

Solutions to CSEC Maths P2 January 2013

Question 1a

Calculate the exact value of

$$\begin{aligned} & (2.67 \times 4.1) - 1.3^2 \\ = & 10.947 - 1.69 \\ & = 9.257 \end{aligned}$$

Question 1b part (i)

Data Given:

Return Air Fare US\$356.00

Hotel Accommodation US\$97.00 per night

Calculate the **TOTAL** Cost of Airfare and Hotel Accommodation for 3 nights:

$$\begin{aligned} & = \$356 + (\$97 \times 3) \\ & = \$356 + \$291 \\ & = \$647 \end{aligned}$$

Question 1b part (ii)

Data Given:

3 Nights Hotel Accommodation plus Return Air Fare EC\$1610.00

US\$1.00 = EC\$2.70

Calculate the cost of the trip (in US dollars) for 3 nights:

$$\begin{aligned} & = \frac{\$1610}{\$2.70} \\ & = \text{US\$}596.296 \\ & = \text{US\$}596.30 \text{ (to the nearest cent)} \end{aligned}$$

Question 1b part (iii)

Angie's Travel Club has the better offer than Petty's Travel Club

Reason: Angie's Travel Club costs less than Petty's Travel Club

Question 1b part (iv)

Data Given: Angie's Travel Club's Cost of EC\$1610.00 includes a sales tax of 15%.

Calculate the Cost of the trip for three nights BEFORE the sales tax was added:

Total Tax AFTER Tax was added = $(100 + 15)\% = 115\%$ for the cost of three nights

$$115\% \text{ Tax} = EC \frac{\$1610}{115} = EC\$14.00$$

Tax BEFORE = 100%

Cost of Trip before Tax of 15% is added:

$$= \frac{EC\$1610}{115\%} \times 100\%$$

$$= EC\$1400$$

Question 2a

Solve for p

$$2(p + 5) - 7 = 4p$$

$$2p + 10 - 7 = 4p$$

$$10 - 7 = 4p - 2p$$

$$3 = 2p$$

$$\frac{3}{2} = p$$

Thus $p = \frac{3}{2}$

Question 2b part (i)

Factorize

$$25m^2 - 1$$

$$25m^2 - 1 = (5m)^2 - (1)^2 \quad \text{[expressed as the difference of two squares]}$$

$$25m^2 - 1 = (5m - 1)(5m + 1)$$

Question 2b

Factorize

$$2n^2 - 3n - 20$$

$$2n^2 - 3n - 20 = (2n + 4)(n - 4)$$

Question 2c part (i)

Data Given:

x represents 1 lollipop y represents 1 toffee

5 lollipops and 12 toffees have a mass of 61 grams

10 lollipops and 13 toffees have a mass of 89 grams

Thus

$$5x + 12y = 61 \quad \dots \text{Equation (1)}$$

$$10x + 13y = 89 \quad \dots \text{Equation (2)}$$

Question 2c part (ii)(a)

Calculate the mass of ONE lollipop

The mass of ONE lollipop is x grams, thus we solve the two equations simultaneously to find the value for x.

Solving Equation (1) and Equation (2) Simultaneously:

From Equation (1)

$$5x + 12y = 61$$

$$y = \frac{61-5x}{12} \quad \dots\text{Equation (3)}$$

Substitute $y = \frac{61-5x}{12}$ into Equation (2) to get:

$$10x + 13\left(\frac{61-5x}{12}\right) = 89$$

Multiply both sides of the Equation by 12:

$$12(10x) + 13(61 - 5x) = 12(89)$$

$$120x + 793 - 65x = 1068$$

$$120x - 65x = 1068 - 793$$

$$55x = 275$$

$$x = \frac{275}{55}$$

$$x = 5$$

Thus, the mass of ONE lollipop is 5 grams

Question 2c part (ii)(b)

Calculate the mass of ONE toffee

The mass of ONE toffee is y grams, thus we solve the two equations simultaneously to find the value for y .

From part (ii)(a), $x = 5$

To find the value of y , substitute $x = 5$ into $y = \frac{61-5x}{12}$

$$y = \frac{61 - 5(5)}{12}$$

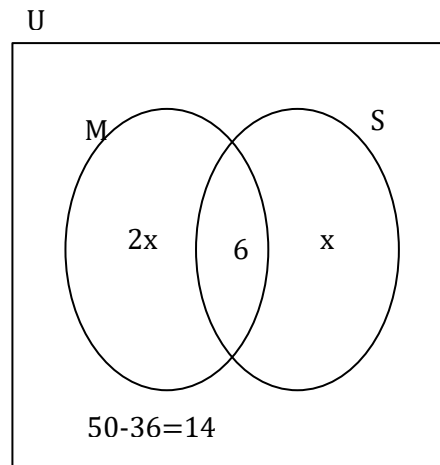
$$y = \frac{61 - 25}{12}$$

$$y = \frac{36}{12}$$

$$y = 3$$

Thus, the mass of ONE toffee is 3 grams

Question 3 part (a)(i)



Question 3 part (a)(ii)

Data Given: Number of Students in Class = 50

Calculate the value of x :

$$2x + 6 + x + 14 = 50$$

$$2x + x + 6 + 14 = 50$$

$$3x + 20 = 50$$

$$3x = 50 - 20$$

$$3x = 30$$

$$x = 10$$

Question 3 part (b)(i)(a)

Data Given: ABC is an isosceles triangle

$$AB = AC$$

$$\angle ABC = 54^\circ$$

Calculate the measure of $\angle BAC$

$$\angle ABC = 54^\circ$$

$$\angle ACB = 54^\circ$$

[Since $AB=AC$ and ABC is isosceles]

$$\angle BAC = 180^\circ - (54^\circ + 54^\circ)$$

[Since the sum of all angles in a triangle = 180°]

$$= 180^\circ - 108^\circ$$

$$= 72^\circ$$

Question 3 part (b)(i)(b)

Data Given: DE is parallel to BC

Calculate $\angle AED$

$$\angle AED = \angle ACB = 54^\circ$$

Question 3 part (b)(ii)

Explain why triangle ABC and ADE are similar but not congruent

Two triangles are **congruent** if both their **corresponding sides** and **angles** are **equal**.

Comparing Corresponding Sides of both Triangles:

Length of AB > Length of AD

Length of BC > Length of DE

Length of AC > Length of AE

Comparing Corresponding Angles of both Triangles:

$$\angle ABC = \angle ADE$$

$$\angle ACB = \angle AED$$

$$\angle BAC = \angle DAE$$

Triangle ABC and ADE are similar since:

1. They are both Isosceles
2. Corresponding Angles are Equal

Triangle ABC and ADE are not congruent since:

1. Corresponding Sides of both Triangles are not Equal

[By Definition of Congruent Triangle]

Question 4a part(i)

Required to make r the subject of the formula:

$$r - h = rh$$

$$r - rh = h$$

$$r(1 - h) = h$$

$$r = \frac{h}{(1 - h)}$$

Question 4a part(ii)

Required to make r the subject of the formula:

$$V = \pi r^2 h$$

$$\frac{V}{\pi h} = r^2$$

$$\sqrt{\left(\frac{V}{\pi h}\right)}$$

Question 4b part (i)

Evaluate:

$$f^{-1}(19)$$

Step 1: Find f^{-1}

$$\text{Let } y = f(x)$$

$$\Rightarrow y = 2x + 5$$

$$y - 5 = 2x$$

$$x = \frac{y - 5}{2}$$

Step 2: Find $f^{-1}(19)$

Replace y by x to get:

$$f^{-1}(x) = \frac{x - 5}{2}$$

$$f^{-1}(19) = \frac{19 - 5}{2}$$

$$= \frac{14}{2}$$

$$= 7$$

Question 4b part (ii)

Evaluate:

$$gf(3)$$

Step 1: Find $f(3)$

$$f(x) = 2x + 5$$

$$f(3) = 2(3) + 5$$

$$= 11$$

Step 2: Find $gf(3)$

$$\begin{aligned} gf(3) &= g(11) \\ &= \frac{11-3}{2} \\ &= \frac{8}{2} \\ &= 4 \end{aligned}$$

Question 4c part (i)

Data Given: The line segment GH has an Equation $3x + 2y = 15$

Determine the gradient of GH

The general equation of a line is $y = mx + c$ where $m = \text{gradient of the line}$

Step 1: Express $3x + 2y = 15$ in the form $y = mx + c$

$$\begin{aligned} 3x + 2y &= 15 \\ 2y &= 15 - 3x \\ 2y &= -3x + 15 \\ y &= \frac{-3x+15}{2} \\ y &= -\frac{3}{2}x + \frac{15}{2} \end{aligned}$$

Step 2: Compare coefficients of x

$$m = -\frac{3}{2}$$

Therefore, the gradient of the line segment GH is $-\frac{3}{2}$

Question 4c part (ii)

Data Given: Another line segment JK is perpendicular to GH

JK passes through the point (4,1)

Determine the Equation of the line JK

The **general form** of the Equation of a Line is $y = mx + c$

The **gradient** of a Line that is **Perpendicular** to GH is $-\frac{1}{m}$

Step 1: Determine Gradient of Perpendicular Line

Since the gradient of the line segment GH is $-\frac{3}{2}$,

Then the gradient of the perpendicular line segment JK = $-\frac{1}{m} = -1 \div -\frac{3}{2}$

(Substituting $m = -\frac{3}{2}$ into $-\frac{1}{m}$ to get the Gradient of JK)

$$\begin{aligned} \text{Gradient of JK} &= -1 \div -\frac{3}{2} \\ &= -1 \times -\frac{2}{3} \\ &= \frac{2}{3} \end{aligned}$$

Step 2: Find for value of c using the point $(4,1)$

Substitute $x = 4, y = 1$ and $m = \frac{2}{3}$ into $y = mx + c$

$$1 = \frac{2}{3}(4) + c$$

$$1 = \frac{8}{3} + c$$

$$c = 1 - \frac{8}{3}$$

$$c = -\frac{5}{3}$$

Step 3: Substitute $m = \frac{2}{3}$ and $c = -\frac{5}{3}$ into $y = mx + c$

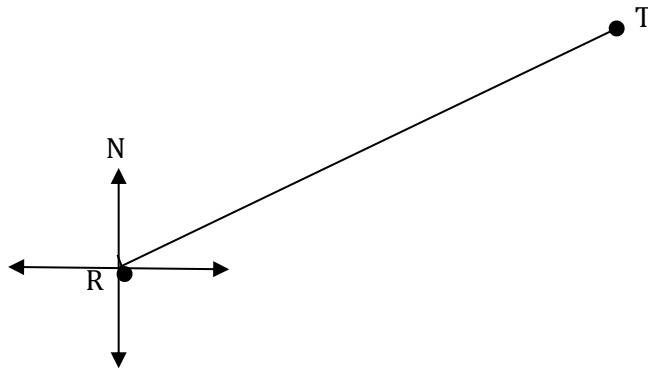
$$y = mx + c$$

$$y = \frac{2}{3}x - \frac{5}{3}$$

Thus the Equation of the line segment JK is $y = \frac{2}{3}x - \frac{5}{3}$

Question 5a part(i)

Data Given: Diagram drawn to scale of 1cm : 30m



Measure and State (in centimetres), the length of RT as drawn in the diagram

Using a Ruler to measure, it is found that the length of the length RT is 5.8cm

Question 5a part (ii)

Measure and State (in degrees), the size of the angle that shows that bearing of T from R

The angle that shows the bearing of T from R is $\angle NRT$

Using a Protractor to measure, it is found that the size of $\angle NRT$ is 65°

Thus, the bearing of T from R is 065° [since a bearing is always expressed as 3 digits]

Question 5a part (iii)

Data Given: Diagram drawn to scale of 1cm : 30m

Calculate the actual distance (in metres), on the playground that RT represents

The length of RT (in cm) = 5.8

Since 1:30

Then $5.8:30 \times 5.8$

$\Rightarrow 5.8:174$

Thus, the actual distance RT = 174m

Question 5b part (i)

Data Given: A point M on the playground is located 300 metres from R on a bearing of 120°

Diagram drawn to scale of 1cm : 30m

Calculate (in centimetres), the length of RM that should be used on the scale drawing

1cm : 30m

Thus $\frac{1}{30} : 1$

To find for 300m:

$$\frac{1}{30} \times 300 : 1 \times 300$$

$$10 : 300$$

Thus 10cm : 300m

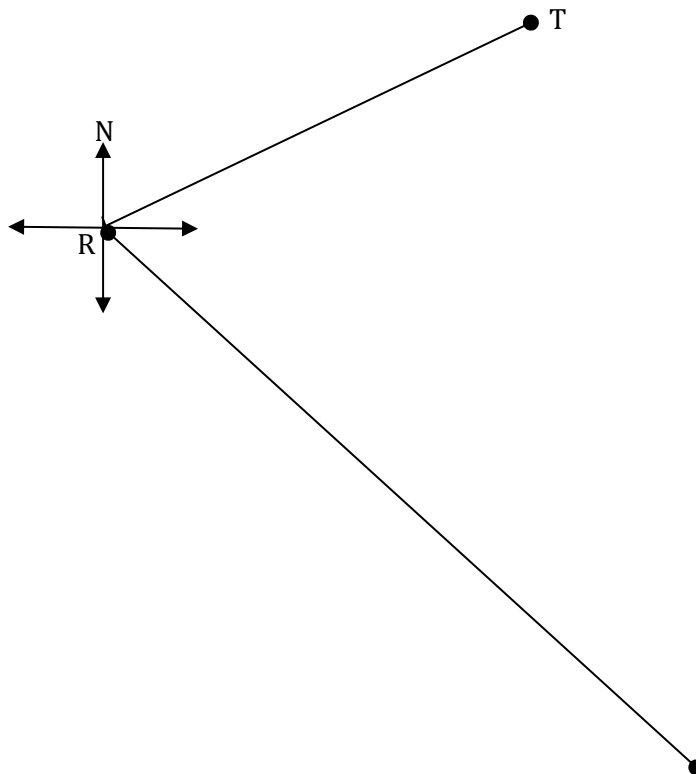
Thus, RM should be drawn to a length of 10cm on the scale drawing

Question 5b part (ii)

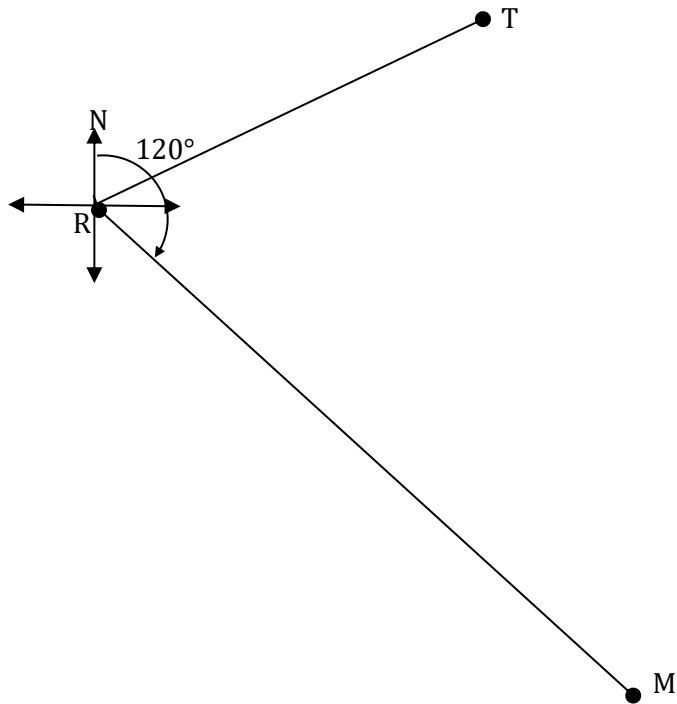
Draw the line RM on the scale drawing (using a Ruler and a pair of Compasses)

The line RM is found to be 10cm long

Point M is located from R 300 metres from R on a bearing of 120°

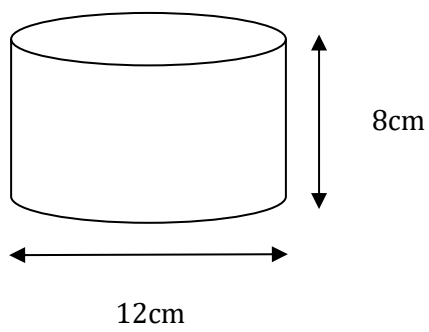


Question 5b part (iii)



Question 6a part (i)

Data Given:



Calculate the radius of the cylinder

$$\text{Diameter} = 12\text{cm}$$

$$\text{Radius} = \frac{\text{Diameter}}{2}$$

$$= \frac{12}{2}$$

$$= 6\text{cm}$$

Thus Radius of the Cylinder = 6cm

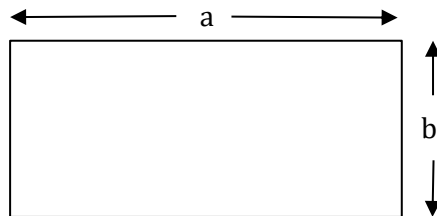
Question 6a part (ii)

Calculate the circumference of the cross section

$$\begin{aligned} \text{Circumference of the Cross - Section} &= 2\pi \times \text{Radius} \\ &= 2(3.14) \times 6 \\ &= 37.68\text{cm} \end{aligned}$$

Question 6b part (i)

Data Given: The rectangle below represents the net of the curved surface of the cylinder shown above



State the values of a and b

a represents the circumference of the cylinder

b represents the height of the cylinder

Thus $a = 37.68\text{cm}$

$$b = 8\text{cm}$$

Question 6b part (ii)

Calculate the area of the curved surface of the cylinder

$$\begin{aligned} \text{Area of the curved surface of the cylinder} &= 2\pi rh \\ &= 2(3.14)(6)(8) \\ &= 301.44\text{cm}^2 \end{aligned}$$

Question 6c

Data Given: 0.5 litres of water are poured into the cylinder

Calculate the height of the water in the cylinder (correct to 1 decimal place)

$$0.5 \text{ litres} = 500 \text{ cm}^3 \quad [\text{since } 1 \text{ litre} = 1000 \text{ cm}^3]$$

$$\text{Volume of a cylinder} = \pi r^2 h$$

Making h the subject of the formula:

$$h = \frac{V}{\pi r^2}$$

$$h = \frac{500}{(3.14)(6^2)}$$

$$h = 4.4 \text{ cm} \quad (\text{to } 1 \text{ decimal place})$$

Question 7a part (i)

Data Given:

Score	Class Mid-Point (x)	Frequency (f)	$f \times x$
0-9	4.5	8	36
10-19	14.5	13	188.5
20-29		25	
30-39		22	
40-49		20	
50-59		12	
Total		100	

State the Modal Class Interval

The Modal Class Interval is the class with the highest frequency

Thus 20-29 is the Modal Class Interval

Question 7a part (ii)

State the class interval in which a score of 19.4 would lie

The score 19.4 would lie in the the class interval 10-19 ($9.5 \leq x < 19.5$)

Question 7b part (i)(a)

Complete the Table to show the class midpoints

Score	Lower Class Boundary & Upper Class Boundary	Class Mid-Point (x)	Frequency (f)	$f \times x$
0-9	$0 \leq x < 9.5$	4.5	8	36

10-19	$9.5 \leq x < 19.5$	14.5	13	188.5
20-29	$19.5 \leq x < 29.5$	$\frac{19.5 + 29.5}{2}$ = 24.5	25	
30-39	$29.5 \leq x < 39.5$	$\frac{29.5 + 39.5}{2}$ = 34.5	22	
40-49	$39.5 \leq x < 49.5$	$\frac{39.5 + 49.5}{2}$ = 44.5	20	
50-59	$49.5 \leq x < 59.5$	$\frac{49.5 + 59.5}{2}$ = 54.5	12	
Total			100	

Question 7b part (i)(b)

Complete the Table to show the value of $f \times x$

Score	Lower Class Boundary & Upper Class Boundary	Class Mid-Point (x)	Frequency (f)	$f \times x$
0-9	$0 \leq x < 9.5$	4.5	8	36
10-19	$9.5 \leq x < 19.5$	14.5	13	188.5
20-29	$19.5 \leq x < 29.5$	$\frac{19.5 + 29.5}{2}$ = 24.5	25	25×24.5 = 612.5
30-39	$29.5 \leq x < 39.5$	$\frac{29.5 + 39.5}{2}$ = 34.5	22	$22 \times 34.5 = 759$
40-49	$39.5 \leq x < 49.5$	$\frac{39.5 + 49.5}{2}$ = 44.5	20	$20 \times 44.5 = 890$
50-59	$49.5 \leq x < 59.5$	$\frac{49.5 + 59.5}{2}$ = 54.5	12	$12 \times 54.5 = 654$
Total			100	

Question 7b part (ii)

Calculate the mean score for the sample

$$\bar{x} = \frac{\sum fx}{\sum f} \quad \text{where } \bar{x} = \text{mean, } \sum fx = \text{sum of } fx, x = \text{midpoint of Class Interval}$$

$$\begin{aligned} \bar{x} &= \frac{36 + 188.5 + 612.5 + 759 + 890 + 654}{100} \\ &= \frac{3140}{100} \\ &= 31.4 \end{aligned}$$

Question 7c

Explain why the value of the mean obtained in part b(ii) is only an estimate of the true value

The mean is obtained using the mid-class interval rather than the actual scores.

Since the mean is being calculated using an interval, and the score can be found anywhere in between the range of values for the interval, then the exact value will not be found, but rather, only an estimation of the value.

Question 7d

Data Given: A Student must score AT LEAST 40 points to qualify for the next round

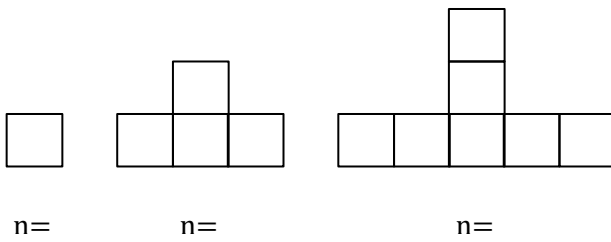
Calculate the probability that a student selected at random qualifies for the second round

$$\begin{aligned} P(\text{Student scores 40 or more points}) &= \frac{\text{Number of Students who scored 40 or more points}}{\text{Total Number of Students}} \\ &= \frac{20 + 12}{100} \\ &= \frac{32}{100} \\ &= \frac{8}{25} \end{aligned}$$

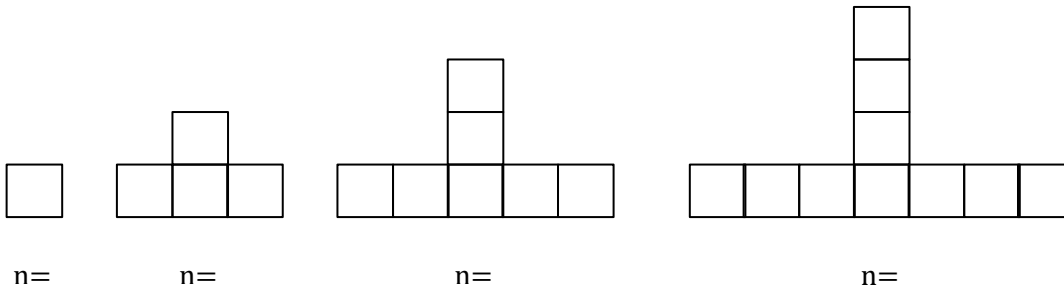
Thus the probability that a student selected at random qualifies for the second round is $\frac{8}{25}$

Question 8a (i)

Data Given:



Draw the fourth diagram in the sequence



Question 8b part (i)

Data Given:

Diagram (n)	Number of Squares
1	1
2	4
3	7
4	a
10	b
c	40

Determine the value of a

We find the number of squares using $3n - 2$, where n is the diagram number

When $n = 4$, the number of squares = $3(4) - 2$

$$= 10$$

Thus, the value of $a = 10$

Question 8b part (ii)

Data Given:

Diagram (n)	Number of Squares
1	1
2	4
3	7
4	a
10	b
c	40

Determine the value of b

Using $3n - 2$, where n is the diagram number

When $n = 10$, the number of squares = $3(10) - 2$

$$= 28$$

Thus, the value of $b = 28$

Question 8b part (iii)

Data Given:

Diagram (n)	Number of Squares
1	1
2	4

3	7
4	a
10	b
c	40

Determine the value of c

Using Number of squares = $3n - 2$, where n is the diagram number

The number of squares = 40

Thus $3n - 2 = 40$

$$3n = 42$$

$$n = \frac{42}{3}$$

$$n = 14$$

Thus, the value of $c = 14$

Question 8c

Write (in terms of n), the number of squares in the n^{th} diagram of the sequence

$$\text{Number of Squares in the } n^{\text{th}} \text{ diagram} = 3n - 2$$

Question 9a part (i)

Data Given: $y = \frac{3}{x}, x \neq 0$

$x(\text{sec})$	0.25	0.5	1	2	3	4	5	6
$y(\text{m/s})$	12		3	1.5		0.75		0.5

Complete the table for the function y

When $x = 0.5, y = \frac{3}{0.5} = 6$

When $x = 3, y = \frac{3}{3} = 1$

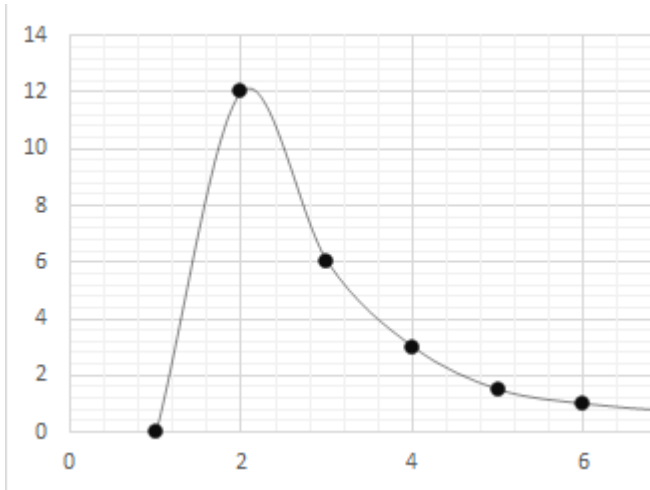
When $x = 5, y = \frac{3}{5} = 0.6$

$x(\text{sec})$	0.25	0.5	1	2	3	4	5	6
$y(\text{m/s})$	12	6	3	1.5	1	0.75	0.6	0.5

Question 9a part (ii)

Plot the points from the table, drawing a smooth curve through ALL points

Using a scale of 2cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis



Question 9b part (i)

Write $f(x) = 3x^2 - 5x + 1$ in the form $a(x - h)^2 + k$, where a , h and k are constants

Step 1: Expand $a(x - h)^2 + k$

$$\begin{aligned} a(x - h)^2 + k &= a(x - h)(x - h) + k \\ &= a(x^2 - 2hx + h^2) + k \\ &= ax^2 - 2ahx + ah^2 + k \end{aligned}$$

$$3x^2 - 5x + 1 \text{ is of the form } ax^2 - 2ahx + ah^2 + k$$

Step 2: Comparing Coefficients of x^2 and x

Equating coefficients of x^2

$$a = 3$$

Equating coefficients of x

$$-5 = -2ah$$

$$-5 = -2(3)h$$

$$-5 = -6h$$

$$\text{Thus } h = \frac{5}{6}$$

Equating the constant terms

$$1 = ah^2 + k$$

Substitute $a = 3$ and $h = \frac{5}{6}$ to find the value k

$$1 = (3)\left(\frac{5}{6}\right)^2 + k$$

$$1 = (3)\left(\frac{25}{36}\right) + k$$

$$1 = \left(\frac{25}{12}\right) + k$$

$$1 - \left(\frac{25}{12}\right) = k$$

$$-\frac{13}{12} = k$$

$$\text{Thus, } k = -\frac{13}{12}$$

Step 4: Substitute $a = 3$, $h = \frac{5}{6}$ and $k = -\frac{13}{12}$ into $a(x - h)^2 + k$ to get:

$$3\left(x + \frac{5}{6}\right)^2 - \frac{13}{12}$$

Thus, $f(x) = 3x^2 - 5x + 1$ written in the form $a(x + h)^2 + k$ is $3\left(x - \frac{5}{6}\right)^2 - \frac{13}{12}$

Question 9b part (ii)

Determine the minimum value of $f(x)$ and the value of x for which $f(x)$ is a minimum

$$a(x + h)^2 + k = 3\left(x - \frac{5}{6}\right)^2 - \frac{13}{12}$$

$$\text{Since } \left(x - \frac{5}{6}\right)^2 \geq 0, \forall x$$

$$\text{Then } 3\left(x - \frac{5}{6}\right)^2 \geq 0$$

$$\text{Thus, the minimum value of } f(x) = 0 - \frac{13}{12}$$

$$= -\frac{13}{12}$$

Now, the value of x for which $f(x)$ is a minimum

$$f(x) \text{ has a minimum value when } 3\left(x - \frac{5}{6}\right)^2 = 0$$

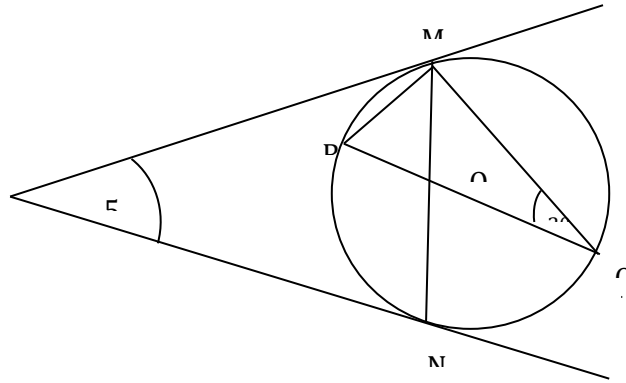
$$3\left(x - \frac{5}{6}\right)^2 = 0$$

$$\Rightarrow x = \frac{5}{6}$$

Thus, the minimum value of $f(x) = -\frac{13}{12}$ and the value of x for which $f(x)$ is a minimum is $\frac{5}{6}$

Question 10a part(i)

Data Given:



Calculate $\angle MRQ$

Triangle MRQ is a Right-Angled Triangle

$$\Rightarrow \angle RMQ = 90^\circ$$

Since the sum of all angles in a triangle = 180°

$$\text{Then } \angle MRQ = 180^\circ - (90^\circ + 20^\circ)$$

$$= 70^\circ$$

Question 10a part(ii)

Calculate $\angle PMR$

$$\angle PMR = 20^\circ \text{ since}$$

1. The angle made a tangent to the circle (PM)
2. The angle made a chord to the circle (MR)

Thus, the angle at M is equal to the angle in the alternate segment at Q

Question 10a part(iii)

Calculate $\angle PMN$

$$PM = PN \quad [\text{since they are both tangents to the circle, with the same starting point}]$$

\Rightarrow Triangle PMN is isosceles

Since the sum of all angles in a triangle = 180°

$$\text{Then } \angle PMN = \angle PNM$$

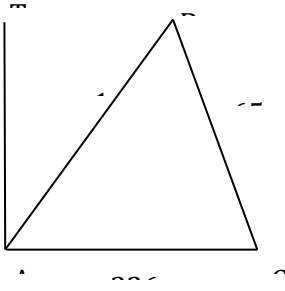
$$= \frac{180^\circ - 54^\circ}{2}$$

$$= \frac{126^\circ}{2}$$

$$= 63^\circ$$

Question 10b part(i)(a)

Data Given:



Calculate the measure of angle ABC

Using the Cosine Rule: $a^2 = b^2 + c^2 - 2ac \cos \hat{B}$

$$a = 65 \quad b = 226 \quad c = 174$$

$$a^2 = b^2 + c^2 - 2ac \cos \hat{B}$$

$$(65)^2 = (226)^2 + (174)^2 - 2(65)(174) \cos \hat{B}$$

$$51076 = 4225 + 30276 - 22620 \cos \hat{B}$$

$$51076 = 34501 - 22620 \cos \hat{B}$$

$$-\frac{51076 - 34501}{22620} = \cos \hat{B}$$

$$\cos \hat{B} = -0.73276$$

Since $\cos \hat{B}$ is negative,

$\Rightarrow \hat{B}$ is obtuse

Let $\cos(180^\circ - x) = -\cos x$

$$\hat{B} = 180^\circ - \cos^{-1}(0.73276)$$

$$= 180^\circ - 42.881^\circ$$

$$= 137.119^\circ$$

$$= 137.1^\circ \text{ (to the nearest } 0.1^\circ \text{)}$$

Question 10b part(i)(b)

Calculate the area of Triangle ABC

$$\text{Area of Triangle ABC} = \frac{1}{2}(\text{side})(\text{side})(\text{sine of the included angle})$$

$$= \frac{1}{2}(174)(65)\sin 137.119^\circ$$

$$= 3848.14m^2$$

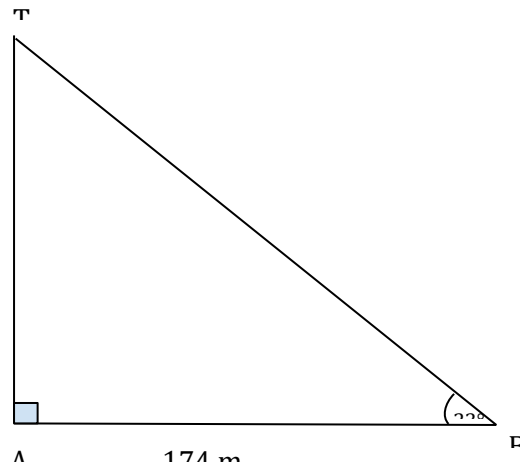
$$= 3848.1 \text{ (to 1 decimal place)}$$

Question 10b part(ii)(a)

Data Given: The line TA represents a vertical lighthouse

The angle of elevation from B is 25

Draw the triangle TAB showing the angle of elevation



Question 10b part(ii)(b)

Calculate the height, TA, of the lighthouse

Using the Ratio of the Tangent:

$$\tan 23^\circ = \frac{TA}{174}$$

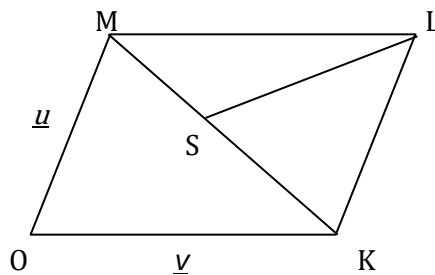
$$174 \tan 23^\circ = TA$$

$$TA = 73.858 \text{ m}$$

$$= 73.86 \text{ m (to 2 decimal places)}$$

Question 11a part (i)

Given Data:



Express MK in terms of \underline{u} and \underline{v}

Using the Vector Triangle Law applied to triangle MOK

$$MK = MO + OK$$

$$= -(u) + v$$

$$= -u + v$$

Question 11a part (ii)

Express SL in terms of \underline{u} and \underline{v}

Using the Vector Triangle Law applied to triangle MOK

$$SL = SK + KL$$

Since $MS = 2 SK$,

$$\text{Then } SK = \frac{1}{3}MK$$

$$= \frac{1}{3}(-u + v)$$

Since the opposite sides of a parallelogram are parallel and equal,

Then, $KL = OM$

$$\overrightarrow{KL} = \underline{u}$$

$$\text{Thus, } SL = \frac{1}{3}(-u + v) + u$$

$$= -\frac{1}{3}u + \frac{1}{3}v + u$$

$$= \frac{1}{3}(2u + v)$$

Question 11a part (iii)

Express OS in terms of \underline{u} and \underline{v}

Using the Vector Triangle Law applied to triangle MOK

$$OS = OK + KS$$

$$= v + -\left(\frac{1}{3}(-u + v)\right)$$

$$= v + \frac{1}{3}u - \frac{1}{3}v$$

$$= \frac{1}{3}(u + 2v)$$

Question 11b

Data Given: The matrix $J = \begin{bmatrix} 0 & -1 & 1 & 0 \end{bmatrix}$ represents a single transformation

The image of the Point P under transformation J is (5,4)

Determine the coordinates of P

Step 1: Multiply the coordinates of P by the matrix J

$$[0 \ -1 \ 1 \ 0][x \ y] = [5 \ 4]$$

$$[(0 \times x) + (-1 \times y) \ (1 \times x) + (0 \times y)] = [5 \ 4]$$

$$[-y \ x] = [5 \ 4]$$

Step 2: Equating Matrices to find the values of x and y

$$x = 4, y = -5$$

Thus the coordinates of P are found to be $(4, -5)$

Question 11c part (i)

Write a matrix H , which represents an enlargement with scale factor 3, about the origin.

The matrix which represents an enlargement with center O and scale factor k is $[k \ 0 \ 0 \ k]$

Thus, the matrix which represents an enlargement with center O and scale factor 3 is $[3 \ 0 \ 0 \ 3]$

Therefore, $H = [3 \ 0 \ 3 \ 0]$

Question 11c part (ii)

Find the coordinates of the image point when the object point $(5, -7)$ undergoes a combined transformation H followed by J

The transformation H followed by J is written as JH

Let $A = (5, -7)$

Step 1: Find the image of A under the transformation H

$$\begin{aligned} [3 \ 0 \ 0 \ 3][5 \ -7] &= [(3 \times 5) + (0 \times -7) \ (0 \times 5) + (3 \times -7)] \\ &= [15 \ -21] \end{aligned}$$

Step 2: Using the image of A under the transformation H , we find the image under the transformation J

$$\begin{aligned} [0 \ -1 \ 1 \ 0][15 \ -21] &= [(0 \times 15) + (-1 \times -21) \ (1 \times 15) + (0 \times -21)] \\ &= [21 \ 15] \end{aligned}$$

Thus, the coordinates of the image point when the object point $(5, -7)$ undergoes a combined transformation H followed by J is $(21, 15)$

Question 11d part (i)

Data Given: Model A costs \$40
Model B costs \$55
Model C costs \$120

Weekly sales in June:

Week 1	Week 2
2 Model A	no model A
5 Model B	6 model B
3 Model C	10 model C

Write down a matrix of 3×2 which represents the sales for two weeks

[2 0 5 6 3 10]

Question 11d part (ii)

Write a 1×3 matrix representing the cost of the phones

[40 55 120]

Question 11d part (iii)

Write the multiplication of the two matrices which represents the superstore's takings from the sale of the call phones for the two weeks

[40 55 120][2 0 5 6 3 10] = [e_{11} e_{12}]