Solutions to CSEC Maths P2 January 2016

Question 1a

Using a calculator, $(3.6 + \sqrt{51.84}) \div 3.75 = 2.88$

Question 1b

In Jar A, 150g of peanut butter $= \$2.14$

$150\text{g} = \$2.14$

$1\text{g} = \frac{2.14}{150} = \$0.014$

Jar B, 400g of peanut butter $= \$6.50$

$1\text{g} = \frac{6.50}{400} = \$0.0016$

Jar A is cheaper per gram, hence is the better buy.

Question 1c part (i)

Principal $= \$1498$ Rate $= 6\%$ Time $= \frac{6}{12} = \frac{1}{2}$ year

$$\text{Simple Interest} = \frac{Principal \times Rate \times Time}{100} = \frac{1498 \times 6 \times \frac{1}{2}}{100} = \$44.94$$

Question 1c part (ii)

Interest after 3 years $= \frac{1498 \times 6 \times 3}{100} = \$269.64$

Total amount $=$ Principal $+$ Interest Earned

Total amount $= \$1498 + \$269.94$

$= \$1767.67$

Question 1c part (iii)

$\$449.40 = \frac{1498 \times 6 \times T}{100}$

$$T = \frac{449.40 \times 100}{1498 \times 6} = 5 \text{ years}$$

Question 2a part (i)

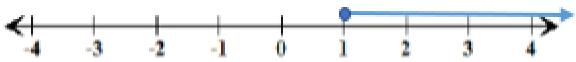
$8 - x \leq 5x + 2$

$8 - 2 \leq 5x + x$

$6 \leq 6x$

$x \geq 1$

Question 2a part (ii)



Question 2b

$2x\ (x + 5) - 3\ (x\text{-}4) = 2x^2 + 10x - 3x + 12$

$= 2x^2 + 7x + 12$

Question 2c

$\frac{3x^2 \times 4x^3}{2x} = \frac{12x^{2+3}}{2x} = 6x^4$

Question 2d

$\frac{x+1}{2} + \frac{5-x}{5} = \frac{5(x+1)+2(5-x)}{10}$

$= \frac{5x+5+10-2x}{10}$

$= \frac{3x+15}{10} = \frac{3(x+5)}{10}$

Question 2e

$4x^2\text{-}4 = (2x - 2)\ (2x + 2)$

$= 2\ (x\ \text{-}1)\ (x\ +1)$

(Difference of two squares)

Question 3a part (i)

Students who visited only Dominica = 10 students

Question 3a part (ii)

Total number of students who visited Canada = 3 + x

Question 3a part (iii)

Total number of students in form = 25

$$3 + x + 10 + 2x = 25$$

$$3x = 12$$

$$x = 4 \text{ students}$$

Question 3a part (v)

$C \cup D = 3 + 4 + 10 = 17$ students

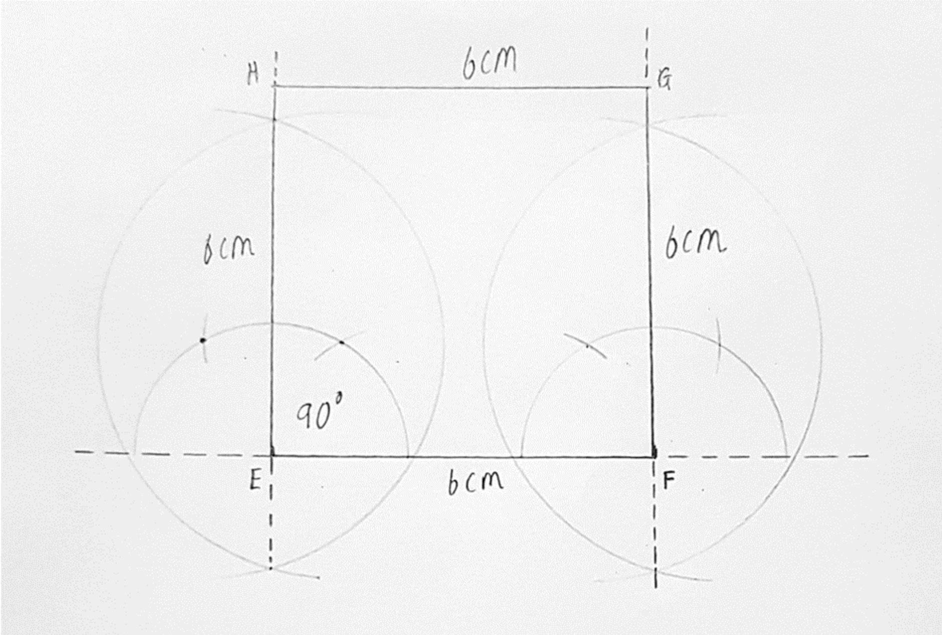
$C \cap D = x = 4$ students

$(C \cup D)' = 25 - (C \cup D)$

$= 25 - 17$

$= 8$ students

Question 3b part (i)



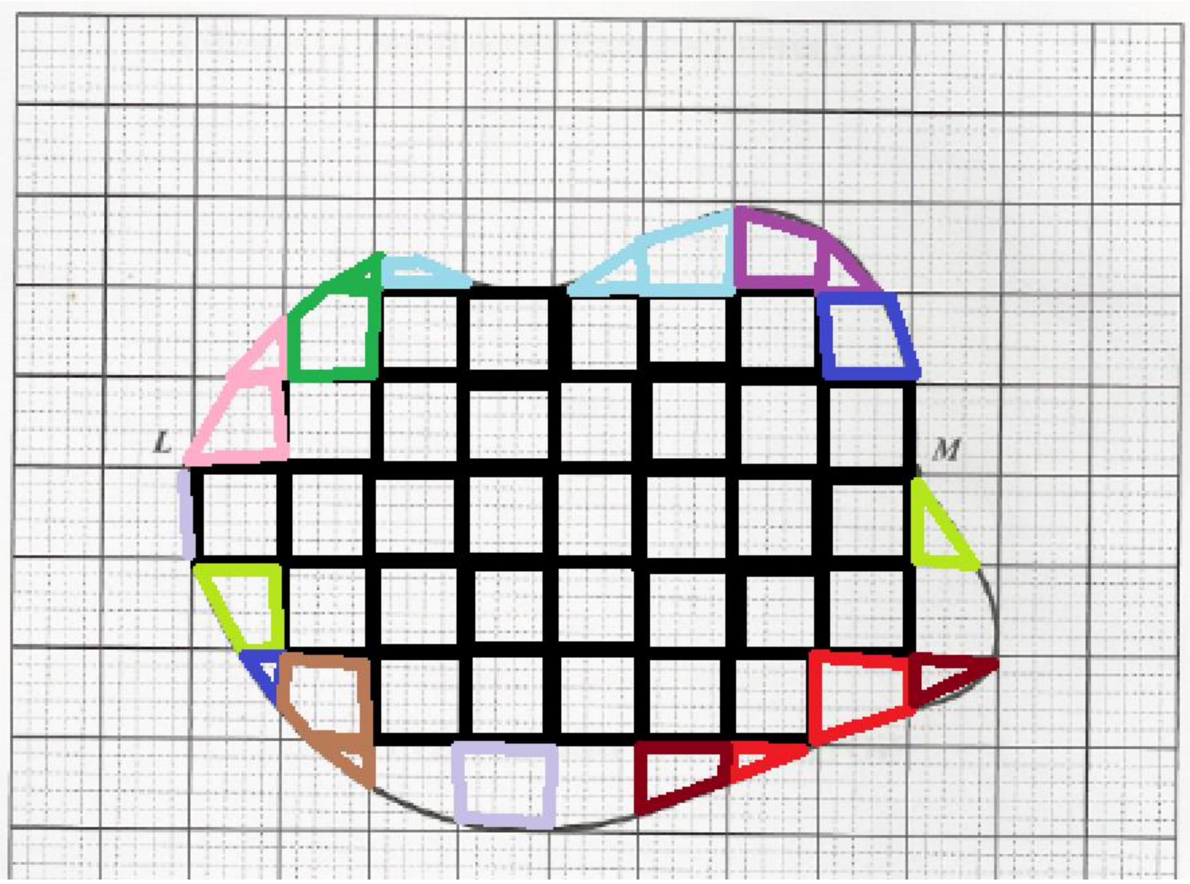
Question 3b part (ii)

FH = 8.5 cm

Question 4a part (i)

LM = 8cm

Question 4a part (ii)



Total no. of squares = 45

Area= $45 \times 1\ cm^2 = 45\ cm^2$

Question 4a part (iii)

LM = 20km

$8\text{cm} = 20\text{km}$

$1 \text{ cm} = \frac{20}{8} = 2.5 \text{ km}$

Question 4a part (iv)

$1\text{cm} = 2.5 \text{ km}$

$2.5 \text{ km} = (2.5 \times 100{,}000) \text{ cm}$

$x = 250{,}000 \text{ cm}$

Scale: 1: 250,000

Question 4a part (v)

$3\text{cm} = 3 \times 2.5\text{km}$

$= 7.5 \text{ km}$

(vi) $3 \text{ cm}^2 = 3\ (2.5)^2$

$= 3 \times 6.25$

$= 18.75 \text{ km}^2$

Question 4b part (i)

Area of Cross section = Area of Rectangle + Area of Semi Circle

$= (5 \times 6) + \frac{1}{2}\pi(3^2)$

$= 30 + 14.13$

$= 44.13 \text{ cm}^2$

Question 4b part (ii)

Volume of prism = Cross sectional area of the prism × length

Volume of Prism $\leq 900 \text{ cm}^3$

Area of Cross Section $= 44.13\text{cm}^3$

Let the length = l

Volume of prism = Cross sectional area of the prism × length

$900 \leq 44.13\ l$

$l \leq \frac{900}{44.13} = 20.39 \text{ m}$

$l \leq 20.39\text{m}$

Question 5a part (i)

The length RS

Using Pythagoras' Theorem:

$11.2^2 + RT^2 = 14.8^2$

$RT^2 = 14.8^2 - 11.2^2$

$RT = \sqrt{14.8^2 - 11.2^2}$

$RT = 9.67$ m

RS = RT – ST

RS = 9.67 – 6

RS = 3.67m

Question 5a part (ii)

<RTW

Let <RTW = x

$\sin x = \frac{11.2}{14.8}$

$x = \sin^{-1}\left(\frac{11.2}{14.8}\right)$

$x = 49.2^\circ$

$<RTW = 49.2^\circ$

Question 5b part (i)

ΔABC coordinates: A (3,2) B (6,2) C (3,4)

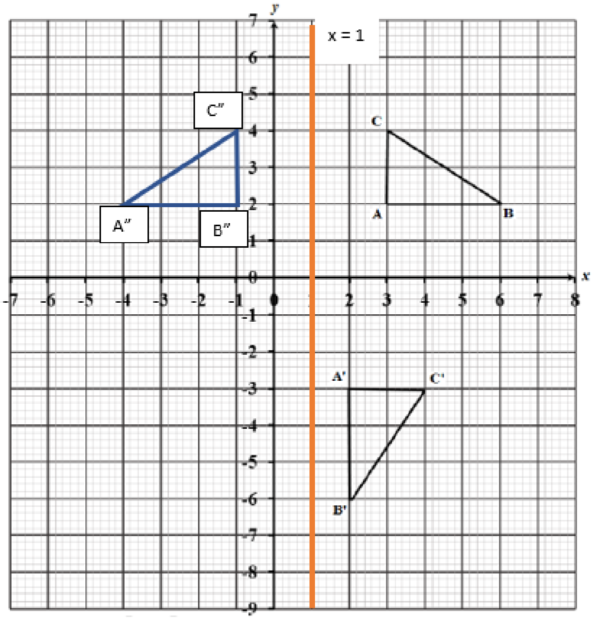
Question 5b part (ii)

ΔA'B'C' coordinates: A'(2,-3) B'(2,-3) C'(4,-3)

Question 5b part (iii)

$\Delta A'B'C'$ maps as a 90^{o} clockwise rotation of ΔABC about the origin.

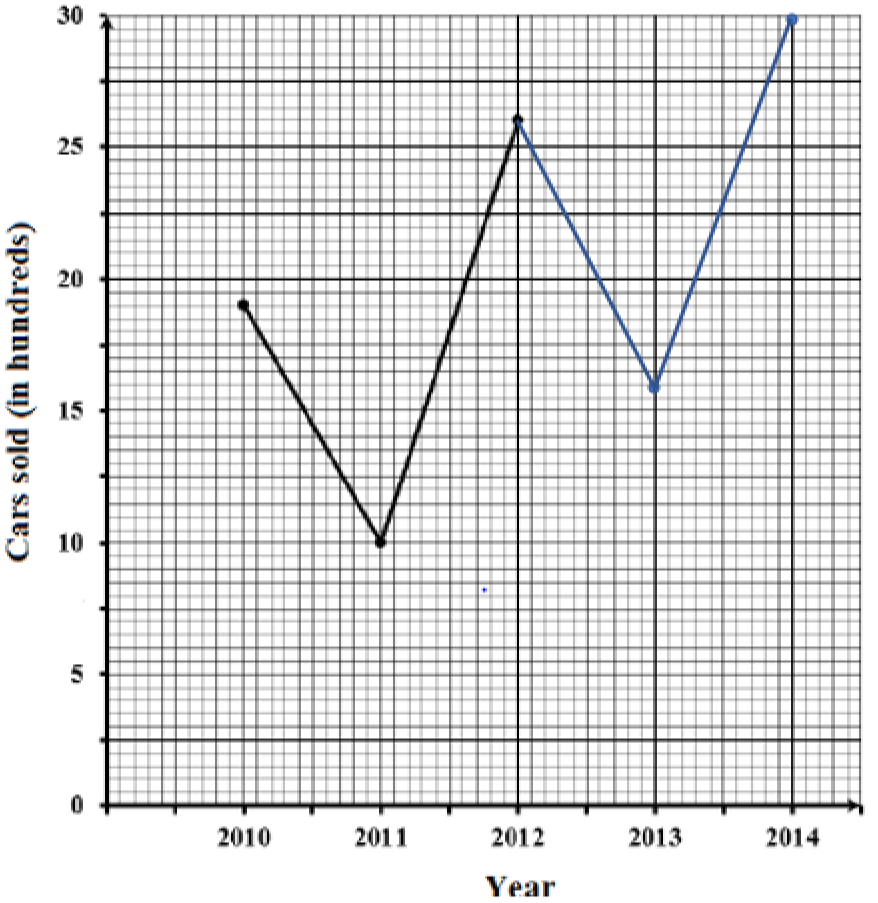
Question 5b part (iv)



Question 5b part (v)

All three triangles are congruent.

Question 6a part (i)



Question 6a part (ii)

Between 2011 and 2012, there was the greatest increase in cars sold, by 1600 cars.

Question 6a part (iii)

Total number of cars sold (In hundreds) $= 19 + 10 + 26 + 16 + 30 = 101$

Total number of cars sold $= 10100$ cars.

Question 6a part (iv)

Mean (in hundreds) $= 22.5$

(Sum of all car sales from 2010 to 2015) $\div 6 = 22.5$

Let sales in 2015 $= x$

$(101 + x) \div 6 = 22.5$

$101 + x = 135$

$x = 135 - 101$

$x = 34$

Number of cars sold in 2015 $= 3400$ cars

Question 6b part (i)

JK: $2y = 5x + 6$

To determine gradient, we must put the equation in the form $y = mx + c$

Dividing by two:

$y = 2.5x + 3$

m, being the gradient $= 2.5$

Question 6b part (ii)

The gradient of two lines perpendicular to each other are the negative inverses of each other.

So, the gradient of GH $= \frac{-1}{2.5} = \frac{-2}{5}$

Question 6b part (iii)

Using the point (5,-1) and $m = \frac{-2}{5}$

$y = mx + c$

$-1 = \frac{-2}{5}(5) + c$

$c = -1 + 2$

$c = 1$

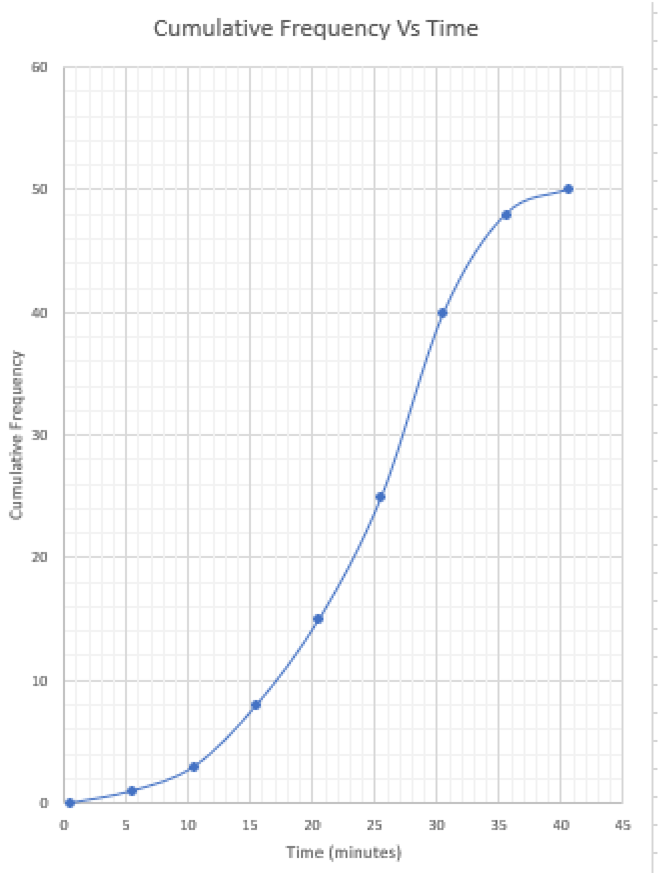
In the form $y = mx + c$:

$y = \frac{-2}{5}x + 1$

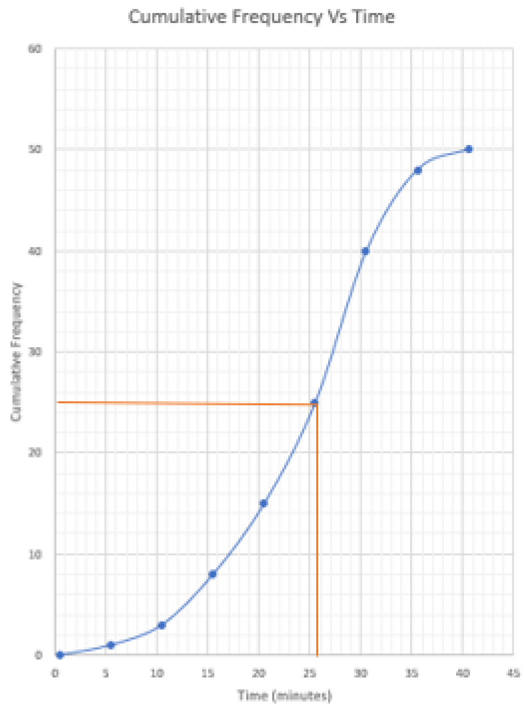
Question 7a

Time (minutes)	Number of Students who Completed (Frequency)	Cumulative Frequency
1-5	1	1
6-10	2	3
11-15	5	8
16-20	7	15
21-25	10	25
26-30	15	40
31-35	8	48
36-40	2	50

Question 7b



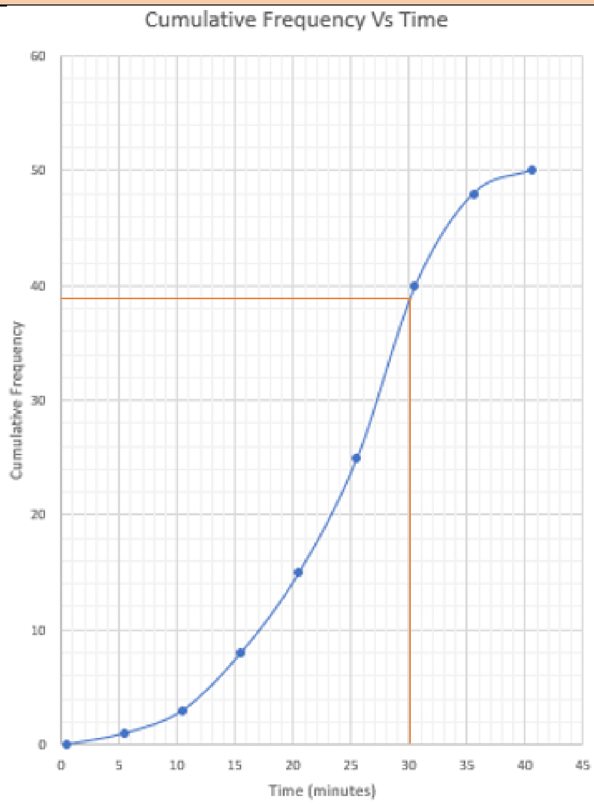
Question 7c part (i)



The median time taken= Time at $(50/2) = 25$

Median Time $= 25.5$ minutes

Question 7c part (ii)

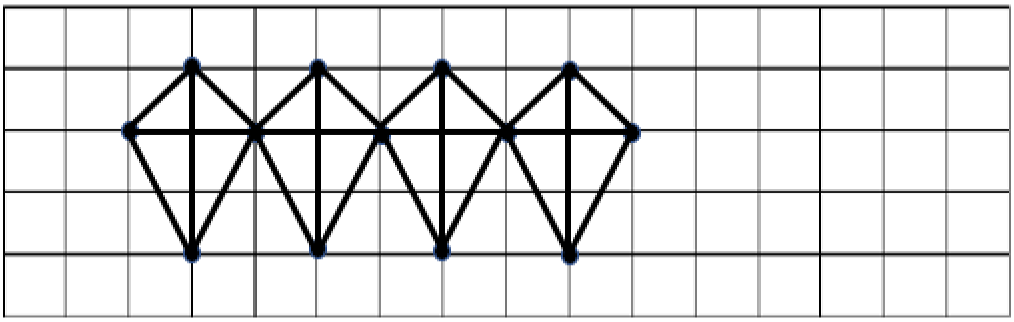


No. of students who took 30 minutes or less $= 39$

Probability of a student taking 30 minutes or less $= \frac{\#\ of\ students\ who\ took\ 30\ min\ or\ less}{Total\ no. of\ students}$

$$= \frac{39}{50} = 0.78$$

Question 8a



Question 8b

Figure	Number of Dots	Number of Lines
1	4	6
2	7	11
3	10	16
4	13	21
	Omitted	
10	31	51
	Omitted	
16	49	81
	Omitted	
N	3N + 1	5N + 1

Question 9a part (i)

The two inequalities are: $x \geq 2$ and $x+y \leq 10$

Question 9a part (ii)

The three pairs of values are: (2,3), (2,8) and (7,3)

Question 9a part (iii)

$P = 5x + 2y - 3$

Substituting (2,3):

$P = 5(2) + 2(3) - 3 = 13$

Substituting (2,8):

$P = 5(2) + 2(8) - 3 = 23$

Substituting (7,3):

$P = 5(7) + 2(3) - 3 = 38$

P has the maximum value at (7,3).

Question 9b part (i)

$g(x) = x^2$

$g(-\frac{1}{2}) = (-\frac{1}{2})^2 = \frac{1}{4}$

$fg(-\frac{1}{2}) = f(\frac{1}{4}) = \frac{3}{2\left(\frac{1}{4}\right)+1} = \frac{3}{1\frac{1}{2}} = 2$

Question 9b part (ii)

$f(x) = \frac{3}{2x+1}$

let y $= \frac{3}{2x+1}$

Make x the subject:

$2x+1 = \frac{3}{y}$

$2x = \frac{3}{y} - 1$

$x = \frac{\frac{3}{y} - 1}{2}$

$f^{-1}(x) = \frac{\frac{3}{x} - 1}{2} = \frac{3 - x}{2x}$

Question 10a part (i)

Area of minor sector $= \frac{40}{360} \times \frac{22}{7} \times 21^2 = 154\text{cm}^2$

Question 10a part (ii)

Area of Triangle $= \frac{1}{2}(21)(21)\sin 40$

$= 141.73cm^2$

Question 10a part (iii)

Area of shaded segment = Area of minor sector – Area of Triangle

$= 154 - 141.73$

$= 12.27\ cm^2$

Question 10b part (i)

$<ADC = 90^{\circ}$ (The angle in a semi-circle is 90°)

Question 10b part (ii)

$<ACD = 36^{\circ}$ (The angle at the centre is twice the angle at the circumference.

Question 10b part (iii)

$<CAD = 54^{\circ}$ (Angles in a triangle add up to 180°

Question 10b part (iv)

$<OEA = 28^{\circ}$

$<OAE = 90^{\circ}$ (Angle made by a tangent to a circle and a radius make a right angle.

$<OEA = 180 - (72 + 90) = 18^{\circ}$

Question 11a part (i)

The given coordinates expressed in the form $(x\ y)$ is shown below,

- $\vec{OB} = (4\ 2)$
- $\vec{AB} = -\vec{OA} + \vec{OB} = -(-2\ 8) + (4\ 2) = (6\ \ -6)$
- $\vec{OM} = \vec{OA} + \vec{AM} = (-2\ 8) + 0.5 \times (6\ -6) = (1\ 5)$

Question 11a part (ii)

To show that $\vec{AC}$ and $\vec{OB}$ are parallel we must first determine $\vec{AC}$.

$\vec{AC} = -\vec{OA} + \vec{OC} == -(-2\ 8) + (0\ 9) = (2\ 1)$

If two vectors are parallel, then one will be a multiple of the next. We can clearly see that $2\vec{AC} = \vec{OB}$ therefore, $\vec{AC}$ and $\vec{OB}$ are parallel.

Question 11b part (i)

If a matrix is singular the determinant is zero. Hence

$$|M| = 0$$

To determine the value of p we must equate the determinant to zero. Hence:

$$(2p \times 1) - \big(4 \times (-3)\big) = 0$$

$$2p + 12 = 0$$

$$p = 6$$

The matrix sum of $2A + B$ is calculated below:

$$2A + B = 2(1\ 2\ \ -4\ 3) + (5\ -1\ 0\ 3)$$

$$2A + B = (2\ 4\ \ -8\ 6) + (5\ -1\ 0\ 3) = (7\ 3\ \ -8\ 9)$$

Question 11b part (ii)

To determine B^{-1},

The adjoint matrix of B is provided below,

$$A_{adj} = (3\ 1\ 0\ 5\ )$$

The determinant of B is $|B| = (5 \times 3) - \left(0 \times (-1)\right) = 15$

The inverse of matrix B is therefore,

$$B^{-1} = \frac{1}{15}(3\ 1\ 0\ 5\ ) = (3/15\ \ 1/15\ 0\ 5/15\ ) = (1/5\ \ 1/15\ 0\ 3/5\ )$$

Question 11b part (iii)

Given:

$$(5\ \ -1\ 0\ 3\ )(x\ y\ ) = (9\ \ 3\ )$$

To solve for x and y we pre-multiply both sides of the equation by B^{-1}**.** Hence,

$$(1/5\ \ 1/15\ 0\ 3/5\ )(5\ \ -1\ 0\ 3\ )(x\ y\ ) = (1/5\ \ 1/15\ 0\ 3/5\ )(9\ 3\ )$$

Solving:

$$(1\ 0\ 0\ 1\ )(x\ y\ ) = \left(\frac{9}{5} + \frac{3}{15}\ 0 + \frac{9}{15}\ \right) = \left(\frac{4}{5}\ \frac{3}{5}\right)$$

Therefore, $x = 4/5$ and $y = 3/5$.