Solutions to CSEC Maths P2 JAN 2012

Question 1a part (i)

Required to Calculate

$$\left(1\frac{3}{4}\right)^2 \div 3\frac{1}{2}$$

$$= \left(\frac{7}{4}\right)^2 \div \frac{7}{2}$$

$$= \frac{49}{16} \div \frac{7}{2}$$

$$= \frac{49}{16} \times \frac{2}{7}$$

$$= \frac{7}{8} \quad \text{[Exact Value]}$$

Question 1a part (ii)

Required to Calculate $\sqrt{0.0529} + 0.216$

Using the Calculator

$\sqrt{0.0529} + 0.216 = 0.23 + 0.216$

$= 0.446$

$= 4.46 \times 10^{-1}$ [Exact Value]

Question 1b part (i)

Data Given: Basic wage of a typist $= \$22.50$ per hour for a 40-hour work week

Overtime Rate $= 1\frac{1}{2}$ the basic hourly rate

Required to Calculate the typist's basic weekly wage

$Basic\ Weekly\ Wage = Basic\ Hourly\ Rate\ \times Number\ of\ hours\ in\ a\ basic\ work\ week$

$= \$22.50 \times 40$

$= \$900.00$

Question 1b part (ii)

Required to Calculate Overtime Wage for one hour of overtime work

$Overtime\ Wage\ for\ one\ hour\ of\ overtime\ work = 1\frac{1}{2} \times basic\ hourly\ rate$

$$= 1\frac{1}{2} \times \$22.50$$

$$= \$33.75$$

Question 1b part (iii)

Required to Calculate Wage earned for overtime if she worked for a total of 52 hours

$Overtime\ Wage\ for\ 52\ hours = Number\ of\ Overtime\ Hours\ \times Overtime\ Rate$

$$= (52 - 40) \times \$33.75$$

$$= \$405.00$$

Question 1b part (iv)

Required to Calculate Number of overtime hours worked to obtain a total wage of $1440.00

$Number\ of\ Overtime\ hours\ worked = \dfrac{Overtime\ Wage}{Overtime\ Rate}$

$$= \dfrac{(Total\ Wage - Basic\ Wage)}{Overtime\ Rate}$$

$$= \dfrac{(\$1440 - \$900)}{\$33.75}$$

$$= \dfrac{\$540}{\$33.75}$$

$$= 16\ hours$$

Question 2a

Data Given: $3x + 2y = 13$

$x - 2y = -1$

Required to Calculate the value of x and y

Using the Method of Substitution

Step 1: Let $3x + 2y = 13$ be Equation 1

Let $x - 2y = 1$ be Equation 2

Step 2: From Equation 2, we find for an expression in terms of y

$x = -1 + 2y$

$x = 2y - 1$ [Equation 3]

Step 3: Substitute Equation 3 into Equation 1

$3(2y - 1) + 2y = 13$

$6y - 3 + 2y = 13$

$8y - 3 = 13$

$8y = 16$

$y = \dfrac{16}{8}$

$y = 2$

Step 4: Substitute $y = 2$ into Equation 2

$x - 2(2) = -1$

$x - 4 = -1$

$x = -1 + 4$

$x = 3$

Thus, $x = 3$ and $y = 2$

Question 2b part (i)

Required to Factorize

$$x^2 - 16$$

Step 1: Express as the difference of two square

$(x)^2 - (4)^2$

Step 2: Factorize

$(x - 4)(x + 4)$

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Question 2b part (ii)

Required to Factorize

$$2x^2 - 3x + 8x = 12$$

Step 1: $2x^2 - 3x + 8x - 12$

Step 2: Factorize

$$2x(x + 4) - 3(x + 4)$$

$$(x + 4)(2x - 3)$$

Question 2c part (i)(a)

Given Data: Adult tickets cost $30.00 each

Children tickets costs $15.00 each

A company bought 28 tickets

Required to Find the number of tickets for children

x tickets were for adults

$Number\ of\ tickets\ for\ children = total\ number\ of\ tickets - number\ of\ tickets\ for\ adults$

$= 28 - x$

Question 2c part (i)(b)

Required to Find the amount spent on tickets for adults

$Amount\ spent\ on\ tickets\ for\ adults = cost\ of\ 1\ adult\ ticket\ \times number\ of\ tickets$

$= \$30 \times x$

$= \$30x$

Question 2c part (i)(c)

Required to Find the amount spent on tickets for children

$Amount\ spent\ on\ tickets\ for\ children = cost\ of\ tickets\ for\ one\ child\ \times number\ of\ tickets$

$= \$15 \times (28 - x)$

$= \$15(28 - x)$

Question 2c part (ii)

Required to Show the amount spent on 28 tickets is $(15x + 420)$

$Total\ amount\ spent\ on\ all\ 28\ tickets$
$\qquad = amount\ spent\ on\ adult\ tickets + amount\ spent\ on\ children\ tickets$

$= 30 + 15(28 - x)$

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$= 30x + 420 - 15x$

$= \$(15x + 420)$

Data Given: Cost of 28 tickets = $660

Required to Calculate the number of adult tickets bought

$Total\ cost\ of\ tickets = \$660$

$660 = 15x + 420$

$660 - 420 = 15x$

$240 = 15x$

$x = \frac{240}{15}$

$x = 16$

Total Number of adult tickets bought is 16

Question 3a part (i)

Data Given: $U = \{51,52,53,54,55,56,57,58,58\}$

$A = \{Odd\ numbers\}$

$B = \{Prime\ numbers\}$

Required to List the members of the set A

$A = \{51,53,55,57,59\}$

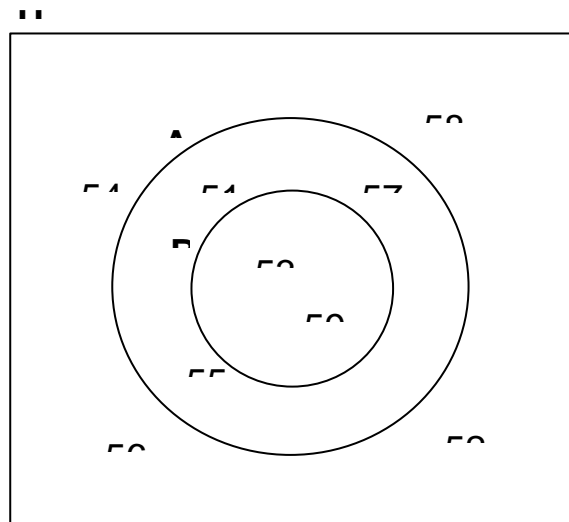| Question 3a part (ii) |
| --- |

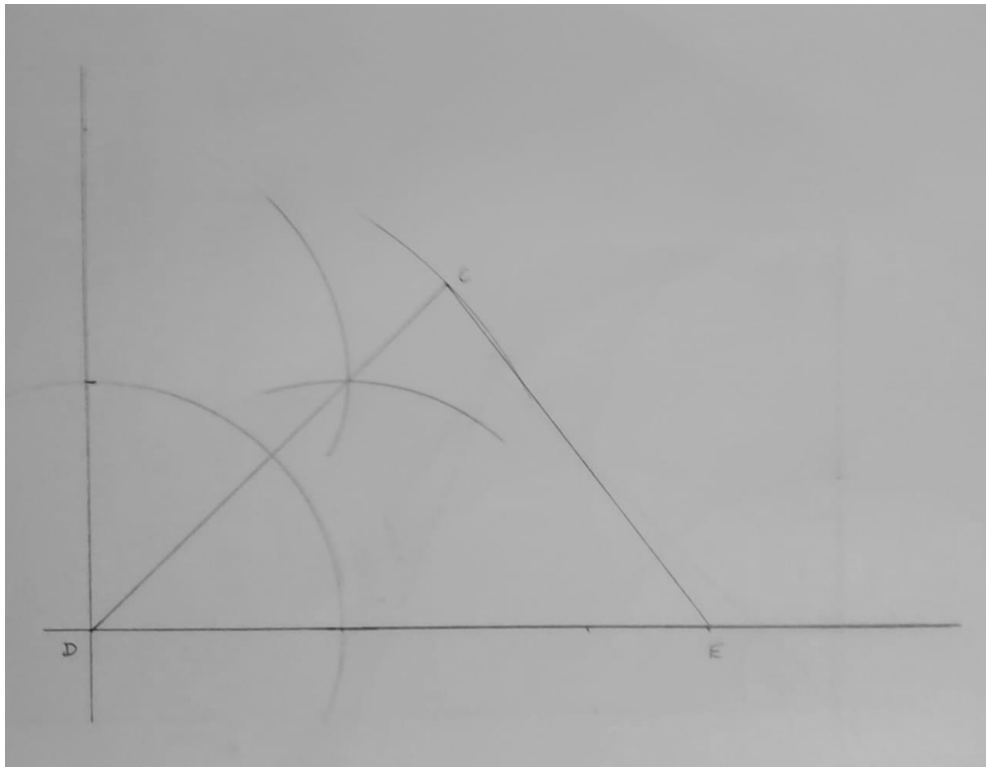**Required to List** the members of the set B

$B = \{53,59\}$

Question 3b part (iii)

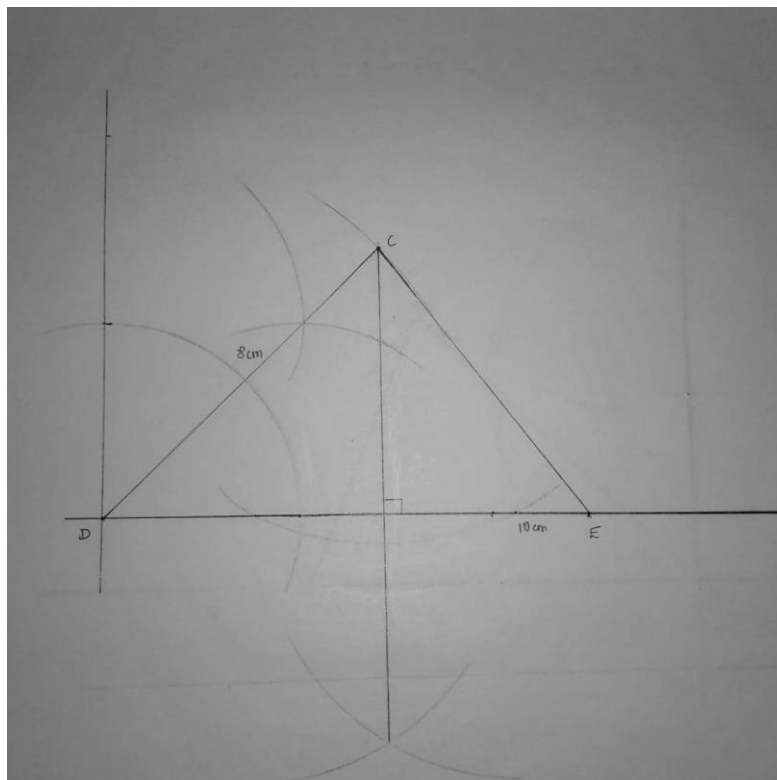**Required to Draw** a Venn Diagram to represent the sets A, B and U

Question 3b part (i)(a)

Required to Construct a triangle CDE in which $DE = 10cm, DC = 8cm$ and angle $CDE = 45°$

Question 3b part (i)(b)

Required to Construct a line, CF, perpendicular to DE such that F lies on DE

Question 3b part(ii)

Required to Measure size of $D\hat{C}E$

Using a Protractor

Angle $DCE = 83°$

Question 4a part(i)

Data Given: Table showing part of a bus schedule

Town	Arrive	Depart
Belleview		6:40 am
Chagvielle	7:35 am	7:45 am
St. Andrews	8:00 am	

Required to Calculate the time spent at Chagville

$Time\ spent\ at\ Chagville = Departure\ time\ from\ Chagville = Arival\ Time\ at\ chagville$

$= 7:45am - 7:35am$

$= 10\ minutes$

Question 4a part(ii)

Required to Calculate the time taken to travel from Belleview to Chagville

Time taken to travel from Belleview to Chagville
= Arrival time at Chagville − Departure time from Belleview

$= 7:35am − 6:40am$

$= 55 \ minutes$

Question 4a part (iii)

Required to Calculate the distance, in km, between Belleview and Chagville, if the bus travelled

at an average speed of $54kmh^{-1}$

Distance between Belleview and CHagville
= time taken to travel from Belleview to Chagville
× Average Speed during the journey

$= \frac{55}{60} \times 54$

$= 49\frac{1}{2} km$

Question 4b part

Data Given: The base area of a cylindrical bucket $= 300cm^2$

4.8 litres of water was poured into the bucket

Required to Calculate the height of the water in the bucket

Volume of water in the cylindrical bucket $= 300 \times h$

$4800cm^3 = 300 \times h$

$h = \frac{4800}{300}$

$h = 16cm$

Question 4c part (i)

Data Given: Length of cuboid = 13cm

Width of cuboid = 4cm

Height of cuboid = $h$cm

Required to Find an expression for the area of the shaded face

Area of the shaded face $= h \times w$

$= 4 \times h$

$= 4h \ cm^2$

Question 4c part (ii)

Required to Write an expression for the volume of the cuboid, in terms of h

$Volume\ of\ the\ cuboid\ = length\ \times width\ \times height$

$= 13 \times 4 \times h$

$= 52h\ cm^3$

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Question 4c part (iii)

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Required to Calculate $h,$ if the volume of the cuboid is $286cm^3$

$Volume\ of\ the\ cuboid\ in\ cm^3 = 286cm^3$

$286 = 52h$

$h = \frac{286}{52}$

$h = 5.5$

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Question 5a part(i)

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Given Data: Two triangles JKL and MLP

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JK is parallel to ML $\qquad$ LM = MP $\qquad$ KLP is a straight line

Angle JLM = 22° $\qquad$ Angle LMP = 36°

Required to find $M\hat{L}P$

$M\hat{L}P = 180° - 36°$

$M\hat{L}P = 144°$

$= \frac{144°}{2}$

$= 72°$

Question 5a part (ii)

Required to find $L\hat{J}K$

$L\hat{J}K = 22°$

Question 5a part (iii)

Required to find JKL

$J\hat{K}L = 72°$

Question 5a part (iv)

Required to find $K\hat{L}J$

$K\hat{L}J = 180° - (22° + 72°)$

$= 86°$

Question 5b part (i)

Data Given: $\qquad$ Diagram showing PQR and its image $P'Q'R'$

Required to State the coordinates of P and Q

$P = (2,1)$ and $Q = (4,3)$

Question 5b part (ii)

Required to Describe Fully the transformation that maps triangle PQR onto triangles $P'Q'R'$

Triangles PQR is mapped onto triangle $P'Q'R'$ by a reflection in the $x - axis$

Question 5b part (iii)

Required to Write the coordinates of images P and Q under the translation $(3 \ -6)$

<u>Step 1</u>: $P \longrightarrow P''$

$P''(2 \ 1) + (3 \ -6)$

$= (5 \ -5)$

$\therefore P'' = (5, -5)$

<u>Step 2:</u> $Q \rightarrow Q''$

$Q'' = (4 \ 3) + (3 \ -6)$

$= (7 \ -3)$

$\therefore Q'' = (7, -3)$

Question 6a

Data Given: An incomplete table with corresponding values of x and y for the function

$y = x^2 - 2x - 3$ for integer values from -2 to 4
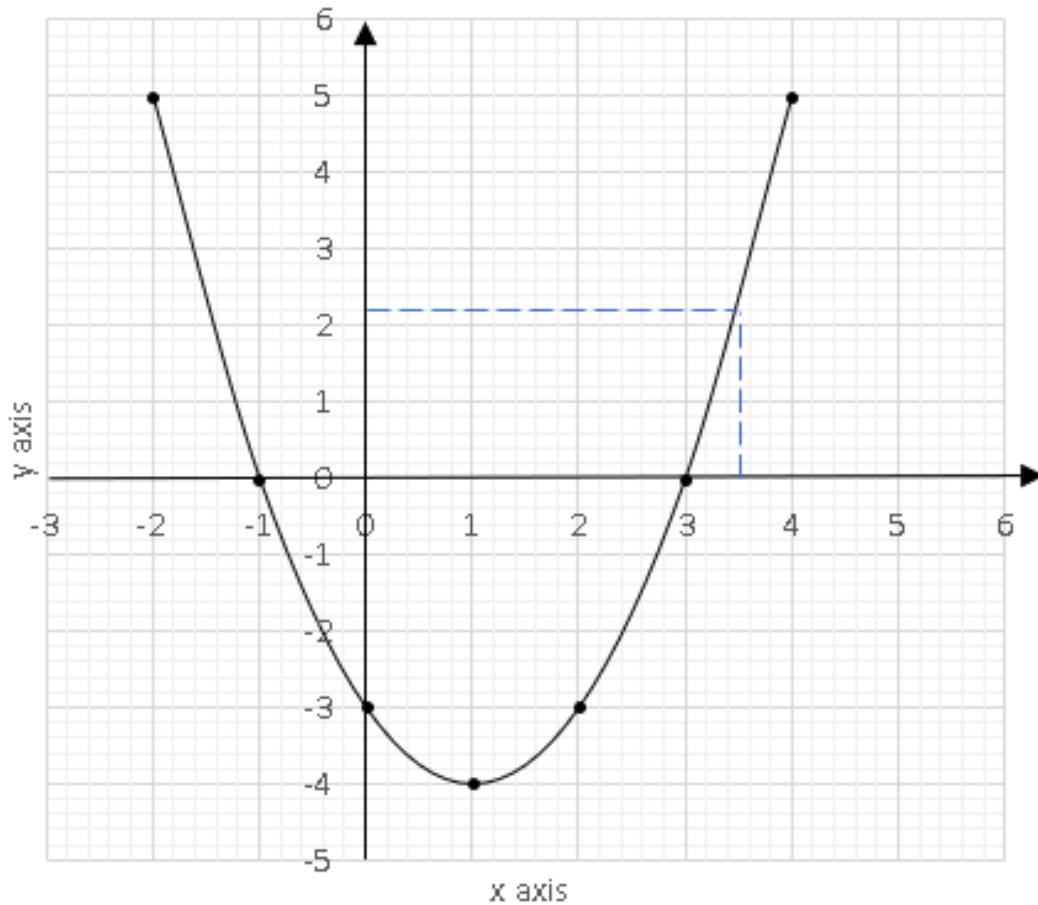
Required to Copy and Complete the Table

x	-2	-1	0	1	2	3	4
y	5	**0**	-3	-4	**-3**	0	5

When $x = -1$ $y = (-1)^2 - 2(-1) - 3$

                     $= 1 + 2 - 3$

                     $= 0$

When $x = 2$ $y = (2)^2 - 2(2) - 3$

                     $y = 4 - 4 - 3$

                     $y = -3$

Question 6b

Required to Plot the graph of $y = x^2 - 2x - 3$ for $-2 \leq x \leq 4$

Required to Use Graph to Estimate the value of y when $x = 3.5$

When $x = 3.5$ $\quad\quad\quad y = 2.2$

Required to Write the equation of the axis of symmetry

Equation of the axis of symmetry is $x = 1$

Required to Estimate the minimum value of the function y

The minimum value of the function y is $y = -4$

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Question 6b part (iii)

Required to State the solutions of the equation $x^2 - 2x - 3 = 0$

The solutions of the equation $x^2 - 2x - 3 = 0$ is $x = 1$ and $x = 3$

Question 7a

Data Given: Histogram showing distribution of heights of seedlings in a sample

Required to Copy and Complete the table

Height in cm, x LCL-UCL	LCB$\leq x \leq$ UCB	Midpoint or Mid-Class Interval	Frequency, f
$1 - 10$	$0.5 \leq x \leq 10.5$	5.5	18
$11 - 20$	$10.5 \leq x \leq 20.5$	15.5	25
$21 - 30$	$20.5 \leq x \leq 30.5$	25.5	23
$31 - 40$	$30.5 \leq x \leq 40.5$	35.5	20
$41 - 50$	$40.5 \leq x \leq 505$	45.5	14
$\sum fx = 2420$			$\sum f = 100$

Question 7b part (i)

Required to Determine the modal class interval

Modal Class Interval is $11 - 20$

Question 7b part (ii)

Required to Determine the number of seedlings in the sample

The number of seedlings $= 18 + 25 + 23 + 20 + 14$

$$= 100$$

Question 7b part (iii)

Required to Determine the mean height of the seedlings

Mean Height of Seedlings $= \dfrac{\sum fx}{\sum f}$

$\sum fx = (5.4 \times 18) + (15.4 \times 25) + (25.5 \times 23) + (3.5 \times 20) + (45.5 \times 14)$

$= 2420$

$\dfrac{\sum fx}{\sum f} = \dfrac{2420}{100}$

$= 24.2 \; cm$

Question 7b part (iv)

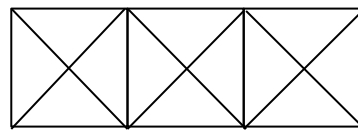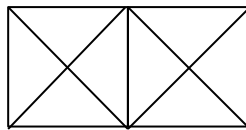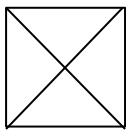**Required to Determine** the probability that a seedling chosen at random has a height that is greater than $30cm$

$$P(Seedling\ is\ greater\ than\ 30\ cm) = \frac{Number\ of\ seedlings\ greater\ than\ 30\ cm}{Total\ number\ of\ seedlings}$$
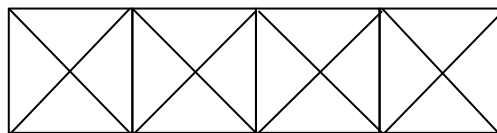
$$= \frac{34}{100}$$

$$= \frac{17}{50}$$

Question 8a

Data Given: Table of values and diagrams showing a sequence of shapes

Required to Draw the 4th shape in the pattern

Question 8b (i)

Required to Copy and Complete the table for Figure 4

	Total Number of Straws	
Figure	Formula	Number
1	$1(6) - 0$	6
2	$2(6) - 1$	11
3	$3(6) - 2$	16
4	$4(6) - 3$	21

10		

Question 8b(ii)

Required to Copy and Complete the table for Figure 10

	Total Number of Straws	
Figure	**Formula**	**Number**
1	$1(6) - 0$	6
2	$2(6) - 1$	11
3	$3(6) - 2$	16
4	$4(6) - 3$	21
10	$10(6) - 9$	51

Question 8c part

Required to Find the figure in the sequence which uses 106 straws

$Total\ Number\ of\ straws = (Figure\ Number\ \times 6) - (Figure\ Number - 1)$

$(n \times 6) - (n - 1) = 106$

$6n - n + 1 = 106$

$5n + 1 = 106$

$5n = 105$

$n = 21$

Thus, figure 21 has 106 $straws$

Question 8d	

Required to Find an expression, in n, for the number of straws in the nth pattern

$Total\ number\ of\ straws\ used\ in\ the\ nth\ pattern = n(6) - (n-1)$

$= -5n + 1$

Question 9a part (i)

Data Given: $y = \frac{2x+3}{x-4}$

Required to make x the subject of the formula

$y(x - 4) = 2x + 3$

$xy - 4y = 2x + 3$

$xy - 2x = 3 + 4y$

$(y - 2)x = 4y + 3$

$x = \frac{4y+3}{y-2}$

Question 9a part (ii)

Required to Determine the inverse of $f(x) = \frac{2x+3}{x-4}, x \neq 4$

$f(x) = \frac{2x+3}{x-4}$

Let $y = f(x)$

$y = \frac{4x+3}{x-2}$

$f^{-1}(x) = \frac{4x+3}{x-2}$

Question 9a part (iii)

Required to Find the value of x for which $f(x) = 0$

Let $\frac{2x+3}{x-4} = 0$

$2x + 3 = 0$

$2x = -3$

$x = -\frac{3}{2}$

Thus, when $f(x) = 0$, $x = -\frac{3}{2}$

Question 9b part (i)

Data Given: Diagrams showing the graphs of lines $x = 6$, $x + y = 40$ and $3y = x$

Required to State the other two inequalities which define the shaded region

$x \geq 6$ and $x + y \leq 40$

Question 9b part (ii)

Required to Identify the three pairs of values for which p has a maximum or minimum value

$(6,2), (6,34), (30,10)$

Question 9b part (iii)

Required to Identify the pair of values which makes p a maximum

When $x = 6$ and $y = 2$ $\qquad p = 4(6) + 3(2)$

$\qquad\qquad\qquad\qquad\qquad\qquad = 30$

When $x = 6$ and $y = 34$ $\qquad p = 4(6) + 3(34)$

$\qquad\qquad\qquad\qquad\qquad\qquad = 126$

When $x = 30$ and $y = 10$ $\qquad p = 4(30) + 3(10)$

$\qquad\qquad\qquad\qquad\qquad\qquad = 150$

Thus, the pair of values which makes p a maximum is $(30,10)$

Question 10a part(i)

Data Given: Diagram showing a regular hexagon with center O and $AO = 8cm$

Required to Determine the size of angle AOB

Angle $AOB = 60°$

[Each interior angle is $60°$ in an equilateral triangle

Question 10a part(ii)

Required to Calculate to the nearest whole number, the area of the hexagon

Let x be $\frac{1}{2}$ of the perimeter of Triangle AOB

$x = \frac{5+5+5}{2}$

$= 7.5$

Using Heron's Formula

$Area = \sqrt{7.5(7.5 - 5)(7.5 - 5)(7.5 - 5)}$

$= \sqrt{7.5 \times 2.5 \times 2.5 \times 2.5}$
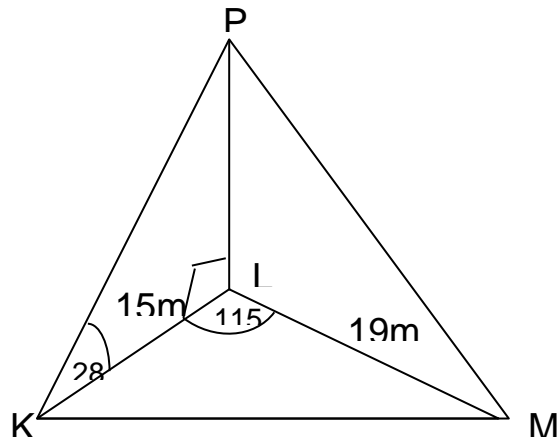
$= \sqrt{117.1875}$

$= 6 \times \sqrt{117.1875}$

$= 64.9$

$\approx 65cm^2$

Question 10b part(i)

Data Given: Diagram showing a vertical pole PL standing on a horizontal plane KLM, where the angle of elevation of P from K is 28°. $KL = 15m, LM = 19m$ and $K\hat{L}M = 115°$

Required to Copy the diagram showing the angle of elevation and one right angle

Angle of Elevation $= 28°$

Question 10b part(ii)(a)

Required to Calculate PL

$tan\theta = \frac{opp}{adj}$

$tan28° = \frac{PL}{15}$

$PL = 15 tan28°$

$PL = 7.97m$

$= 8.0\ m$ [to 2 significant figures]

Question 10b part(ii)(b)

Required to Calculate KM

Using the Cosine Rule

$KM^2 = LM^2 + KL^2 - 2(LM)(KL)\ cos\ cos\ K\hat{L}M$

$= (19)^2 + (15)^2 - 2(19)(15)cos115°$

$= 586 + 240.89$

$= 826.89$

$KL = \sqrt{826.89}$

$= 28.7 \, m$

$\approx 29m$ [to 2 significant figures]

Question 10b part(ii)(c)

Required to Calculate the angle of elevation of P from M

$tan P\widehat{M}L = \dfrac{PL}{LM}$

$= \dfrac{8}{19}$

$P\widehat{M}L = \left(\dfrac{8}{19}\right)$

$\approx 22.7°$

$\approx 23°$ [to 2 significant figures]

Question 11a part(i)(a)

Given Data: Diagram showing position vectors OA and OB

Required to Find OA in the form $(x \ y)$

$OA = (-1 \ 3)$ is of the form $(x \ y)$ where $x = -1$ and $y = 3$

Question 11a part(i)(b)

Required to Find OB in the form $(x \ y)$

$OA = (5 \ 1)$ is of the form $(x \ y)$ where $x = 5$ and $y = 1$

Question 11a part(i)(c)

Required to Find BA in the form $(x \ y)$

Using the Vector Triangle Law

$BA = BO + OA$

$= (5\ 1) + (-1\ 3)$

$= (-5 - 1\ \ -1 + 3)$

$= (-6\ 2)$ is of the form $(x\ y)$, where $x = -6$ and $y = 2$

Question 11b part(ii)(a)

Data Given: G is the midpoint of the line AB

Required to Find BG in the form $(x\ y)$

$BG = \frac{1}{2}BA$

$\frac{1}{2}BA = \frac{1}{2}(-6\ 2)$

$= (-3\ 1)$ is of the form $(x\ y)$, where $x = -3$ and $y = 1$

Question 11b part(ii)(b)

Required to Find OG in the form $(x\ y)$

$OG = OB + BG$

$= (5\ 1) + (-3\ 1)$

$= (2\ 2)$ is of the form $(x\ y)$, where $x = 2$ and $y = 2$

Question 11b part (i)

Data Given: $L = (3\ 2\ 1\ 4)$ and $M = (-1\ 3\ 0\ 2)$

Required to Evaluate $L + 2M$

$L + 2M$

$= (3\ 2\ 1\ 4) + 2(-1\ 3\ 0\ 2)$

$= (3\ 2\ 1\ 4) + (-2\ 6\ 0\ 2)$

$= (1\ 8\ 1\ 8)$

Question 11b part (ii)

Required to Evaluate LM

LM

$= (3\ 2\ 1\ 4\ )(-1\ 3\ 0\ 2\ )$

$= \big((3 \times -1) + (2 \times 0)\ (3 \times 3) + (2 \times 2)\ (1 \times -1) + (4 \times 0)\ (1 \times 3) + (4 \times 2)\ \big)$

$= LM = (-3\ 13\ -1\ 11\ )$

Question 11c part (i)

Data Given: $\quad Q = (4\ 2\ 1\ 1\ )$

Required to find Q^{-1}

$Q^{-1} = \frac{1}{|Q|}(d\ -b\ -c\ a\ )$

$Q^{-1} = \frac{1}{ad-bc}(1\ -2\ -1\ 4\ )$

$= \frac{1}{(1)(4)-(-2)(-1)}(1\ -2\ -1\ 4\ )$

$= \frac{1}{2}(1\ -2\ -1\ 4\ )$

$= \left(\frac{1}{2}\ -1\ -\frac{1}{2}\ 2\ \right)$

Question 11c part (ii)

Required to find the value of x and y in the equation $(4\ 2\ 1\ 1\ )(x\ y\ ) = (8\ 3\ )$

Step 1: Multiply Matrices

$((4x + 2y)\ (1x + 1y)\ ) = (8\ 3\ )$

Step 2: Equating Entries

$4x + 2y = 8 \quad$ [Equation 1]

$x + y = 3 \qquad$ [Equation 2]

Step 4: Multiply Equation 2 by 4

$4(x + y) = 4(3)$

$4x + 4y = 12 \quad$ [Equation 3]

Step 5: Subtract Equation 1 from Equation 2

$4x + 4y = 12\ -$

$\underline{4x + 2y = 8}$

$\underline{\qquad 2y = 4}$

$y = \frac{4}{2}$

$y = 2$

Step 6: Substitute $y = 2$ into Equation 2

$x + y = 3$

$x + 2 = 3$

$x = 3 - 2$

$x = 1$

Thus, $x = 1$ and $y = 2$

$y = \frac{4}{2}$