)	Perimeter (units)
A	10	2	20	24
B	9	3	27	24
C	8	4	32	24
D				
E				

Question 8b

Data Given

Rectangle D has a perimeter of 24 cm, with length, l, and width, w.

Determine the values of l and w for the area of the rectangle D to be as large as possible

$Area\ of\ rectangle, A = l \times w$

$Perimeter\ of\ rectangle = 2l + 2w = 24\ units$

$A = l \times w$ ……………… Equation 1

$2l + 2w = 24$ ………….. Equation 2

From Equation 2

$l = 12 - w$………………Equation 3

Substituting Equation 3 into Equation 1

$A = (12 - w) \times w$

$A = 12w - w^2$

At Maximum

$\frac{dA}{dw} = 0$

$12 - 2w = 0$

$2w = 12$

$w = 6$

When w = 6;

$l = 12 - w$

$= 12 - 6$

$= 6$

Since the maximum area occurs at l = 6 and w = 6 rectangle D is actually a square.

$Maximum\ Area\ =\ S \times S$

$= 6 \times 6$

$= 36\ square\ units$

Question 8c

Data Given

Rectangle E has a perimeter of 36 cm, with length, l, and width, w.

Determine the values of l and w for the area of the rectangle E to be as large as possible

For rectangle E to have a maximum value then it is also a square.

Therefore $l\ = w = S$

$Perimeter\ of\ rectangle = 4w = 36\ units$

$w = 9\ units$

$l = 9\ units$

$Maximum\ Area\ =\ S \times S$

$= 9 \times 9$

$= 81\ square\ units$

Rectangle	**Length**	**Width**	**Area (square units)**	**Perimeter (units)**
A	10	2	20	24
B	9	3	27	24

C	8	4	32	24
D	6	6	36	24
E	9	9	81	36

Question 9a part (i)

Data Given

$$f(x) = \frac{2x - 7}{x} \ and \ g(x) = \sqrt{x + 3}$$

Evaluate $f(5)$

$$f(x) = \frac{2x - 7}{x}$$

$$f(5) = \frac{2(5) - 7}{5}$$

$$= \frac{3}{5}$$

Question 9a part (ii)

(a) **Write** an expression in terms of x for $f^{-1}(x)$

$$Let \ y = f(x)$$

$$y = \frac{2x - 7}{x}$$

$$Switch \ x \ and \ y$$

$$x = \frac{2y - 7}{y}$$

$$xy = 2y - 7$$

$$2y - xy = 7$$

$$y(2 - x) = 7$$

$$y = \frac{7}{2 - x}$$

$\therefore f^{-1}(x) = \frac{7}{2-x}, \quad x \neq 2$

(b) **Write** an expression in terms of x for $gf(x)$

Replacing x in $g(x\ )$ with $f(x)$

$$gf(x) = \sqrt{\frac{2x-7}{x} + 3}$$

$$= \sqrt{\frac{2x-7}{x} + \frac{3}{x}}$$

$$= \sqrt{\frac{2x-7+3x}{x}}$$

$$= \sqrt{\frac{5x-7}{x}}$$

$$= \sqrt{5 - \frac{7}{x}} \quad , x \neq 0$$

Question 9b part (i)

Express the quadratic function $1 - 6x - x^2$, in the form $k - \ a(x+h)^2$

$$1 - 6x - x^2 \equiv \ k - \ a(x+h)^2$$

$$1 - 6x - x^2 \equiv \ k - \ [a\ (x+h)(x+h)]$$

$$1 - 6x - x^2 \equiv \ k - \ [a\ (x^2 + 2hx + \ h^2)]$$

$$1 - 6x - x^2 \equiv \ k - ax^2 - 2ahx - \ ah^2$$

Equating coefficients

$x^2; \quad -a = -1$

$a = 11$

$x; \quad -2ah = -6$

$-2(1)h = -6$

$-2h = -6$

$h = 3$

$constant; \quad k - ah^2 = 1$

$k - (1)(3)^2 = 1$

$k - 9 = 1$

$k = 10$

$\therefore 1 - 6x - x^2 \equiv 10 - (x+3)^2$ in the form $k - a(x+h)^2$ where $k = 10, a = 1$ and $h = 3.$

Question 9b part (ii)

(a) **State** the maximum value of $1 - 6x - x^2$

$$1 - 6x - x^2 \equiv 10 - (x+3)^2$$

$$Maximum\ value\ of\ \ 1 - 6x - x^2 = 10 - 0$$

$$= 10$$

(b) **State** the equation of the axis of symmetry

$Let \quad y = 1 - 6x - x^2$

Rearranging gives

$y = -x^2 - 6x + 1 \qquad which\ is\ in\ the\ form\ y = ax^2 + bx + c$

$$Equation\ of\ the\ axis\ of\ symmetry; \quad x = \frac{-b}{2a}$$
$$= \frac{-(-6)}{2(-1)}$$
$$= \frac{6}{-2}$$
$$= -3$$

$Equation\ of\ the\ axis\ of\ symmetry\ is\ x = -3$

Question 9b part (iii)

Determine the roots of $1 - 6x - x^2$

$$1 - 6x - x^2 = 0$$

$$Recall \qquad 1 - 6x - x^2 \equiv 10 - (x+3)^2$$

$$10 - (x+3)^2 = 0$$

$$(x+3)^2 = 10$$

$$x + 3 = \pm\sqrt{10}$$

$$x = \pm\sqrt{10} - 3$$

$$x = +\sqrt{10} - 3 \qquad or \qquad x = -\sqrt{10} - 3$$

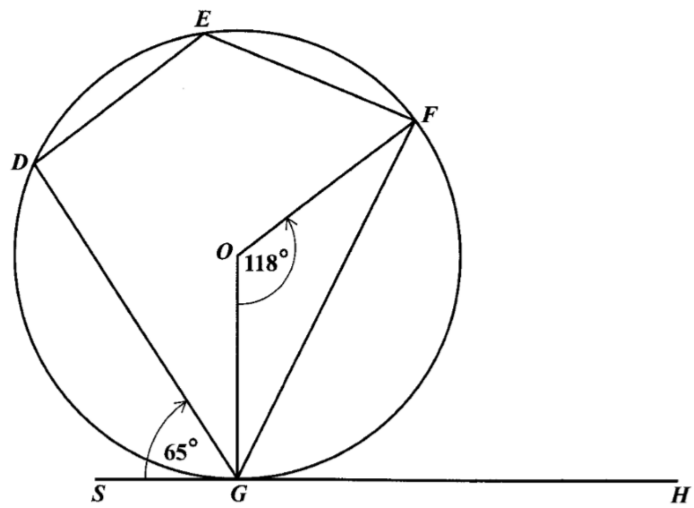
$x = 3.162 - 3 \quad or \quad x = -3.162 - 3$

$x = 0.162 \quad or \quad x = -6.162$

$x = 0.16 \; or - 6.16 \quad \textbf{\textit{to 2 decimal places}}$

Question 10a part (i)

Data Given



Calculate $< OGF$

Triangle OGF is an isosceles triangle since the radius of a circle is constant;

$$Length\ of\ OG = Length\ of\ OF = radius\ of\ the\ circle$$

Therefore the base angles would be equivalent;

$$O\hat{G}F = O\hat{F}G$$

$$= \frac{180° - 118°}{2} \quad \text{since the total sum of angles in a triangle} = 180°$$

$$= \frac{62°}{2}$$

$$= 31°$$

Question 10a part (ii)

Calculate $< DEF$

SGH is a tangent to the circle so an angle of 90° is created where it comes into contact with the circle.

Based on the given diagram; $O\hat{G}S = 90°$

$$O\hat{G}D = O\hat{G}S - 65°$$

$$O\hat{G}D = 90° - 65°$$

$$= 25°$$

$$D\hat{G}F = O\hat{G}D + O\hat{G}F$$

$$D\hat{G}F = 25° + 31°$$

$$= 56°$$

$$D\hat{E}F = 180° - D\hat{G}F$$

$$D\hat{E}F = 180° - 56°$$

$$= 124°$$

Question 10b part (i)

Data Given

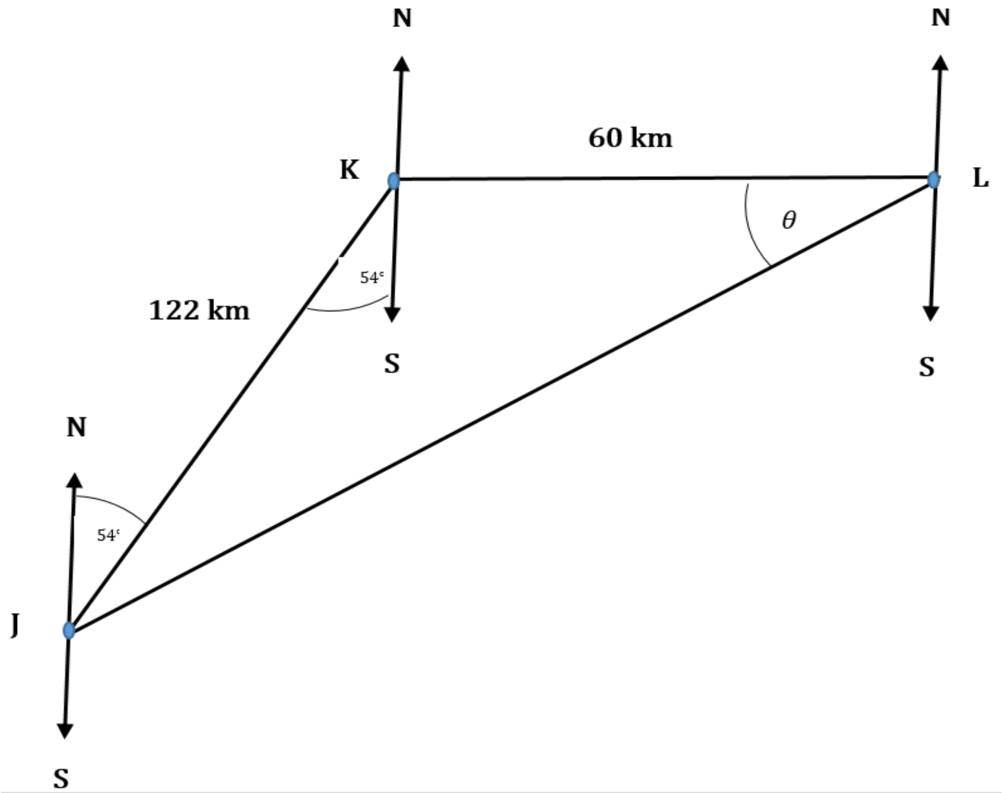
J, K and L are three sea ports.

A ship began its journey at L, sailed to K, then to L and returned to J.

The bearing of K from J is 54° and Z is due east of K.

$JK = 122\ km$ and $KL = 60\ km$

Draw a clearly labelled diagram to represent the data given



Question 10b part (ii)

(a) **Calculate** the measure of angle JKL

$$J\widehat{K}L = 90° + 54°$$

$$= 144°$$

(b) **Calculate** the distance JL

Applying cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

Where

$a = JL, \quad b = JK, \quad c = KL \quad and \quad A = J\widehat{K}L$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$JL^2 = JK^2 + KL^2 - 2\,(JK)(KL)\,\cos(J\widehat{K}L)$$

$$JL^2 = (122)^2 + (60)^2 - 2\,(122)(60)\,\cos(144°)$$

$$JL^2 = 14884 + 3600 - (-11844)$$

$$JL^2 = 30328$$

$$JL = \sqrt{30328}$$

$$JL = 174.149$$

Distance JL = 174. 15 **to 2 decimal places**

(c) **Calculate** the bearing of J from L

Applying sine rule to triangle JKL

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Where

$a = JK, \quad A = K\hat{L}J, \quad b = JL \quad$ and $\quad B = J\widehat{K}L$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{JK}{\sin(K\hat{L}J)} = \frac{JL}{\sin(J\widehat{K}L)}$$

$$\frac{122}{\sin(K\hat{L}J)} = \frac{174.15}{\sin(\,144°\,)}$$

$$\sin(K\hat{L}J) = \frac{122}{\frac{174.15}{\sin(\,144°\,)}}$$

$$\sin(K\hat{L}J) = \frac{122}{296.282}$$

$$K\hat{L}J = \sin^{-1}\left(\frac{122}{296.282}\right)$$

$$K\hat{L}J = 24.31°$$

$$\begin{aligned} Bearing\ of\ J\ from\ L &= 270° - \ K\hat{L}J \\ &= 270° - 24.31° \\ &= 245.7° \end{aligned}$$

Question 11a part (i)

Data Given

Under a matrix transformation $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the points V and W were mapped onto V' and W':

$$V\ (3,5) \rightarrow V'(5,-3)$$

$$W\ (7,2) \rightarrow V'(2,-7)$$

Determine the values of $a, b\ , c\ and\ d$

$$M \times V = V'$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} 3a + 5b \\ 3c + 5d \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Equating terms gives

$3a + 5b = 5 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . Equation\ 1$

$3c + 5d = -3 \ldots \ldots . \ldots . . . . \ldots \ldots . \ldots Equation\ 2$

$$M \times W = W'$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} 7 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

$$\begin{pmatrix} 7a + 2b \\ 7c + 2d \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

Equating terms gives

$7a + 2b = 2 \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . Equation\ 3$

$7c + 2d = -7 \ldots \ldots . \ldots . . . . \ldots \ldots . \ldots Equation\ 4$

$3a + 5b = 5 \ldots \ldots . \ldots . . . . \ldots \ldots . \ldots Equation\ 1$

$7a + 2b = 2 \ldots \ldots . \ldots . . . . \ldots \ldots . \ldots Equation\ 3$

Solving simultaneously

From Equation 1

$a = \frac{5-5b}{3}$

Substituting into Equation 3

$$7\left(\frac{5-5b}{3}\right) + 2b = 2$$

$$\frac{35-35b}{3} + 2b = 2$$

Multiplying throughout by 3

$$35 - 35b + 6b = 6$$

$$-29b = -29$$

$$b = 1$$

$$a = \frac{5-5b}{3} = \frac{5-5(1)}{3} = 0$$

$3c + 5d = -3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \textit{Equation } 2$

$7c + 2d = -7 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \textit{Equation } 4$

Solving simultaneously

From Equation 2

$c = \frac{-3-5d}{3}$

Substituting into Equation 4

$$7\left(\frac{-3-5d}{3}\right) + 2d = -7$$

$$\frac{-21-35d}{3}+2d=-7$$

Multiplying throughout by 3

$$-21-35d+6d=-21$$

$$-29d=0$$

$$d=0$$

$$c=\frac{-3-5d}{3}=\frac{-3-5(0)}{3}=-1$$

Therefore, $M=\begin{pmatrix} a & b \\ c & d \end{pmatrix}=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Question 11a part (ii)

State the coordinates of Z such that $Z\ (x,y)\rightarrow Z'(5,1)$ under the transformation, M

$$Z\ (x,y)\rightarrow Z'(5,1)$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0x+1y \\ -1x+0y \end{pmatrix}=\begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} y \\ -x \end{pmatrix}=\begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

Since the matrices are both 2×1, we can equate terms

$$y=5$$

$$-x=1$$

$$x = -1$$

$Z\ (x, y) = (-1, 5).$

Question 11a part (iii)

Describe the geometric transformation, M

The transformation matrix, $M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ rotates the matrix it is applied to in a clockwise manner about the origin O at an angle of 90°.

Question 11b part (i)

Data Given

$\overrightarrow{OP}$ and $\overrightarrow{OR}$ are position vectors with respect to the origin, O.

Point, P = (2, 7) and $\overrightarrow{PR} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

(a) **Write** $\overrightarrow{OP}$ $in\ the\ form\ \begin{pmatrix} a \\ b \end{pmatrix}$

Point, P = (2, 7)

Hence

$$\overrightarrow{OP} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

$Which\ is\ in\ the\ form\ \begin{pmatrix} a \\ b \end{pmatrix}\ where\ a = 2\ and\ b = 7.$

(b) **Write** $\overrightarrow{OR}$ $in\ the\ form\ \begin{pmatrix} a \\ b \end{pmatrix}$

$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$$

$$\overrightarrow{OR} = \begin{pmatrix} 2 \\ 7 \end{pmatrix} + \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

$Which\ is\ in\ the\ form\ \binom{a}{b}\ \ where\ \ a\ = 6\ and\ b\ = 4.$

Question 11b part (ii)

Data Given

Point, $S = (14, -2)$

(a) **Find** $\overrightarrow{RS}$

$$S = (14, -2)$$

$$\overrightarrow{OS} = \binom{14}{-2}$$

$$\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS}$$

$$\overrightarrow{OR} = -\binom{6}{4} + \binom{14}{-2}$$

$$= \binom{8}{-6}$$

(b) **Show** P, R and S are collinear.

$$\overrightarrow{PS} = \overrightarrow{PR} + \overrightarrow{RS}$$

R is a common point as $\overrightarrow{PR}\ and\ \overrightarrow{RS}$ are parallel to each other and will lie on the line PS.

Using $\overrightarrow{PR}\ and\ \overrightarrow{RS}$; in order for the points to be collinear

$$\overrightarrow{PR} = \mathbf{k}\left(\overrightarrow{RS}\right) \quad where\ k\ is\ a\ scalar\ multiple$$

$$\overrightarrow{PR} = \binom{4}{-3}$$

$$\overrightarrow{RS} = \binom{8}{-6} = 2\binom{4}{-3} \qquad where\ k = 2$$

Therefore P, R and S are collinear.