Solutions to CSEC Maths P2 January 2015

Question 1a

Using a calculator, or otherwise, calculate the EXACT value of

$(12.8)^2 - (30 \div 0.375)$ [3 marks]

Using a calculator:

$$(12.8)^2 - (30 \div 0.375)$$

$$163.84 - 80 = \underline{83.84}$$

Question 1b part (i)

Mark spends 3/8 of his monthly income on housing. Of the REMAINDER, he spends 1 3 on food and saves what is left.

Calculate the fraction of his monthly income spent on food. [2 marks]

$$\frac{1}{3} \times [1 - \frac{3}{8}]$$

$$\frac{1}{3} \times \frac{5}{8} = \frac{5}{24}$$

Question 1b part (ii)

Calculate the fraction of his monthly income that he saved. [2 marks]

$$1 - \left(\frac{3}{8} + \frac{3}{24}\right)$$

$$1 - \left(\frac{(3 \times 3) + 5}{24}\right)$$

$$1-\left(\frac{14}{24}\right)$$

$$\frac{24}{24}-\frac{14}{24}=\frac{10}{24}$$

$$\frac{10}{24}=\frac{5}{12}$$

Question 1c part (i)

At Bank A, US$1.00=BD$ 1.96. Calculate the value of US$700 in BD$. [2 marks]

US$1.00=BD$ 1.96

US$700 = 700 × 1.96

US$700 = BD$ 1,372

Question 1c part (ii)

At Bank B, the value of US$700 is BD$ 1 386.

Calculate the value of US$1.00 in BD$ at this bank. [2 marks]

1,386 ÷ 700 = BD$ 1.98

US$1.00=BD$ 1.98 at Bank B

Total 11 marks

Question 2a

Simplify $p^{3}q^{2} \times pq^{5}$ [2 marks]

$$p^{3+1} \times q^{2+5}$$

$$p^{4}q^{7}$$

Question 2b

Express as a single fraction in its simplest form: [2 marks]

$$\frac{a}{5} + \frac{3a}{2}$$

$$\frac{(2 \times a) + (5 \times 3a)}{10}$$

$$\frac{17a}{10}$$

Question 2c part (i)

Factorize completely:

$x^{2} - 5x + 4$ [2 marks]

$$x^{2} - 5x + 4$$

$$x^{2} - x - 4x + 4$$

$$x(x-1) - 4(x-1)$$

$$(x-1)(x-4)$$

Question 2c part (ii)

$m^{2} - 4n^{2}$ [2 marks]

Difference of 2 Squares:

(m+2n)(m-2n)

Question 2d part (i)

Solve for x

$2x - 7 \leq 3$ [1 mark]

$$2x - 7 \leq 3$$
$$2x \leq 3 + 7$$
$$2x \leq 10$$
$$x \leq \frac{10}{2}$$
$$x \leq 5$$

Question 2d part (ii)

If x is a positive integer, list the possible values of x. [1 mark]

Possible values of x = 1, 2, 3, 4, 5

Question 2e

Find the value of $2\pi\sqrt{\frac{l}{g}}$ where $\pi = 3.14$, $l = 0.625$ and g = 10. [2 marks]

$$2\pi\sqrt{\frac{l}{g}}$$
$$2(3.14)\sqrt{\frac{0.625}{10}}$$
$$6.28 \times 0.25$$
$$= 1.5$$

Question 3a part (i)

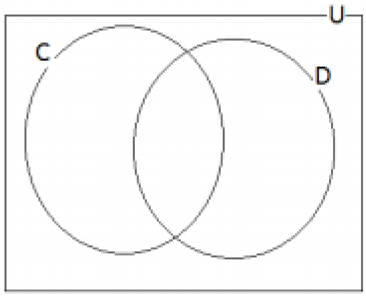
In a survey of 30 families, the findings were that: 15 families owned dogs 12 families owned cats x families owned BOTH dogs and cats 8 families owned NEITHER dogs NOR cats (i) Given that:

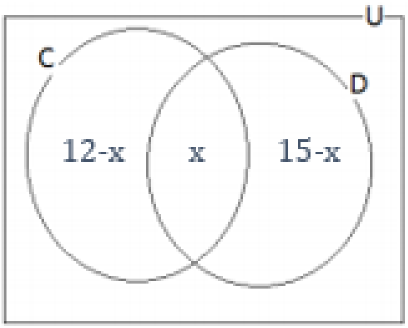
U = {families in the survey}

C = {families who owned cats}

D = {families who owned dogs}

Use the given information copy and complete the Venn diagram below. [4 marks]





Question 3a part (ii)

Write an expression, in x, which represents the TOTAL number of families in the survey.

[1 mark]

$$12 - x + x + 15 - x + 8$$

$$35 - x$$

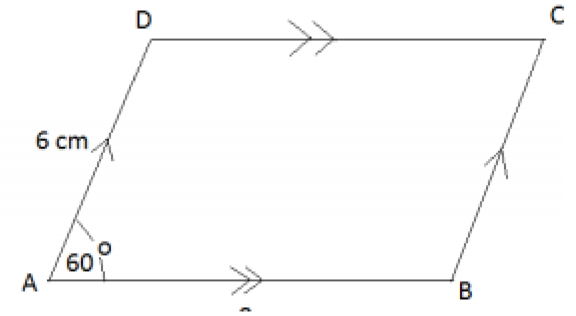
Question 3a part (iii)

Write an equation which may be used to solve for x. [1 mark]

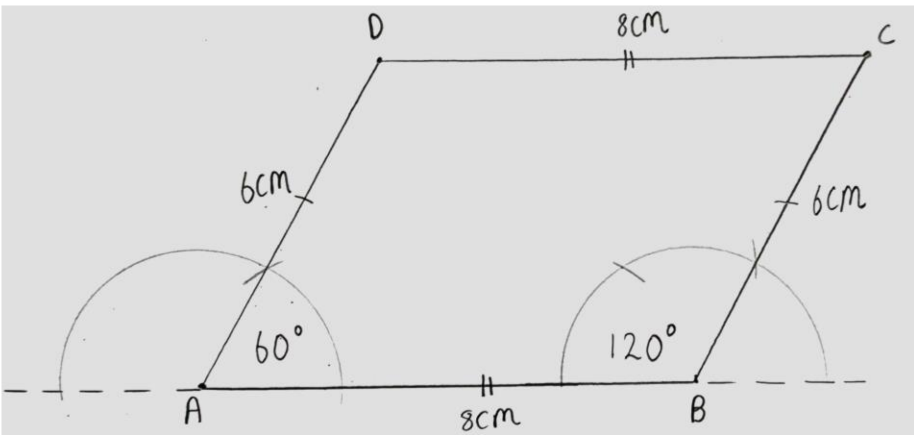
$30 = 35 - x$

Question 3b

The diagram below, not drawn to scale, shows parallelogram ABCD.



Using a ruler, a pencil and a pair of compasses only, construct a parallelogram ABCD with AB = 8 cm, AD = 6 cm and $< DAB = 60o$. Marks will be awarded for construction lines clearly shown. [6 marks]



Question 4a

An electrician charges a fixed fee for a house visit plus an additional charge based on the length of time spent on the job.

The total charges, y, are calculated using the equation $y = 40x + 75$, where x represents the time in hours spent on the job.

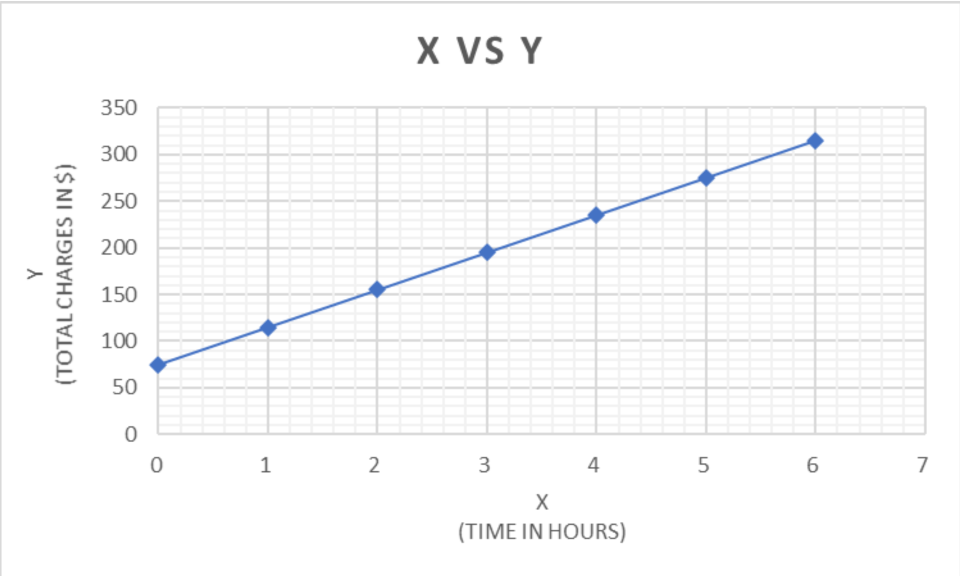
Complete the table of values for the equation $y = 40x + 75$.

x **(time in hours)**	0	1	2	3	4	5	6
y **(total charges in $)**	75	115		195		275	315

x **(time in hours)**	0	1	2	3	4	5	6
y **(total charges in $)**	75	115	155	195	235	275	315

Question 4b

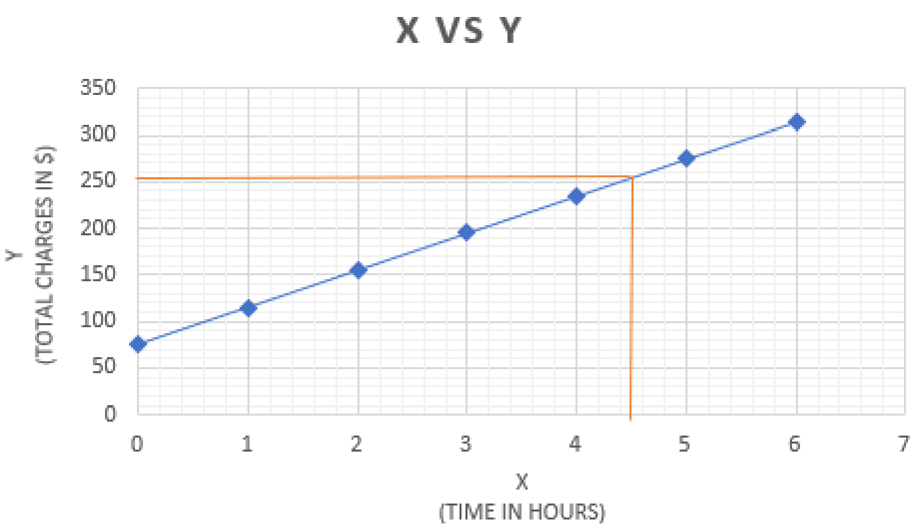
On graph paper, using a scale of 2cm to represent one hour on the x-axis and 2cm to represent 50 dollars on the y-axis, plot the 7 pairs of values shown in your completed table. Draw a straight line through all plotted points. [5 marks]



Question 4c part (i)

Using your graph, determine,

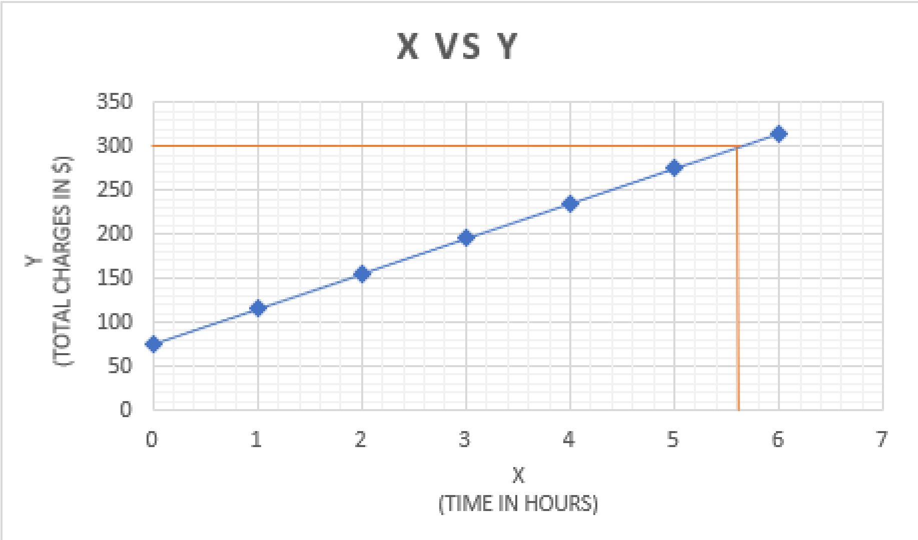
the total charges when the job took 4.5 hours. [2 marks]



When x =4.5, y = $255

Question 4c part (ii)

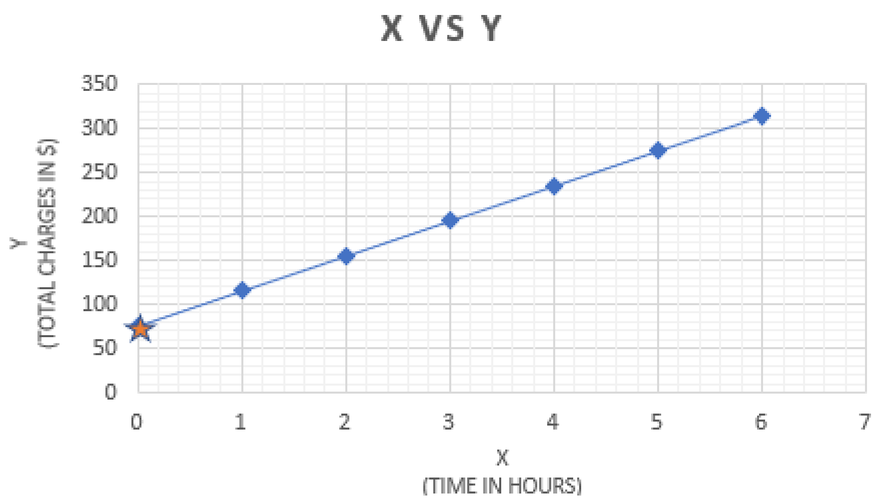
the time, in hours, spent on a job if the total charges were $300. [2 marks]



When $y = \$\,300, x = 5.6$ hrs

Question 4c part (iii)

the fixed charges for a visit. [1 mark]

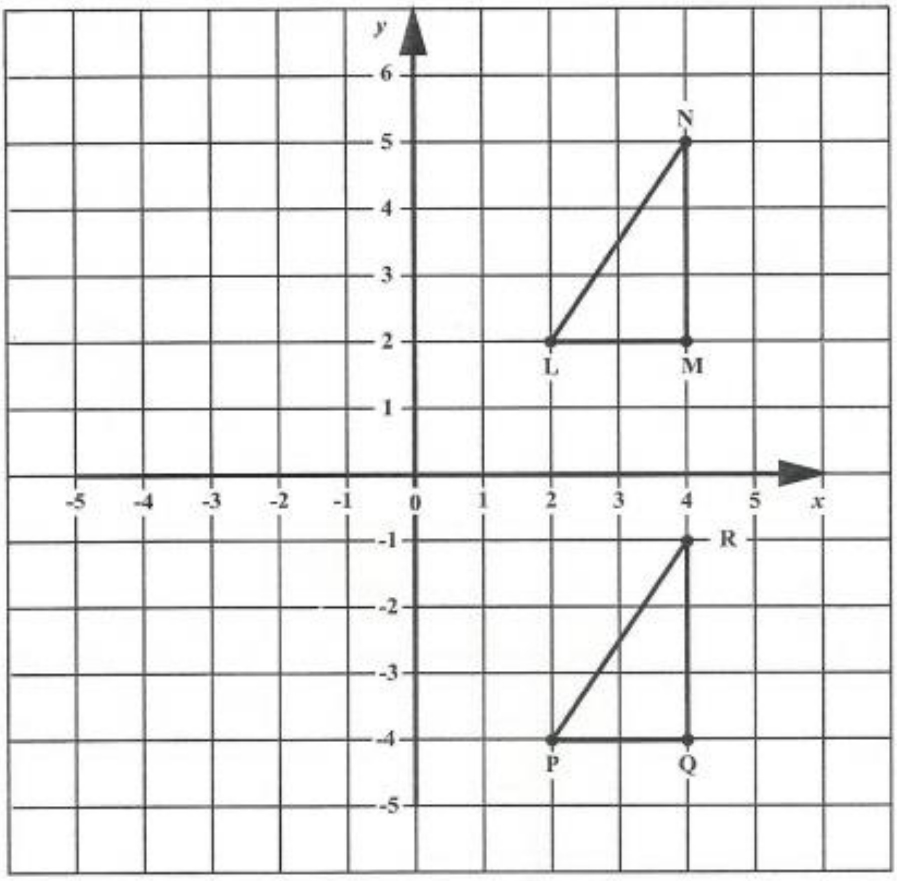


Fixed charges = when $x = 0$

When $x = 0, y = \$75$

Question 5 part(i)

5. The diagram below shows $\Delta\, LMN$ and its image $\Delta\, PQR$ after a transformation.



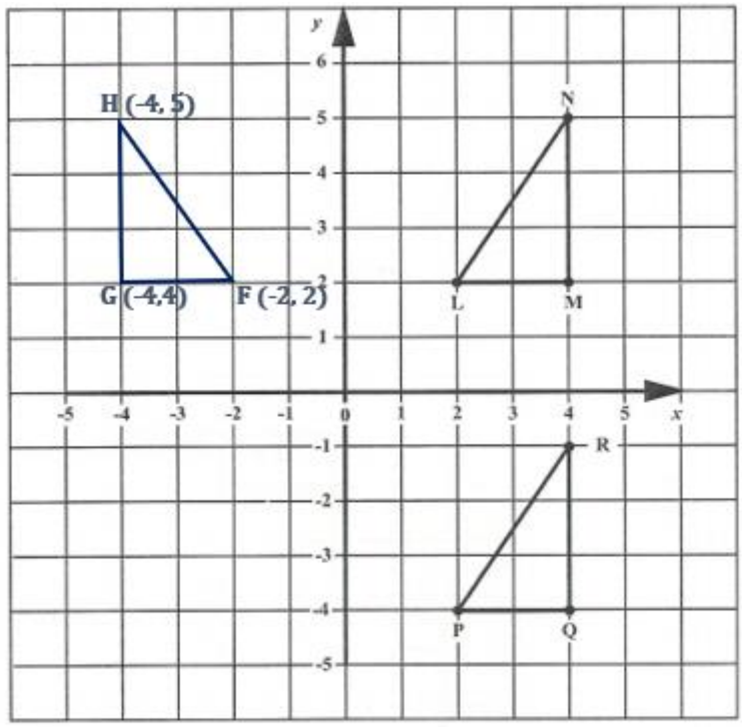
(i) Write down the coordinates of N. .... **(1 mark)**

N (4, 5)

Question 5 part(ii)

(ii) On the grid above, draw $\Delta\, FGH$, the reflection of $\Delta\, LMN$ in the y-axis.

(4 marks)



Question 5 part(iii)

Using vector notation, describe the transformation which maps Δ LMN onto ΔPQR

(2 marks)

$$P\begin{pmatrix}2\\-4\end{pmatrix} - L\begin{pmatrix}2\\2\end{pmatrix} = \begin{pmatrix}0\\-6\end{pmatrix} \quad Q\begin{pmatrix}4\\-4\end{pmatrix} - M\begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}0\\-6\end{pmatrix} \quad R\begin{pmatrix}4\\-1\end{pmatrix} - N\begin{pmatrix}4\\5\end{pmatrix} = \begin{pmatrix}0\\-6\end{pmatrix}$$

Question 5 part(iv)

Complete the following statement:

Δ PQR is mapped onto ΔFGH by a combination of two transformations. First, Δ PQR is mapped onto Δ LMN by a __________, parallel to the __________; then ΔFGH by a __________, in the ______________.

(3 marks)

Δ PQR is mapped onto Δ FGH by a combination of two transformations. First, Δ PQR is mapped onto Δ LMN by a translation of +6 units, parallel to the y-axis; then Δ FGH by a reflection, in the y-axis

Question 5 part(v)

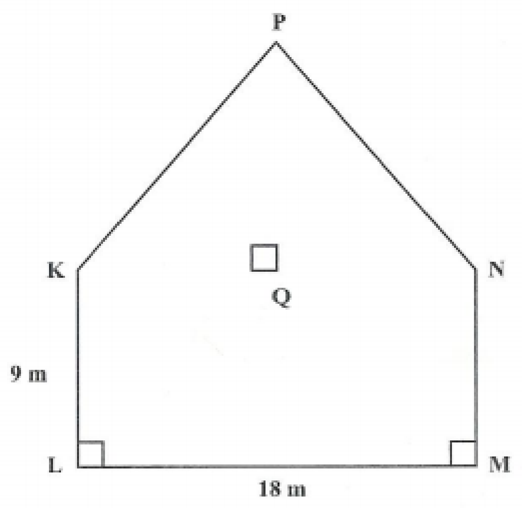
Δ PQR and Δ FGH are congruent. State two reasons why they are congruent.

(2 marks)

1. The corresponding sides of each triangle have the same length
2. The corresponding angles of each triangle are the same.

Question 6a part(i)

The diagram below is a scale drawing of the side view of a building. Q is the midpoint of KN, and <KLM = <LMN = 90°



Measure and state the length of PQ on the drawing.

(1 mark)

PQ = 3.5 cm

Question 6a part(ii)

Determine the scale of the drawing.

(2 marks)

$$3\text{cm} = 9\text{m}$$

Changing to cm throughout:

$$3\text{cm} = 900\text{cm}$$

$$1\text{cm} = \frac{900}{3}\,cm$$

$$1\text{cm} = 300\text{cm}$$

$$\text{Scale} = 1{:}300$$

Question 6a part(ii)

Calculate the actual area of LMNPK of the building.

(4 marks)

This is the sum of the area of Triangle PKN + Rectangle KLMN

$$\text{Area of Triangle PN} = \frac{bh}{2}$$

$$\text{h} = 3.5 \text{ x } 300 = 1050 \text{ cm} = 10.5\text{m}$$

$$\text{b} = 18\text{m}$$

$$\text{Area of Triangle PN} = \frac{18 \times 10.5}{2}$$

$$= 94.5 \text{ m}^2$$

$$\text{Area of Rectangle KLMN} = \text{l} \times \text{b}$$

$$\text{l} = 18\text{m}$$

$$\text{b} = 9\text{m}$$

$$\text{Area of Rectangle KLMN} = 18 \times 9$$

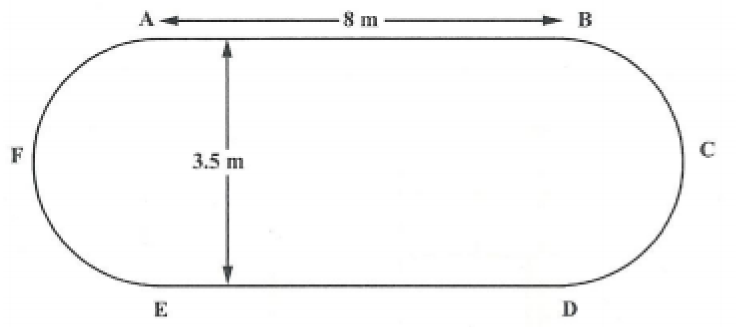
$$= 162 \text{ m}^2$$

$$\text{Area of Triangle PKN + Rectangle KLMN} = 94.5 + 162$$

$$\text{Area of LMNPK} = 256.5\text{m}^2$$

Question 6b part(i)

The diagram below, not drawn to scale, shows the plan of a swimming pool in the shape of a rectangle and two semicircles. The rectangle has dimensions of 8 metres by 3.5 metres. Use $\pi = \frac{22}{7}$



State the length of the diameter of the semicircle, AFE.

(1 mark)

$$\text{Diameter of AFE} = 3.5\text{m}$$

Question 6b part(ii)

Calculate the perimeter of the swimming pool.

(3 marks)

$$\text{Perimeter} = 8\text{m} + 8\text{m} + (\pi d)$$

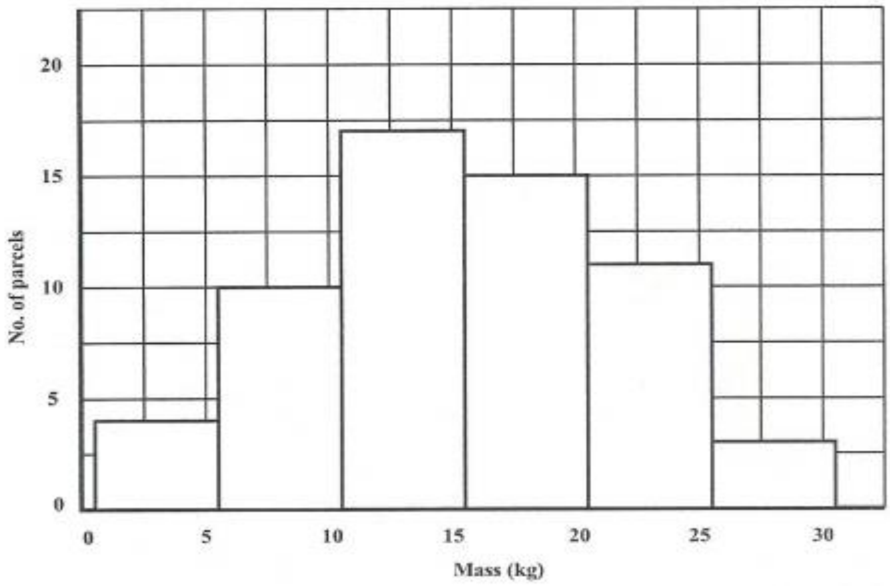
$$\text{Perimeter} = 16\text{m} + \left(\frac{22}{7} \times 3.5\right)$$

$$= 16\text{m} + 11\text{m}$$

$$\text{Perimeter} = 27\text{m}$$

Question 7a part(i)

The masses of 60 parcels collected at a post office were grouped and recorded as shown in the histogram below.



Complete the table below to show the information given in the histogram.

(2 marks)

Mass (kg)	No. of Parcels	Cumulative Frequency
1-5	4	4
6-10	10	14
11-15	17	31
16-20	15	46
21-25	11	
26-30	3	60

Question 7a part(ii)

Complete the column headed "Cumulative Frequency".

(1 mark)

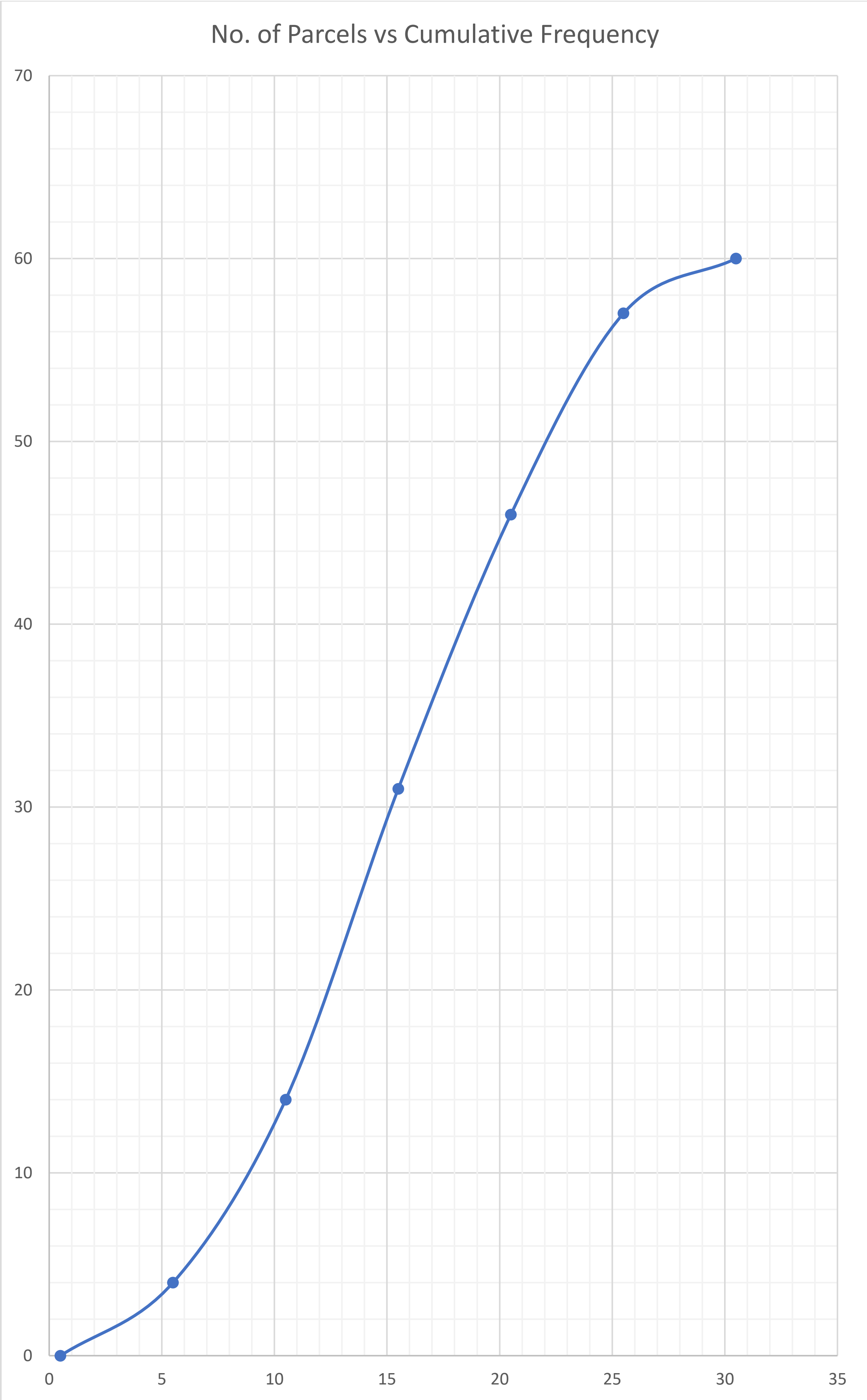
Mass (kg)	No. of Parcels	Cumulative Frequency
1-5	4	4
6-10	10	14
11-15	17	31
16-20	15	46
21-25	11	57
26-30	3	60

Question 7b

On the grid provided, using a scale of 2cm to represent 5kg on the x axis, and 2cm to represent 10 parcels on the y-axis, draw the cumulative frequency curve for the data.

(5 marks)

Mass (kg)	No. of Parcels	Cumulative Frequency	Upper Class Limit
0	0	0	0.5
1-5	4	4	5.5
6-10	10	14	10.5
11-15	17	31	15.5
16-20	15	46	20.5
21-25	11	57	25.5
26-30	3	60	30.5

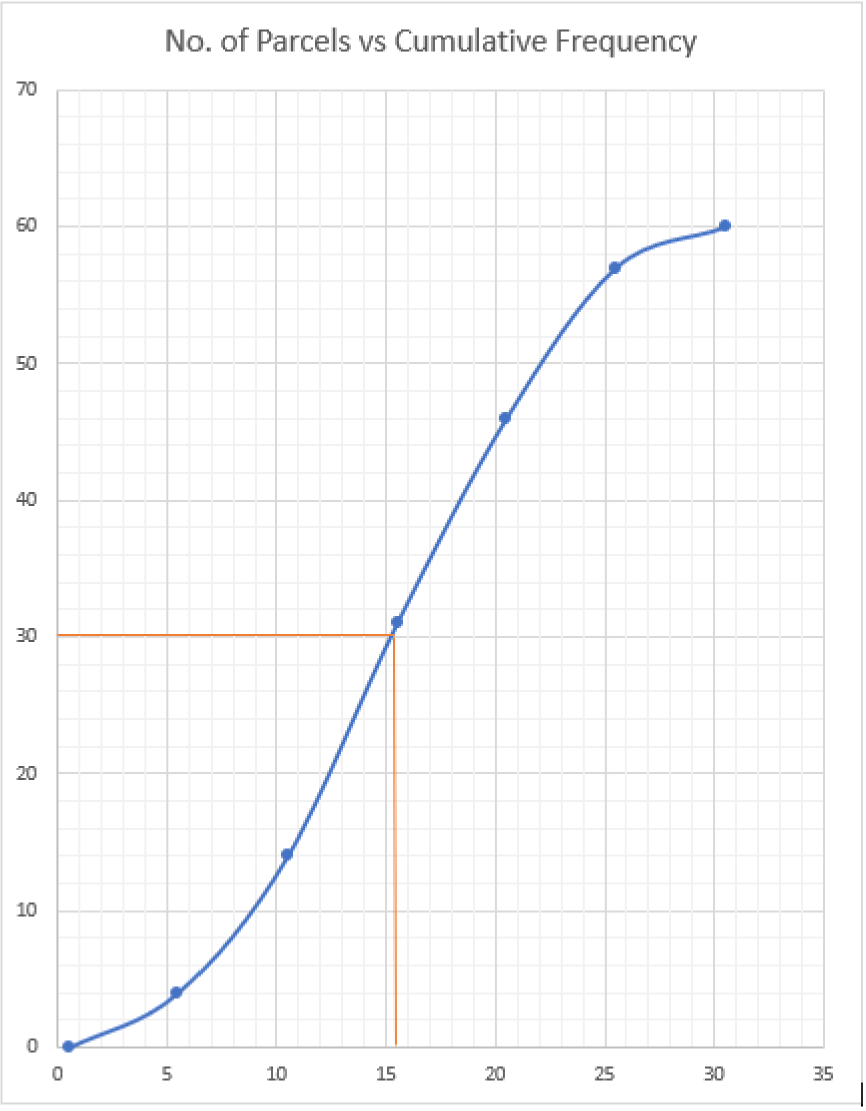

No. of Parcels vs Cumulative Frequency
70
60
50
40
30
20
10
0
0
5
10
15
20
25
30
35

Question 7c

Use the graph drawn at (b) to estimate the median mass of the parcels.

[2 marks]

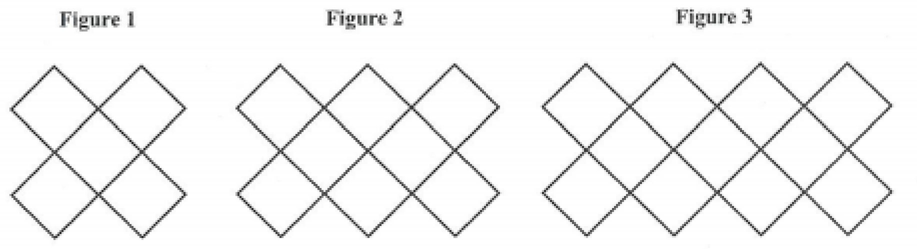
$$\text{Median} = \frac{60}{2} = 30 \text{ parcels}$$



$$\text{Median mass} = 15.5\text{kg}$$

Question 8a

The diagram below shows the first three figures in a sequence of figures.



Draw the fourth figure in the sequence.

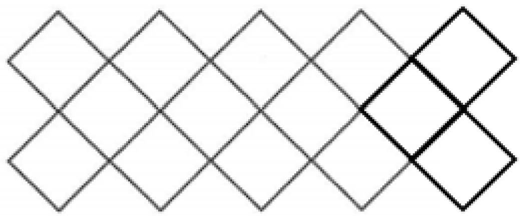


Figure 4

Question 8b

The table below shows the number of squares in each figure. Study the pattern in the table and complete the table by inserting the missing values in the rows numbered (i), (ii), (iii) and (iv).

	Figure (n)	No. of Squares	
	1	5	
	2	8	
	3	11	
(i)	4	14	(1 mark)
(ii)	10	32	(2 marks)
(iii)	16	50	(2 marks)
(iv)	n	3n +2	(3 marks)

Question 9a part (i)

The functions f (x) and g (x) are defined as: $f(x) = \frac{5x-4}{3}$ $g(x) = x^2 - 1$

(i) Evaluate f (7).

[1 mark]

$$f(x) = \frac{5x-4}{3}$$

$$f(7) = \frac{5(7)-4}{3}$$

$$= \frac{31}{3} = 10\frac{1}{3}$$

Question 9a part (ii)

Write an expression, in terms of x, for $f^{-1}(x)$.

[2 marks]

$$f(x) = \frac{5x-4}{3}$$

$$\text{let } y = \frac{5x-4}{3}$$

$$\frac{3y+4}{5} = x$$

$$\text{Now, } f^{-1}(x) = \frac{3x+4}{5}$$

Question 9a part (iii)

Write an expression, in terms x, for f g (x).

[2 marks]

$$f\,g\,(x) = f\,(x^2 - 1)$$

$$\frac{5(x^2 - 1) - 4}{3}$$

$$\frac{5x^2 - 9}{3}$$

Question 9b part (i)

Express the quadratic function $f(x) = 3x^2 + 6x - 2$, in the form $a\,(x + h)^2 + k$, where a, h and k are constants.

[3 marks]

$$f(x) = 3x^2 + 6x - 2$$

$$f(x) = 3(x+1)^2 + k$$
$$= 3x^2 + 6x + 3 + k$$
$$3x^2 + 6x + 3 - 5 = 3x^2 + 6x - 2$$
$$f(x) = 3(x+1)^2 - 5$$

Question 9b part (ii)

Hence, or otherwise, state the minimum value of $f(x) = 3x^2 + 6x - 2$. [1 mark]

When put in the form $a(x+h)^2 + k$, k exhibits the minimum or maximum value.

So, the minimum value = -5

Question 9b part (iii)

State the equation of the axis of symmetry of the function $f(x) = 3x\ 2 + 6x - 2$.

[2 marks]

The equation of the axis of symmetry is the line which cuts the curve in half.

This is the vertical line that runs through the minimum point.

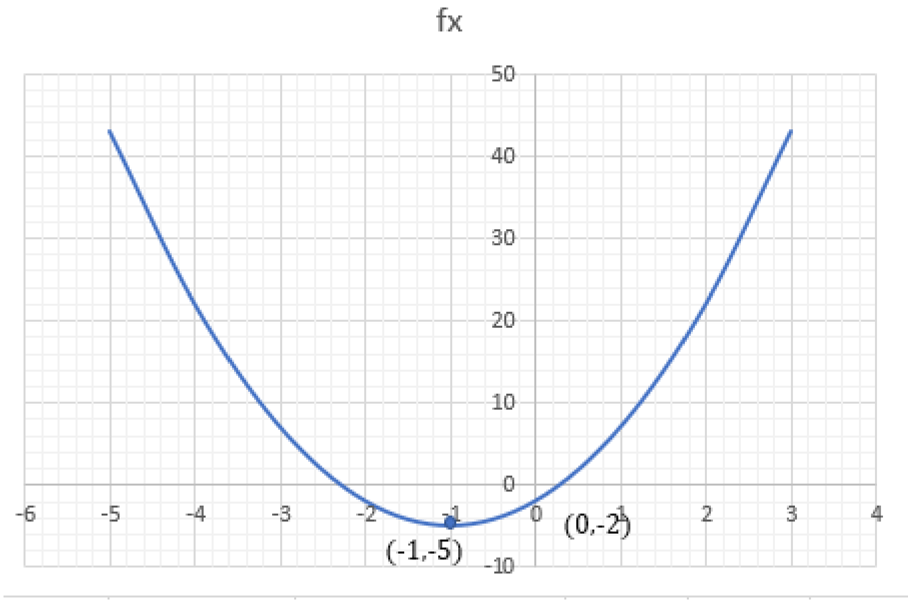
When y = -5:

$$-5 = 3(x+1)^2 - 5$$
$$0 = 3(x+1)^2$$
$$0 = (x+1)^2$$
$$0 = x+1$$
$$x = -1$$

The equation of the axis of symmetry: x = -1

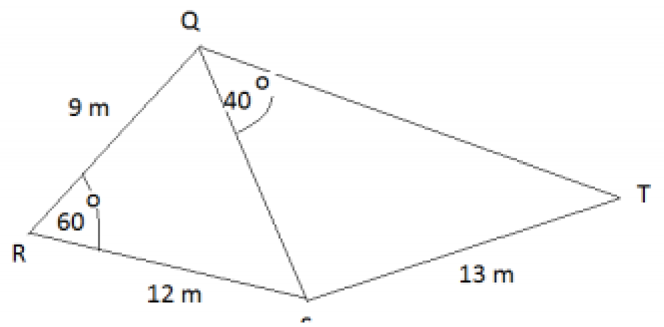
Question 9b part (iv)

Sketch the graph of $y = 3x\ 2 + 6x - 2$, showing on your sketch (a) the intercept on the y-axis (b) the coordinates of the minimum point. [4 marks]



Question 10a part (i)

On the diagram below, not drawn to scale, RQ = 9 m, RS = 12 m, ST = 13 m, $< QRS = 60^o$ and $< SQT = 40^o$.



Calculate, correct to 1 decimal place,

the length QS

[2 marks]

Using the cosine rule

$QS^2 = 9^2 + 12^2 + 2(9)(12)\ Cos60^o$

$= 10.8$ (1 dp)

Question 10a part (ii)

the measure of $< QTS$

[2 marks]

Using the sine rule

$$\frac{10.8}{sinT} = \frac{13}{sin40^o}$$

$$\text{So, } \frac{sin sin\ T}{10.8} = \frac{sin40^o}{13}$$

$$\text{sinT} = \frac{sin40^o}{13} \times 10.8$$

$T = sin^{-1}0.534$

$T = 32.5^o$

Question 10a part (iii)

the area of triangle QRS

[2 marks]

$$\text{Area} = \frac{1}{2}\ side\ \times side\ \times sin\ sin\ (angle)$$

$$\text{Area} = \frac{1}{2}\ 9 \times 2 \times sin60$$

$= 46.765\ m^2$

Question 10a part (iv)

the perpendicular distance from Q to RS.

[1 mark]

$$\sin\theta = \frac{opposite}{hypothenuse}$$

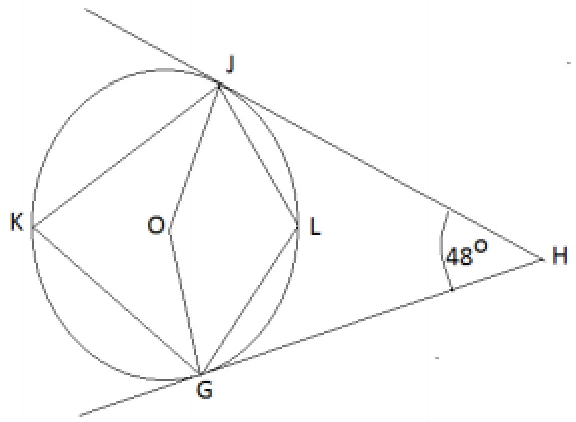
$$\sin 60 = \frac{opposite}{9}$$

$$\text{opposite} = 9\sin 60$$

$$\text{Perpendicular distance} = 9\sin 60 = 7.8\text{cm}$$

Question 10b part (i)

The diagram below, not drawn to scale, shows a circle with centre O. HJ and HG are tangents to the circle and $< JHG = 48^o$.



Calculate , giving the reason for each step of your answer, the measure of:

$< OJH$

[2 marks]

This is the angle between the tangent and the radius.

$$<\text{OJH} = 90^o$$

Question 10b part (ii)

$< JOG$

[2 marks]

The opposite angles are supplementary.

$180^{o} - 48^{o} = 132^{o}$

Question 10b part (iii)

$< JKG$

[2 marks]

The angle at the centre of the circle is twice the size of the angle at the circumference.

$$< JKG = \frac{132}{2} = 66°$$

Question 10b part (iv)

$< JLG$

[2 marks]

The opposite angles of a cyclic quadrilateral are supplementary.

$180^{o} - 66^{o} = 114^{o}$

Question 11a part (i)

Write the following equations

$3x + 2y = -1$

$5x + 4y = 6$

in the form AX =B, where A, X and B are matrices.

[2 marks]

$3x + 2y = -1$

$5x + 4y = 6$

$$= (3\ 2\ 5\ 4)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 6 \end{pmatrix} \text{ (Form AX = B)}$$

Question 11a part (ii)

Use a matrix method to solve for x and y.

[4 marks]

$$\text{If } AX = B$$

$$X = A^{-1}B$$

$$A^{-1} = \frac{1}{|A|} \times adjA$$

$$= \frac{1}{(3\times4)-(2\times5)} \times (4\ \ -2\ \ -5\ 3\ )$$

$$= \frac{1}{2}(4\ \ -2\ \ -5\ 3\ )$$

$$X = \frac{1}{2}(4\ \ -2\ \ -5\ 3\ )\begin{pmatrix} -1 \\ 6 \end{pmatrix}$$

$$= \frac{1}{2}(4 \times -1\ +\ -2 \times 6\ \ -5 \times -1\ \ +\ 3 \times 6\ )$$

$$= \frac{1}{2}(-4\ \ +\ -12\ 5\ \ \ \ +\ 18\ )$$

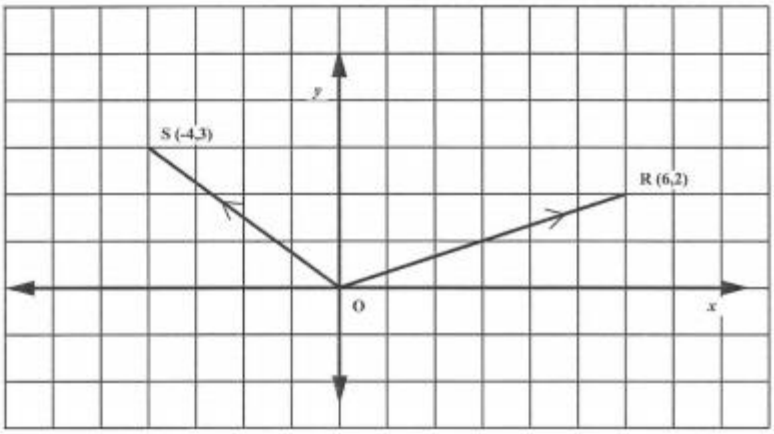
$$= \frac{1}{2}\begin{pmatrix} -16 \\ 23 \end{pmatrix}$$

$$= \begin{pmatrix} -8 \\ 11.5 \end{pmatrix}$$

x = -8, y= 11.5

Question 11b part (i)

The diagram below shows two position vectors $\overrightarrow{OR}$ and $\overrightarrow{OS}$ such that R (6,2) and S (-4,3)



Write as a column vector in the form $\binom{x}{y}$:

$\overrightarrow{OR}$

[1 mark]

$$\binom{6}{2}$$

Question 11b part (ii)

$\overrightarrow{OS}$

[1 mark]

$$\binom{-4}{3}$$

Question 11b part (iii)

$\overrightarrow{SR}$

[2 marks]

$$\overrightarrow{SR} = \overrightarrow{SO} + \overrightarrow{OR}$$

$$= \binom{4}{-3} + \binom{6}{2}$$

$$= \binom{10}{-1}$$

Question 11b part (iv)

Find $|\overrightarrow{OS}|$

[1 mark]

$$|\overrightarrow{OS}| = \sqrt{(-4)^2 + 3^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25} = 5 \text{ units}$$

Question 11b part (v)

Given that $OT = \binom{2}{5}$, prove that OSTR is a parallelogram.

[4 marks]

$$OT = \binom{2}{5}$$

If OSTR is in fact a parallelogram then OS and RT would be parallel and equal in magnitude, and so will OR and ST

$$\mathrm{RT} = \mathrm{RO} + \mathrm{OT}$$
$$= \binom{-6}{-2} + \binom{2}{5}$$
$$= \binom{-4}{3}$$

This proves RT and OS are parallel and equal

$$\mathrm{ST} = \mathrm{SO} + \mathrm{OT}$$
$$= \binom{4}{-3} + \binom{2}{5}$$
$$= \binom{6}{2}$$

This proves ST and OR are parallel and equal

Hence, OSTR is a parallelogram.