Solutions to CSEC Maths P2 January 2014

Question 1(a)

$$\left(1\frac{3}{4}-\frac{1}{8}\right)+\left(\frac{5}{6}\div\frac{2}{3}\right)$$

$$\frac{13}{8}+\frac{5}{4}$$

$$\frac{23}{8}$$

Question 1(b)

$By\ calculator$

2.399

$= 2.40(correct\ to\ 2\ decimal\ places)$

Question 1(c)(i)

Cost of bracelets in China $= \$6800$

Amount paid in duty $= \$1360$

Total cost $= 6800 + 1360 = \$8160$

Question 1(c)(ii)(a)

Selling price of 165 bracelets $= \$68.85 \times \$165$

$= \$11360.25$

Selling price exceeds cost price, hence a profit was made

Profit = Selling Price – Total Cost

=\$11360.25-\$81600.00

=\$3200.25

Question 1(c)(ii)(b)

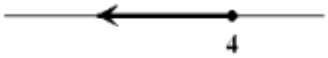
$$Percentage\ Profit = \frac{3200.25}{8160.00} \times 100$$

$= 39.2\% \cong 39\%$

Question 2(a)(i)

$$2(x-6)+3x \leq 8$$
$$2x-12+3x \leq 8$$
$$2x+3x-12 \leq 8$$
$$2x+3x \leq 8+12$$
$$5x \leq 20$$
$$x \leq 4$$

Question 2(a)(ii)



Question 2(b)(i)

$$3x-6y+ax-2ay$$
$$3(x-2y)+a(x-2y)$$
$$(x-2y)(3+a)$$

Question 2(b)(iii)

$$p^2-1$$
$$(p-1)(p+1)$$

Question 2(c)

$$(2k-3)(k-2)$$
$$2k^2-4k-3k+6$$
$$2k^2-7k+6$$

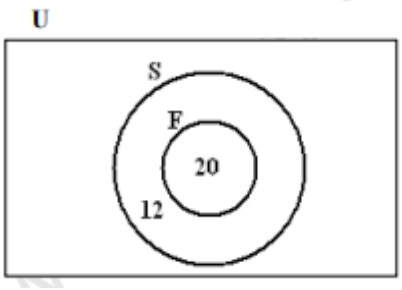
Question 2(d)

$$Substituting\ x=1\ in\ 3x+y=2$$
$$y=-1$$
$$Substituting\ x=1\ in\ 4x-2y=6$$
$$y=-1$$

Hence the point (1,-1) is the point of intersection of both lines

Question 3(i)

Number of students which study Spanish only = 32-20 = 12



Question 3(ii)

32-20 = 12 students which study Spanish but not French

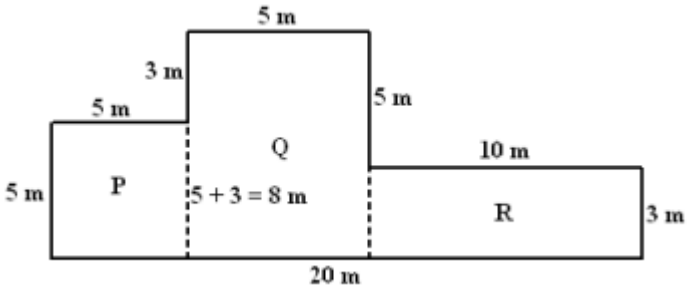
Question 3(b) (i)

$$l = x + 5 + 2x$$
$$l = (3x + 5)m$$

Question 3(b) (ii)(a)

$$Perimeter = x + 3 + 5 + x + 2 + 3 + (3x + 5) + x$$
$$Perimeter = (8x + 16)m$$
$$8x + 16 = 56$$
$$x = 5$$

Question 3(b)(ii)(b)



Area of square, P = 5 x 5 = 25
Area of rectangle, Q = 8 x 5 = 40
Area of rectangle, R = 10 x 3 = 30
Total Area = 25+30+40 = 95m^2

Question 4(a)

Line 1: y=x+2

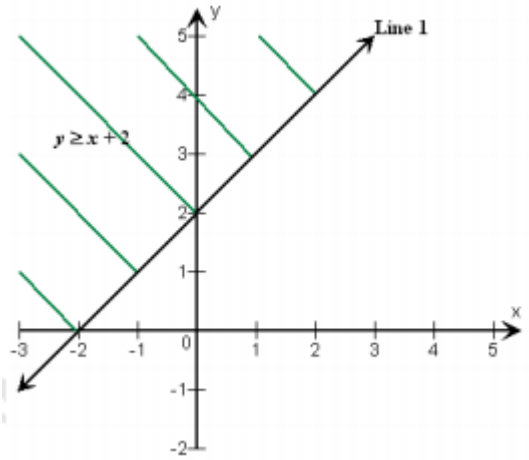
Line 2: y=x

Line 3: y=2

Question 4(b)

$$Using\ points\ (0,0) and\ (2,2)$$
$$m = \frac{2-0}{2-0} = \frac{2}{2}$$
$$m = 1$$

Question 4(c)



Question 4(d)

$$y \leq 2$$
$$y \geq x$$
$$x \leq 0$$

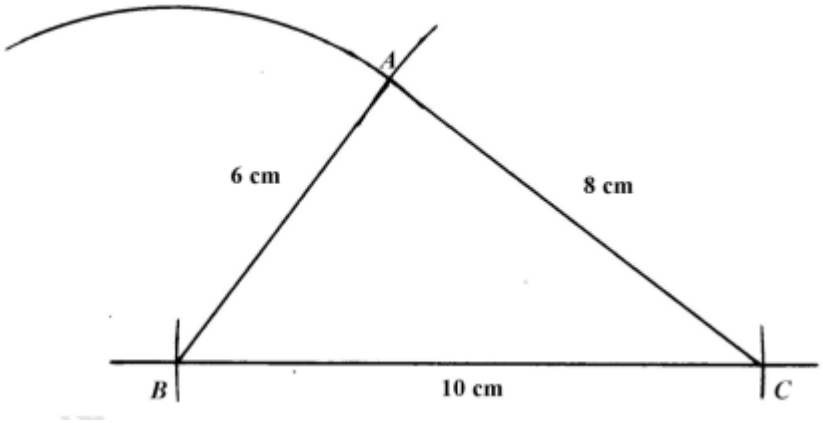
Question 4(e)

Gradient of line 1 is 1

Hence required gradient of the perpendicular line is -1.

Since the required lines passes through the origin, then its equation is y=-x

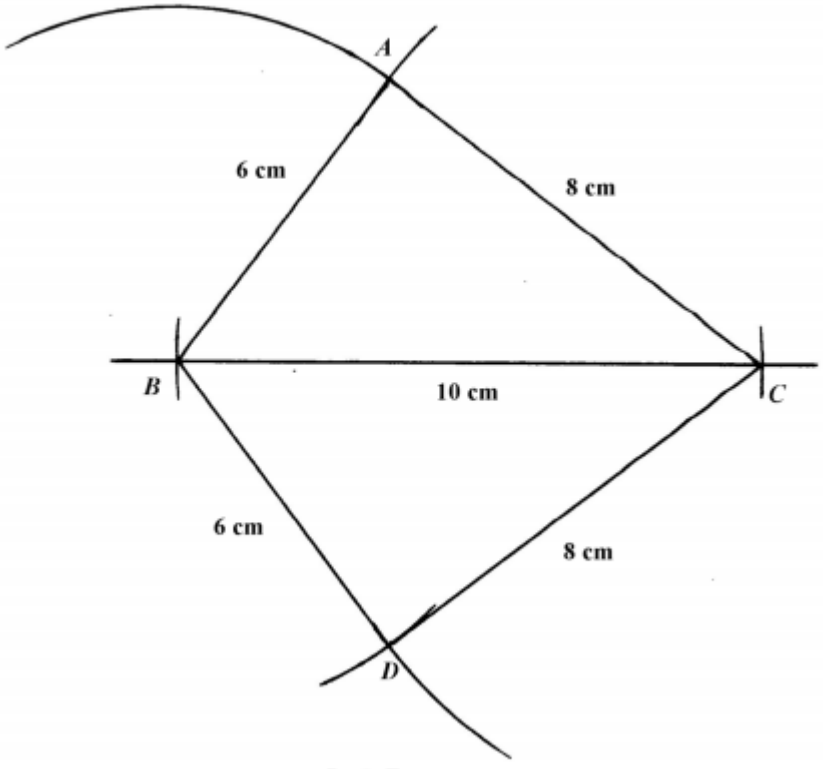
Question 5(a)



Question 5(a)(ii)

53^{o}

Question 5(a)(iii)



Question 5(b)(i)

$$Area\ PQRS = \frac{1}{2}(12+6)15$$
$$= 135cm^2$$

Question 5(b)(ii)

$$Volume\ of\ block = 135 \times 3 = 405cm^2$$

Question 5(b)(iii)

$$Mass\ of\ 1cm^3\ of\ the\ metal = \frac{1.5 \times 1000}{405}$$
$$= 3.703 \cong 3.7g\ (1dp)$$

Question 6(a)(i)

$$x = 28^o (Alternate\ Angles)$$

Question 6(a)(ii)

$$Angle\ y = 180 - \frac{180-28}{2}$$
$$Angle\ y = 104^o$$

Question 6(a)(iii)

$$z = 104^o\ (Vertically\ opposite\ and\ Corresponding\ Angles)$$

Question 6(b)(i)

J has coordinates (-4,1)

Question 6(b)(ii)

K'L' = 2cm

Question 6(b)(iii)

The translation is a reflection

Question 6(b)(iv)

J" = (1,-2)

K"=(4,-2)

L"=(4,0)

Question 7(a)

The number of seedlings is 85

Question 7(b)(i)

The lower class limit is 8

Question 7(b)(ii)

The upper class boundary is 12.5

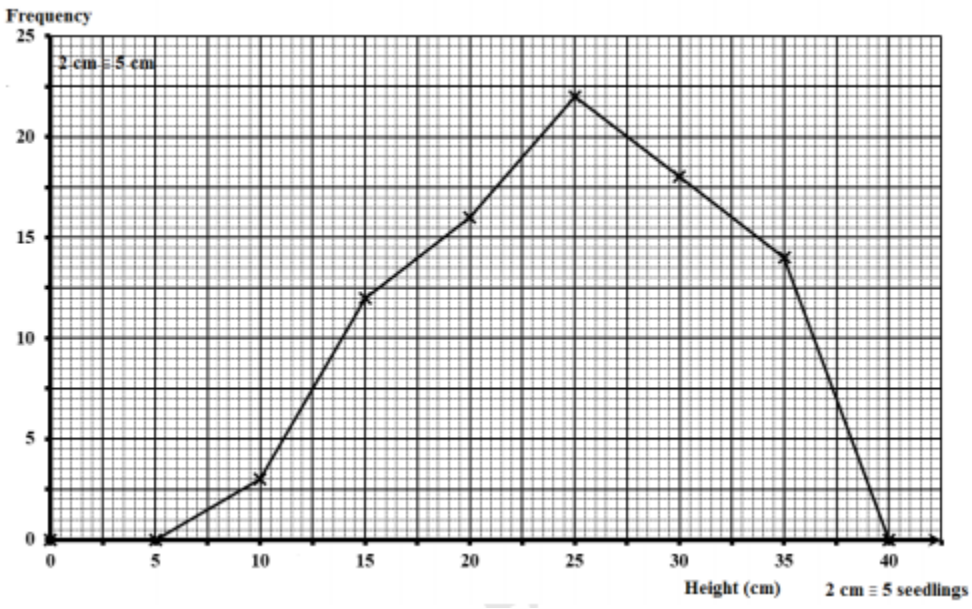
Question 7(b)(iii)

Class width $= 12.5\text{-}7.5 = 5$

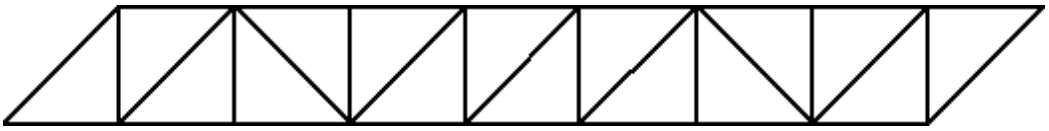
Question 7(c)

Height, x (cm) LCL-UCL	Lower Class Boundary LCB	Upper Class Boundary UCB	Midpoint or Mid-class Interval MCI	Frequency, f	Points to be plotted (MCI , Frequency)
					(0, 0)
3 – 7	$2.5 \le x < 7.5$		$\frac{2.5+7.5}{2} = 5$	0	(5, 0)
8 – 12	$7.5 \le x < 12.5$		$\frac{7.5+12.5}{2} = 10$	3	(10, 3)
13 – 17	$12.5 \le x < 17.5$		$\frac{12.5+17.5}{2} = 15$	12	(15, 12)
18 – 22	$17.5 \le x < 22.5$		$\frac{17.5+22.5}{2} = 20$	16	(20, 16)
23 – 27	$22.5 \le x < 27.5$		$\frac{22.5+27.5}{2} = 25$	22	(25, 22)
28 – 32	$27.5 \le x < 32.5$		$\frac{27.5+32.5}{2} = 30$	18	(30, 18)
33 – 37	$32.5 \le x < 37.5$		$\frac{32.5+37.5}{2} = 35$	14	(35, 14)
				$\sum f = 85$	(40, 0)

Question 7(d)



Question 8(a)



Question 8(b)

	No. of Trapezia (n)	**No. of Triangles**	**No. of Dots**
	n	$4n$	$4n+2$
(i)	4	$4(4)=16$	$4(4)+2=16+2=18$
(ii)	10	$4(10)=40$	$4(10)+2=40+2=42$
(ii)	$\frac{64}{4}=16$	64	$64+2=66$
(iv)	n	$4n$	$4n+2$

When n=1, number of triangles $=4\times1=4$ and the number of dots $=4+2=6$
When n=2, number of triangles $=4\times2=8$ and the number of dots $=8+2=10$

When n=3, number of triangles $=4\times3=12$ and the number of dots $=10+2=12$

Hence if the number of trapezia = n, then the number of triangles will be 4 times this $=4\times n=4n$ and the number of dots will therefore be 2 more than the number of trapezia $4n+2$

Question 9(a)(i)(a)

$$g(x) = 3x - 2$$
$$g(4) = 3(4) - 2$$
$$g(4) = 12 - 2$$
$$g(4) = 10$$

Question 9(a)(i)(b)

$$hg(4) = h(10)$$
$$\frac{10}{10} - 3$$
$$1 - 3$$
$$= -2$$

Question 9(a)(ii)(a)

$$Let\ y = h(x)$$
$$y = \frac{10}{x} - 3$$
$$y + 3 = \frac{10}{x}$$
$$x(y + 3) = 10$$
$$x = \frac{10}{y + 3}$$
$$h^{-1}(x) = \frac{10}{x + 3}, x \neq 3$$

Question 9(a)(ii)(b)

$$gg(x) = 3(3x - 2) - 2$$
$$= 9x - 6 - 2$$
$$= 9x - 8$$

Question 9(b)(i)

Roots are x=-1 and x=5

Question 9(b)(ii)(a)

$$if\ x = -1\ and\ x = 5\ are\ the\ roots\ of\ x^2 + bx + c = y, then$$
$$x^2 + bx + c = \big(x - (-1)\big)(x - 5)$$
$$= (x + 1)(x - 5)$$
$$= x^2 - 4x - 5$$
$$Hence\ c = -5$$

Question 9(b)(ii)(b)

From the above equation, b=-4

Question 9(b)(iii)

$$Minimum\ point\ occurs\ at\ \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$$

$$\left(\frac{-(-4)}{2(1)}, \frac{4(1)(-5)-(-4)^2}{4(1)}\right)$$

$$(2, -9)$$

Question 10(a)(i)

$$F\hat{A}W = 90 - 54 = 36^o$$

Question 10(a)(ii)

$$S\hat{K}F = 180 - 54 = 126^o$$

(Opposite angles of a cyclic quadrilateral are supplementary)

Question10(a)(iii)

Opposite sides SK and AF are parallel
Angles KSA and SAF are co-interior opposite angles, hence they are supplementary

Therefore,

$$54 + 62 + < ASW = 180^o$$
$$< ASW = 64^o$$

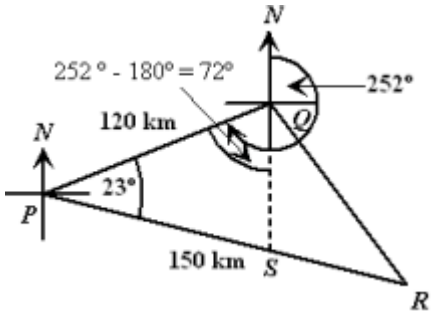
Question 10(b)(i)(a)

$$QR^2 = (120)^2 + (150)^2 - 2(120)(150)cos23$$
$$QR^2 = 3761.83$$
$$QR = 61.33$$
$$\cong 61.3\ (1dp)$$

Question 10(b)(i)(b)

$$Area\ PQR = \frac{1}{2}(120)(150)(sin23)$$
$$= 3516.58$$
$$= 3516.6km^2\,(1dp)$$

Question 10(b)(ii)



$$< PQS = 252 - 180$$
$$< PQS = 72^o$$
$$< NPR = 72 + 23 = 95^o$$
$$Hence\ the\ bearing\ of\ R\ from\ P\ is\ 095^o$$

Question 11(a)(i)

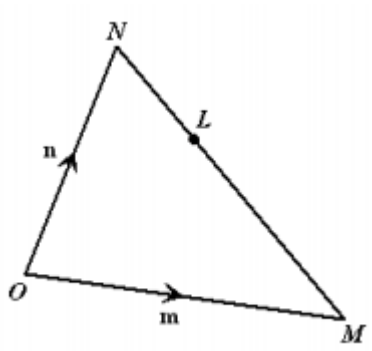
$$\det T = (2 \times 3) - (-1 \times 1)$$
$$= 6 - (-1)$$
$$= 7$$

Question 11(a)(ii)

If (a,b) is mapped onto (4,9) under the translation T, then (4,9) would be mapped onto (a,b) under the translation T^{-1}, therefore we multiply the point (4,9) by the inverse of T to yield the values of a and b respectfully.

$$\begin{pmatrix} \frac{3}{7} & \frac{1}{7} \\ -\frac{1}{7} & \frac{2}{7} \end{pmatrix} \begin{pmatrix} 4 \\ 9 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$$
$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{3}{7} \times 4 & \frac{1}{7} \times 9 \\ -\frac{1}{7} \times 4 & \frac{2}{7} \times 9 \end{pmatrix}$$
$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$
$$a = 3; b = 2$$

Question 11(b)(i)



Question 11(b)(ii)(a)

$$\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{ON}$$
$$\overrightarrow{MN} = -(m) + n$$
$$\overrightarrow{MN} = -m + n$$

Question 11(b)(ii)(b)

$$ML = \frac{2}{3} MN$$
$$\overrightarrow{ML} = -\frac{2}{3} m + \frac{2}{3} n$$

Question 11(b)(iii)

$$\overrightarrow{OL} = \overrightarrow{OM} + \overrightarrow{ML}$$
$$m + \left(-\frac{2}{3} m + \frac{2}{3} n\right)$$

$$\frac{1}{3}m + \frac{2}{3}n$$
$$\frac{1}{3}\begin{pmatrix} 3 \\ 6 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} 9 \\ 0 \end{pmatrix}$$
$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$
$$\begin{pmatrix} 7 \\ 2 \end{pmatrix}$$
$$\overrightarrow{OL} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$$