Solutions to CSEC Maths P2 January 2017

Question 1a part (i)

$\frac{3\frac{1}{2}\times 1\frac{2}{3}}{4\frac{1}{5}} = \frac{\frac{7}{2}\times\frac{5}{3}}{\frac{21}{5}} = \frac{\frac{7}{2}\times\frac{5}{3}}{\frac{21}{5}}$

$$= \frac{7}{2} \times \frac{5}{3} \times \frac{5}{21}$$

$$= \frac{1}{2} \times \frac{5}{3} \times \frac{5}{3}$$

$$= \frac{25}{18} = 1\frac{7}{18}$$

Question 1a part (ii)

$5.47 - \sqrt{\frac{0.1014}{1.5}} = 5.47 - \sqrt{0.0676}$

$= 5.47 - 0.26$

$= 5.21$

Question 1b part (i)

5 Juvenile Tickets at \$P each = \$130.50

1 Juvenile ticket $= \frac{130.50}{5} = \$26.10 = P$

Question 1b part (ii)

14 Youth tickets at \$44.35 each = \$Q

$Q = 14 \times 44.35$

$Q = \$620.90$

Question 1b part (iii)

An adult ticket costs twice the cost of a youth ticket

Adult ticket $= \$44.35 \times 2$

$= \$88.70$

No. of tickets sold, R $= \frac{2483.60}{88.70} = 28$ tickets

(iv) Total amount collected $= \$130.50 + \$620.90 + \$2483.60$

$= \$3235.00$

15% Taxes paid $= 0.15 \times 3235$

$= \$485.25$

Question 2a

$$\frac{2x+3}{3} + \frac{x-4}{4} = \frac{4(2x+3)+3(x-4)}{12}$$

$$= \frac{8x+12+3x-12}{12} = \frac{11x}{12}$$

Question 2b

Let the number be x

Multiplicative inverse of x $= \frac{1}{x}$

$x + \frac{1}{x} = 5x$

Question 2c part (i)

$x^2 - 36 = x^2 - 6^2$

Difference of 2 squares:

$x^2 - 36 = (x+6)(x-6)$

Question 2c part (ii)

$2x^2+5x-12 = (2x-3)(x+4)$

Question 2d

Volume, $V = \pi r^2 h$

$$r^2 = \frac{V}{\pi h}$$

$$r = \sqrt{\frac{V}{\pi h}}$$

Question 2e

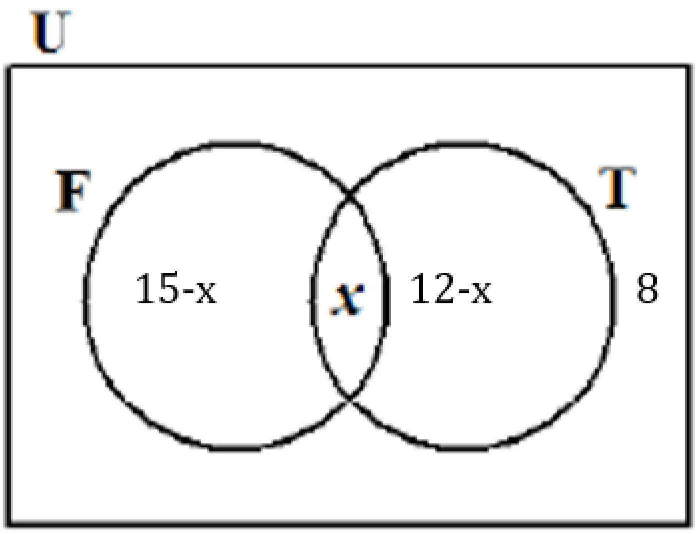
$(x+2)^2 - 3 = (x+2)(x+2) - 3$

$= x^2 + 4x + 4 - 3$

$= x^2 + 4x + 1$

Where a= 4, b = 1

Question 3a part (i)



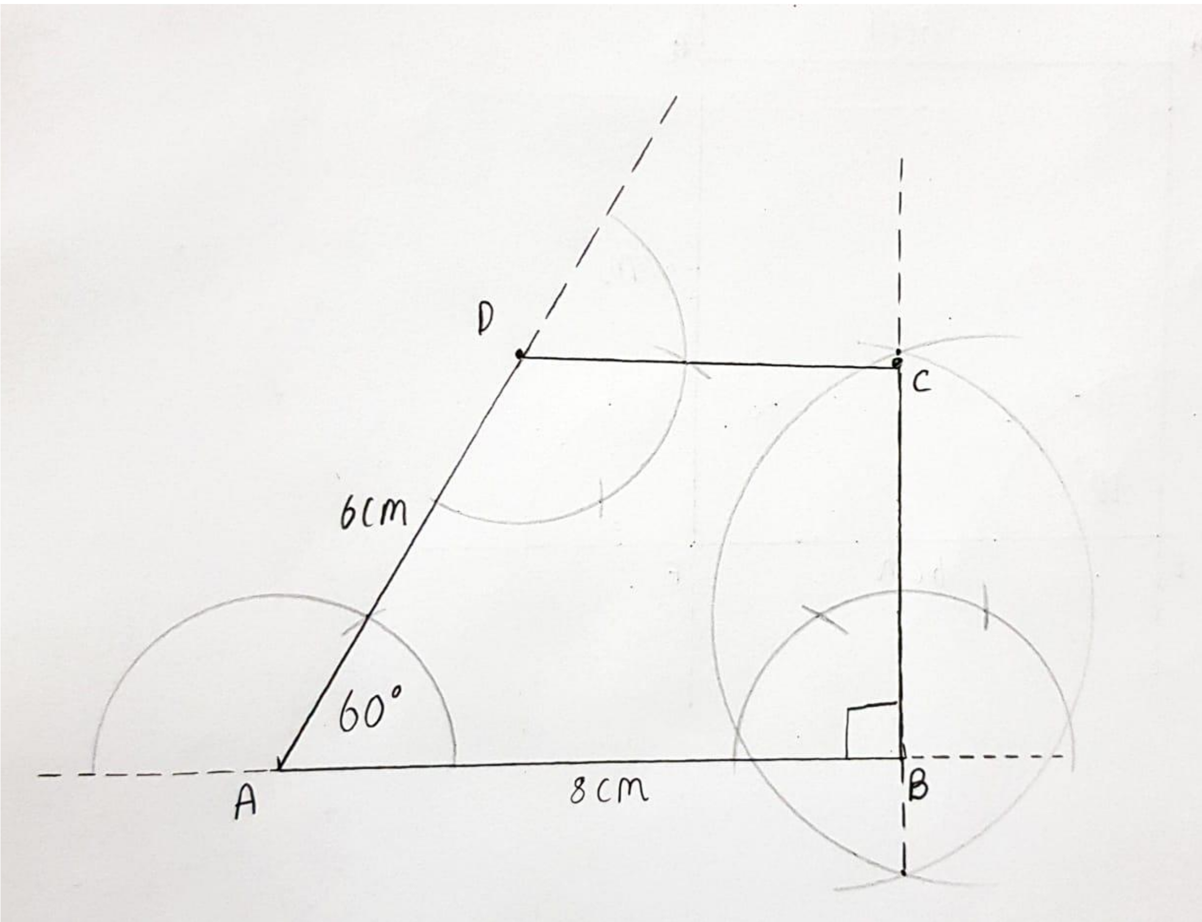
Question 3a part (ii)

$15 - x + x + 12 - x + 8 = 28$

$35 - x = 28$

$x = 7$

Question 3b



Question 4a part (i)

$f(x) = 4x - 7 \qquad g(x) = \frac{3x+1}{2}$

$g(0) + g(5) = \frac{3(0)+1}{2} + \frac{3(5)+1}{2}$

$= \frac{1}{2} + \frac{16}{2} = \frac{17}{2} = 8.5$

Question 4a part (ii)

$fg(5) = 4(g(5)) - 7$

$= 4(\frac{16}{2}) - 7$

$= 32 - 7$

$= 25$

Question 4a part (iii)

$f^{-1}(1)$:

$f(x) = 4x - 7$

Let $y = 4x - 7$

Making x the subject:

$y + 7 = 4x$

$x = \frac{y+7}{4}$

$f^{-1}(x) = \frac{x+7}{4}$

$f^{-1}(1) = \frac{1+7}{4} = 2$

Question 4 b part (i)

P(6,-1) Q(2,7)

Gradient $= \frac{7--1}{2-6} = \frac{8}{-4} = -2$

Question 4b part (ii)

Midpoint $= (\frac{2+6}{2}, \frac{7\pm1}{2})$

$= (4, 3)$

Question 4b part (iii)

Negative inverse of PQ's Gradient $= \frac{1}{2}$

Point on line = (4,3)

y = mx +c

$3 = \frac{1}{2}(4) + c$

3-2 = c

1 = c

Equation of line:

$y = \frac{1}{2}x + 1$

Question 5a part (i)

In the diagram above, the corresponding angles of ΔPQR and ΔSTR are equal and the ratio of their corresponding sides are the same.

Question 5a part (ii)

RS = 15 cm, SP = 9cm, ST = 12cm

$$RP = 15 + 9 = 24 \text{ cm}$$

$$\text{Now, } \frac{RP}{RS} = \frac{PQ}{ST}$$

$$\frac{24}{15} = \frac{PQ}{12}$$

$$PQ = \frac{24 \times 12}{15} = 19.2cm$$

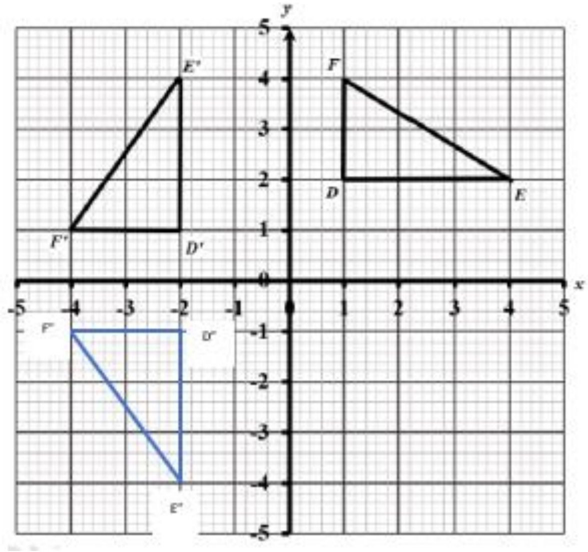
Question 5b part (i)

E (4,2)

Question 5b part (ii)

There was a 90^{o} anti-clockwise rotation about the origin.

Question 5b part (iii)



Question 6a part (i)

Scale: 1:25000

$31.8\text{cm} = 25000 \times 31.8$

$= 795000\text{cm}$

$795000 \text{ cm} = (795000 \div 100{,}000) \text{ km}$

$= 7.95 \text{ km}$

Question 6a part (ii)

$2.75 \text{ km} = (2.75 \times 100{,}000) \text{ cm}$

$= 275{,}000 \text{ cm}$

On the map $= \frac{275000}{25000} = 11cm$

Question 6b part (i)

Splitting the square diagonally, we obtain two right angle triangles. Here, we are allowed to use Pythagoras' theorem.

Side = 11 cm

$AC^2 = 11^2 + 11^2$

$AC^2 = 121 + 121 = 242$

$AC = \sqrt{242} = \sqrt{121 \times 2} = \sqrt{121}\sqrt{2} = 11\sqrt{2}$

Hence proven.

Question 6b part (ii)

Area of a circle, $A = \pi r^2$

$r = \frac{11\sqrt{2}}{2}$

$A = \pi(\frac{11\sqrt{2}}{2})^2 = \pi(\frac{121}{2}) = \pi(60.5) = 190.07 \text{ cm}^2$

Question 6b part (iii)

Area of square = s × s

$= 11 \times 11 = 121 \text{ cm}^2$

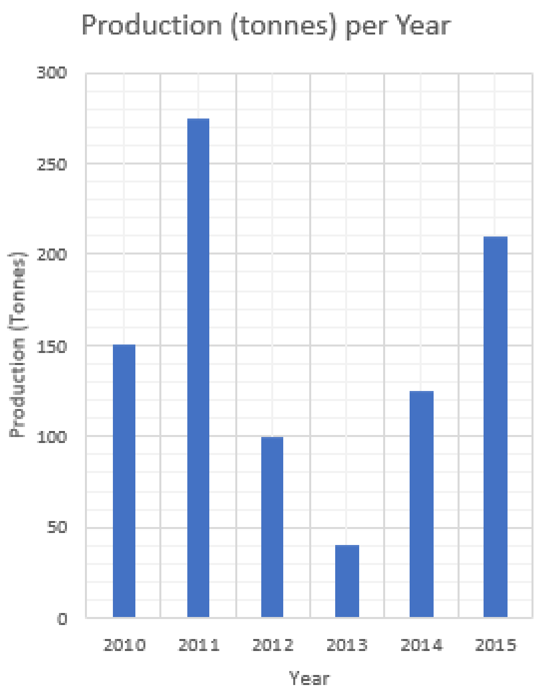
Question 6b part (iv)

$$Shaded\ region = (Area\ of\ Circle - Area\ of\ Square) \div 4$$

$$= (190.07 - 121) \div 4$$

$$= 17.27\text{cm}^2$$

Question 7a



Question 7b

Range = Highest value – Lowest value

= 275 – 40

= 235 tonnes

Question 7c part (i)

2011 had the greatest production of bananas.

Question 7c part (ii)

This is illustrated by the highest bar.

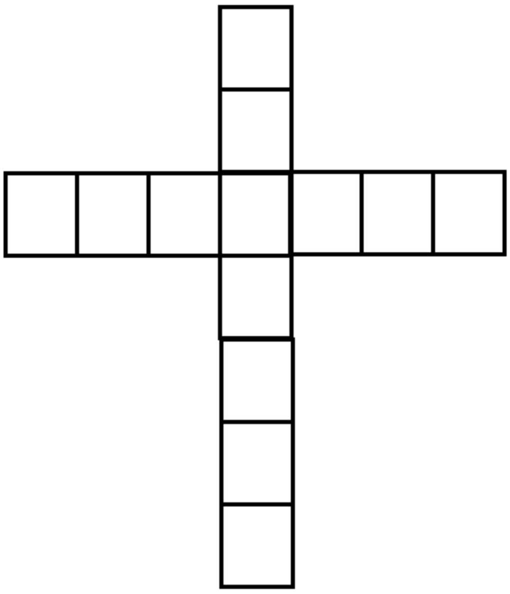
Question 7d part (i)

The greatest change occurred between 2011 and 2012.

Question 7d part (ii)

This is illustrated on the bar graph where there is the most drastic change in height.

Question 8a



Question 8b

Figure	Number of Unit Squares	Perimeter of figure
1	1	4
2	5	12
3	9	20
4	13	28
12	45	92
30	117	236
n	4n-3	8n-4

Question 9a part (i)

When y is inversely proportional to x:

$$y \alpha \frac{1}{x}$$

$$y = \frac{k}{x}$$

Question 9a part (ii)

Using the values: y = 2 and x = 3 from the table:

$$2 = \frac{k}{3}$$

$$k = 6$$

Question 9a part (iii)

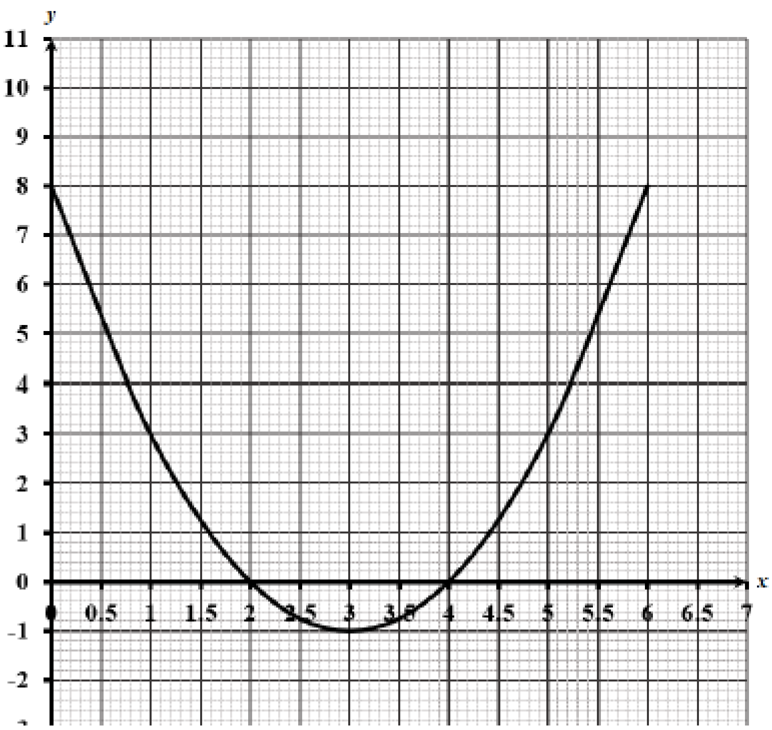
Now the proportionality formula: $y = \frac{6}{x}$

When $y = 1.2$: $x = \frac{6}{1.2} = 5$

When $x = 20$: $y = \frac{6}{20} = 0.3$

$a = 5, b = 0.3$

Question 9 part (i)



The roots are where the graph cuts the x axis. These points are: x = 2 and x = 4

Question 9b part (ii)

The minimum point occurs at the coordinate (3, -1)

Question 9b part (iii)

x^2+6x+8

Using $(x+3)^2+k$

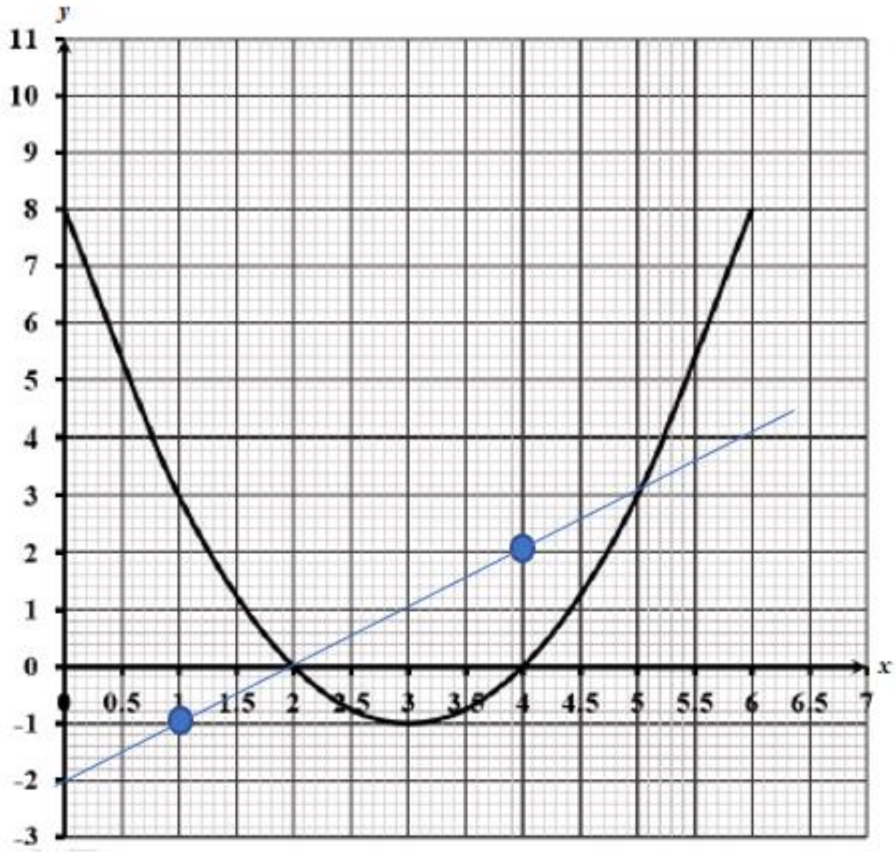
$(x+3)^2 = x^2+6x+9$

$x^2+6x+8 = (x+3)^2-1$

Question 9b part (iv)

$g(x) = x-2$

g(x)	x
-1	1
2	4



Question 9b part (v)

Using the intersecting points of the line and curve:

When x = 2, y = 0 (2,0)

and when x = 5, y = 3 (5,3)

Question 10a part (i)

<HKL = 20° (Angles subtended by a chord, HL at the circumference of the circle and on the same arc are equal.

Question 10a part (ii)

<JOK = 180 – (50+50) = 80° (The sum of angles in a triangle is 180°)

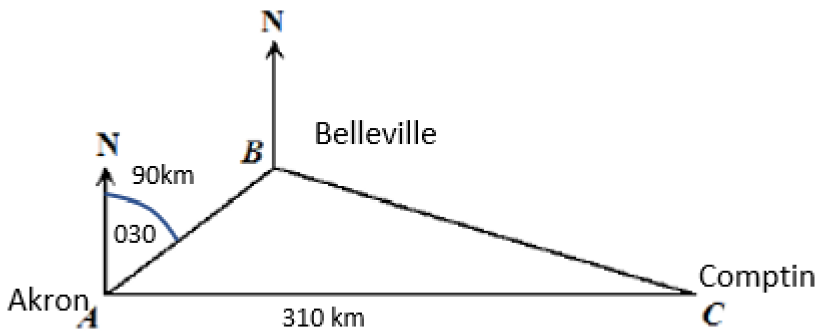
Question 10a part (iii)

<JHK:

<OKP = 90° (Angle made by tangent to a circle and a chord is a right angle.

<JKP = 90-50 = 40°

Question 10b part (i)



Question 10b part (ii)

Using the cosine rule:

$BC^2 = 90^2 + 310^2 - 2(90)\ (310)\ \cos 60$

$= 8100 + 96100 + 55800(0.5)$

$= 104200 - 27900 = 76300$

$BC = \sqrt{76300} = 276.22$ km

Question 10b part (iii)

Using the sine rule:

$$\frac{310}{sinABC} = \frac{276.22}{sin60}$$

$\sin ABC = \frac{310(sin60)}{276.22} = 0.9719$

$\angle ABC = \sin^{-1}(0.9719) = 76.3^{o}$

From the diagram, the angle is obtuse:

$\angle ABC = 180 - 76.3 = 104.7^{o}$

Question 10b part (iv)

Bearing of C from B = $360 - (150+104.7) = 105.3^{o}$

Question 11a part (i)

$T = (c\ 0\ 0\ d\ )\ \ P(2,3)\ Q(2,-3)$

$$\begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} e11 \\ e21 \end{pmatrix}$$

e11 = (c×2) + (0×3) = 2c

e21 = (0×2) + (d×3) = 3d

$$\begin{pmatrix} 2c \\ 3d \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

Now, 2c = 2

c = 1

3d = -3

d = -1

Question 11a part (ii)

$T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -5 \\ 4 \end{pmatrix} = \begin{pmatrix} e11 \\ e21 \end{pmatrix}$$

e11 = (1×-5) + (0×4) = -5

e21 = (0×-5) + (-1×4) = -4

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -5 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ -4 \end{pmatrix}$$

The image transformation is (-5, -4)

Question 11a part (iii)

The transformation $T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ shows a reflection across the x axis.

Question 11a part (iv)

The inverse of T, T^{-1}, will map Q onto P

$T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$|T| = (1\times -1) - (0\times 0)$

$= -1$

$T^{-1} = \frac{1}{-1}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$= \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Question 11b part (i)

OP:

P (4, -1):

$OP = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

QR= QO + OR

Q (0,2): $OQ = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$, $QO = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$

R(3,7): $OR = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$

QR = QO + OR

$= \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

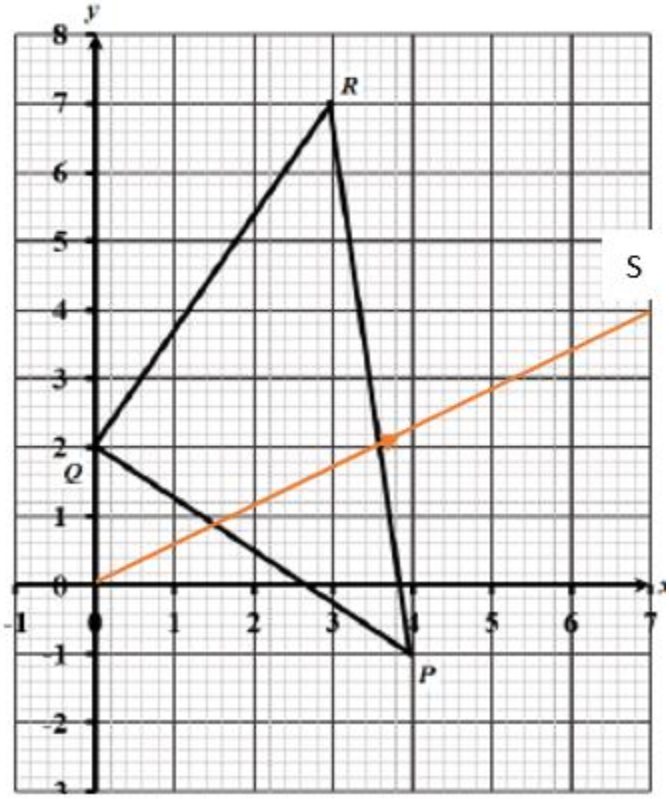
$x = 3, y = 5$

Question 11b part (ii)

$QR = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

$|QR| = \sqrt{3^2 + 5^2} = \sqrt{34} units$

Question 11b part (iii)



$QP = QO + OP$

$= \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$RS = RO + OS$

$= \begin{pmatrix} -3 \\ -7 \end{pmatrix} + \begin{pmatrix} 7 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$QP = RS$, hence they are equal in length and parallel