

## **Solutions to CSEC Maths P2 January 2018**

## Question 1(a)(i)

Required to calculate  $5\frac{1}{2} \div 3\frac{2}{3} + 1\frac{4}{5}$ , giving your answer as a fraction in its lowest terms.

$$\begin{aligned}
 & 5\frac{1}{2} \div 3\frac{2}{3} + 1\frac{4}{5} \\
 &= \left(\frac{11}{2} \div \frac{11}{3}\right) + \frac{9}{5} \\
 &= \left(\frac{11}{2} \times \frac{3}{11}\right) + \frac{9}{5} \\
 &= \frac{3}{2} + \frac{9}{5} \\
 &= \frac{15}{10} + \frac{18}{10} \\
 &= \frac{33}{10} \quad \text{as a fraction in its lowest terms.}
 \end{aligned}$$

## Question 1(a)(ii)

Required to calculate  $165 \times 0.38^2$ , giving your answer in EXACT value.

$$\begin{aligned}
 165 \times 0.38^2 &= 165 \times 0.38 \times 0.38 \\
 &= 23.826 \quad \text{(in exact form)}
 \end{aligned}$$

## Question 1(b)(i)

Required to write the answer in (a)(ii) correct to two decimal places.

$$23.826 = 23.83 \quad \text{(to two decimal places)}$$

## Question 1(b)(ii)

Required to write the answer in (a)(ii) correct to three significant figures.

$$23.826 = 23.8 \quad (\text{to three significant figures})$$

#### Question 1(b)(iii)

Required to write the answer in (a)(ii) correct to the nearest whole number.

$$23.826 = 24 \quad (\text{to the nearest whole number})$$

#### Question 1(c)(i)

Required to determine the simple interest earned.

$$\begin{aligned} \text{Simple Interest earned} &= \text{Amount received} - \text{Principal amount} \\ &= \$5810 - \$5000 \\ &= \$810 \end{aligned}$$

#### Question 1(c)(ii)

Required to determine the annual interest paid by the credit union.

$$\begin{aligned} SI &= \frac{P \times R \times T}{100} \\ 810 &= \frac{5000 \times R \times 3}{100} \\ 810 &= 150R \\ R &= \frac{810}{150} \\ R &= 5.4\% \end{aligned}$$

$\therefore$  The annual interest paid by the credit union is 5.4%

#### Question 1(c)(iii)

Required to determine the length of time it would take for Mr. Adam's investment to be doubled, at the same rate of interest.

The investment is \$5000. For the investment to be doubled, the interest amount will have to be \$5000.

$$SI = \frac{P \times R \times T}{100}$$

$$5000 = \frac{5000 \times 5.4 \times T}{100}$$

$$100 = 5.4T$$

$$T = \frac{100}{5.4}$$

$$T = 18.52 \text{ years} \quad (\text{to 2 decimal places})$$

Question 2(a)(i)

Required to determine the value of  $1 * 2$ .

$$a * b = \sqrt{a + 4b}$$

$$1 * 2 = \sqrt{1 + 4(2)}$$

$$= \sqrt{1 + 8}$$

$$= \sqrt{9}$$

$$= 3$$

Question 2(a)(ii)

Required to determine whether the operation  $*$  is commutative and justify your answer.

$$a * b = \sqrt{a + 4b}$$

$$b * a = \sqrt{b + 4a}$$

Since  $a * b \neq b * a$ , then the operation  $*$  is not commutative.

Question 2(b)(i)

Required to solve the inequality  $3 - 2x > 5$ .

$$3 - 2x > 5$$

$$-2x > 5 - 3$$

$$-2x > 2$$

$$x < \frac{2}{-2}$$

$$x < -1$$

### Question 2(b)(ii)

Required to represent your answer in (b)(i) on the number line.



### Question 2(c)(i)

Required to write two equations in  $x$  and  $y$  to represent the information above.

Let  $x$  represent the cost of an adult ticket.

Let  $y$  represent the cost of a child ticket.

Since 2 adult tickets at  $\$x$  each and 3 children tickets at  $\$y$  each cost  $\$43.00$ , then the equation is :  $2x + 3y = 43$

Since 1 adult ticket at  $\$x$  each and 1 child ticket<sup>1</sup> at  $\$y$  each cost  $\$18.50$ , then the equation is :  $x + y = 18.50$

### Question 2(c)(ii)

Required to solve the equations simultaneously to determine the cost of one adult ticket.

$$2x + 3y = 43 \quad \rightarrow \text{Equation 1}$$

$$x + y = 18.50 \quad \rightarrow \text{Equation 2}$$

Multiplying Equation 2 by 3 gives:

$$3x + 3y = 55.50 \quad \rightarrow \text{Equation 3}$$

Equation 3 - Equation 1 gives:

$$x = 12.50$$

$\therefore$  The cost of 1 adult ticket is \$12.50.

Question 3(a)(i)

Required to state the value of  $n(P \cup R)$ .

$$P \cup R = \{b, v, s, d, e, f, i, g\}$$

$$n(P \cup R) = 8$$

Question 3(a)(ii)(a)

Required to list the members of  $M \cap P$ .

$$M \cap P = \{b, d\}$$

Question 3(a)(ii)(b)

Required to list the members of  $M \cup R'$ .

$$M = \{k, b, i, d\}$$

$$R' = \{k, b, v, s, t, w\}$$

$$M \cup R' = \{k, b, i, d, v, s, t, w\}$$

Question 3(b)(i)

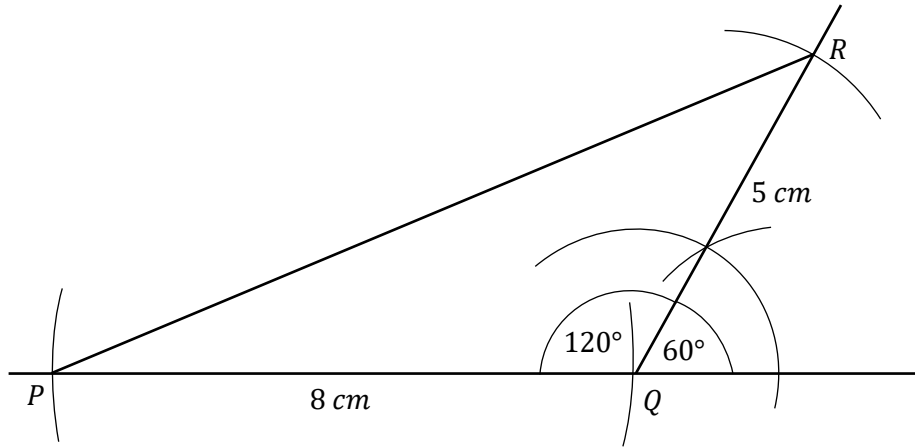
Required to use a ruler, a pencil and a pair of compasses to construct triangle  $PQR$ .



$$PQ = 8 \text{ cm}$$

$$\text{Angle } PQR = 120^\circ$$

$$QR = 5 \text{ cm}$$



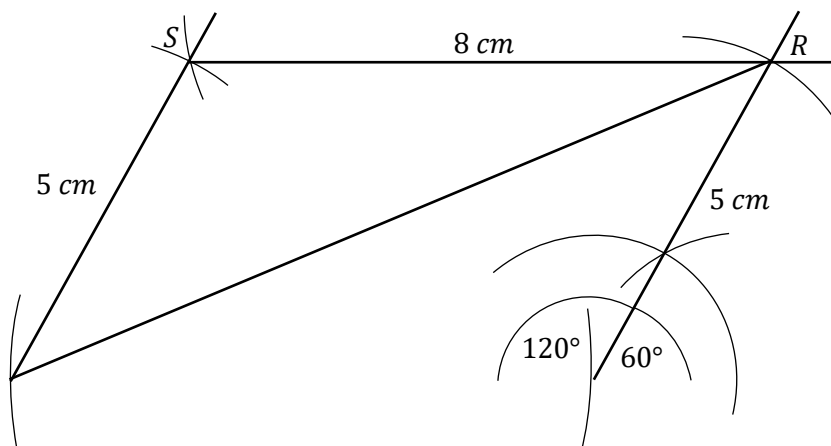
Question 3(b)(ii)

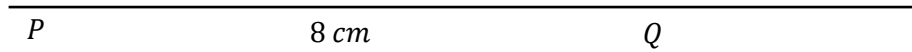
Required to measure and state the length of the side  $PR$ .

Acceptable answers for the length of  $PR$  are within the range  $11.3 \leq PR \leq 11.4 \text{ cm}$ .

Question 3(b)(iii)

Required to construct the point  $S$ , such that  $PQRS$  is a parallelogram.





#### Question 4(a)(i)

Required to write the equation of the line,  $l$ , in the form  $y = mx + c$ .

$$3x - 4y = 5$$

$$-4y = -3x + 5$$

Dividing throughout by  $-4$  gives:

$$y = \frac{3}{4}x - \frac{5}{4}$$

which is of the form  $y = mx + c$ , where  $m = \frac{3}{4}$  and  $c = -\frac{5}{4}$ .

#### Question 4(a)(ii)

Required to determine the gradient of the line,  $l$ .

$$y = \frac{3}{4}x - \frac{5}{4}$$

The gradient of the line,  $l = \frac{3}{4}$ .

#### Question 4(a)(iii)

Required to determine the value of  $r$ .

$$y = \frac{3}{4}x - \frac{5}{4}$$

Substituting point  $P(r, 2)$  gives:

$$2 = \frac{3}{4}r - \frac{5}{4}$$

$$8 = 3r - 5$$

$$3r = 8 + 5$$

$$3r = 13$$

$$r = \frac{13}{3}$$

#### Question 4(a)(iv)

Required to find the equation of the straight line passing through the point  $(6, 0)$  which is perpendicular to  $l$ .

The gradient of  $l = \frac{3}{4}$ .

The gradient of the line perpendicular to  $l = \frac{-1}{\frac{3}{4}}$

$$= -\frac{4}{3}$$

Recall:  $y - y_1 = m(x - x_1)$

Substituting  $m = -\frac{4}{3}$  and the point  $(6, 0)$  gives:

$$y - 0 = -\frac{4}{3}(x - 6)$$

$$y = -\frac{4}{3}x + 8$$

$\therefore$  The equation of the required line is  $y = -\frac{4}{3}x + 8$ .

#### Question 4(b)(i)

Required to draw straight lines  $x + y = 10$  and  $y = x$  on the grid.

Consider  $x + y = 10$ .

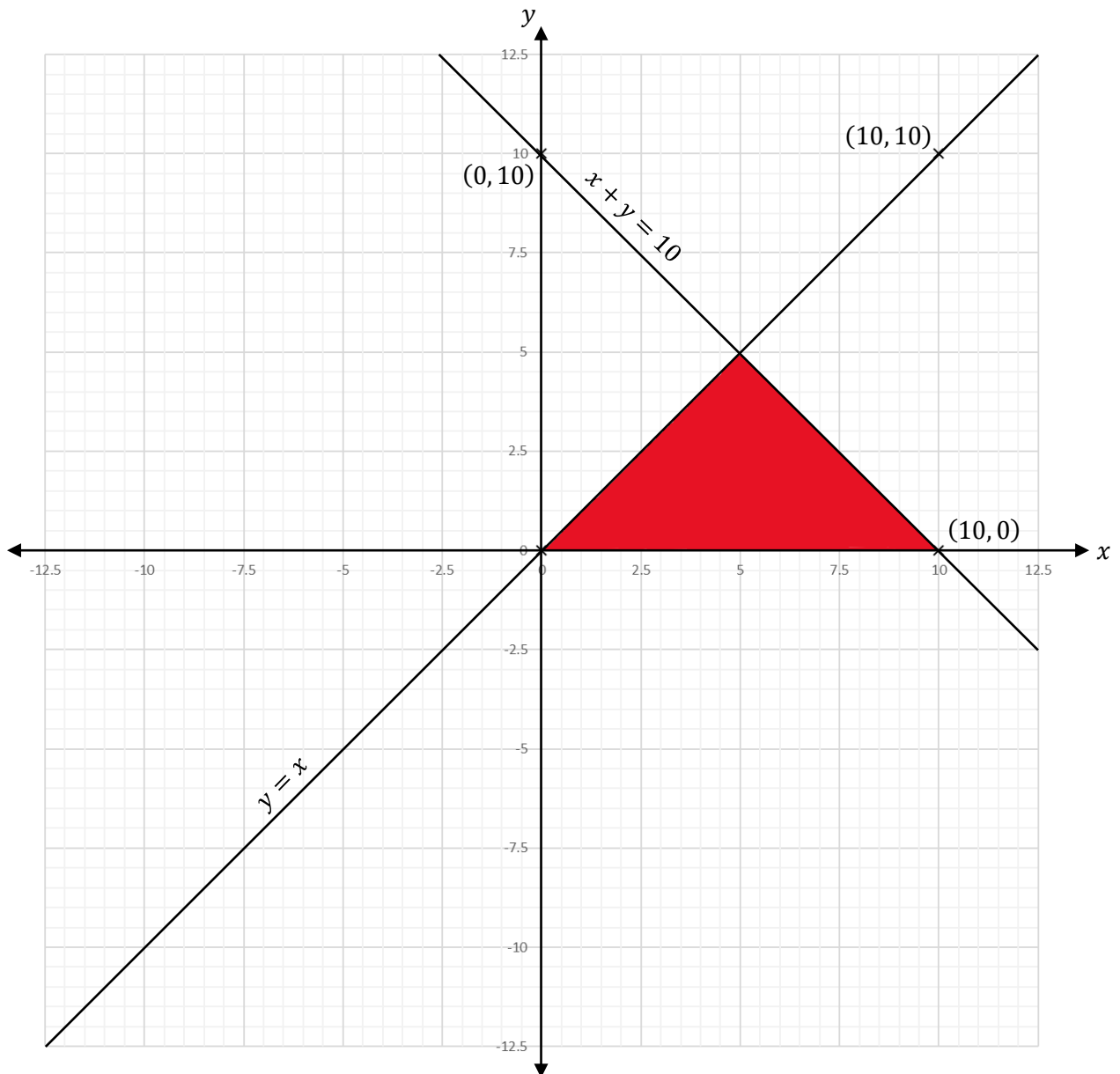
When  $x = 0, y = 10$ .

When  $y = 0, x = 10$ .

Consider  $y = x$ .

When  $x = 0, y = 0$ .

When  $x = 10, y = 10$ .



Question 4(b)(ii)

Required to shade the region which satisfies the four inequalities.

Inequalities:  $x \geq 0$  ,  $y \geq 0$  ,  $x + y \leq 10$  ,  $x \geq y$

(See graph above)

#### Question 5(a)(i)

Required to state the name of the polygon given.

The polygon has 6 sides.

Hence, it is a hexagon.

#### Question 5(a)(ii)

Required to calculate the perimeter of the polygon  $EFGHIJ$ .

$$\begin{aligned}\text{Perimeter of } EFGHIJ &= 6 \times 5 \\ &= 30 \text{ cm}\end{aligned}$$

#### Question 5(a)(iii)

Required to determine the size of each interior angle of the polygon.

The sum of the interior angles of a polygon of  $n$  sides  $= 180^\circ(n - 2)$ .

$$\begin{aligned}\text{The sum of the 6 interior angles of the given hexagon} &= 180^\circ(6 - 2) \\ &= 180^\circ(4) \\ &= 720^\circ\end{aligned}$$

$$\begin{aligned}\text{Each interior angle of the hexagon} &= \frac{720^\circ}{6} \\ &= 120^\circ\end{aligned}$$

#### Question 5(a)(iv)

Required to show that the area of the polygon, to the nearest whole number, is  $65 \text{ cm}^2$ .

$$\begin{aligned}\text{Area of } \triangle OEF &= \frac{1}{2} (OE)(OF) \sin E\hat{O}F \\ &= \frac{1}{2} (5)(5) \sin 60^\circ\end{aligned}$$

The hexagon comprises of 6 of these triangles.

$$\begin{aligned}\therefore \text{Area of hexagon} &= 6 \times \frac{1}{2} (5)(5) \sin 60^\circ \\ &= 64.95 \\ &= 65 \text{ cm}^2 \quad (\text{to the nearest whole number})\end{aligned}$$

#### Question 5(b)(i)

Required to determine the capacity of the tank, in litres.

$$\begin{aligned}\text{Cross-sectional area of the tank} &= \text{Area of the hexagon} \\ &= 64.95 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{After 52 seconds, the volume of water poured} &= 75 \times 52 \\ &= 3900 \text{ cm}^3\end{aligned}$$

The tank is  $\frac{2}{5}$  full.

Hence,

$$\frac{2}{5}V = 3900$$

$$V = \frac{3900}{\frac{2}{5}}$$

$$V = 9750 \text{ cm}^3$$

Now,

$$1000 \text{ cm}^3 = 1 \text{ l}$$

$$1 \text{ cm}^3 = \frac{1}{1000}$$

$$\begin{aligned} 9750 \text{ cm}^3 &= \frac{1}{1000} \times 9750 \\ &= 9.75 \text{ l} \end{aligned}$$

$\therefore$  The capacity of the tank is 9.75 litres.

#### Question 5(b)(ii)

Required to calculate the height,  $h$ , in metres, of the tank.

$$V = \text{Cross-sectional area} \times h$$

$$V = 64.95h$$

Now,

$$\frac{2}{5}V = 3900$$

$$\frac{2}{5} \times 64.95 \times h = 3900$$

$$25.98 \times h = 3900$$

$$h = \frac{3900}{25.98}$$

$$h = 150 \text{ cm} \quad (\text{to the nearest whole number})$$

$$100 \text{ cm} = 1 \text{ m}$$

$$1 \text{ cm} = \frac{1}{100}$$

$$150 \text{ cm} = 1.50 \text{ m} \quad (\text{to 2 decimal places})$$

∴ The height,  $h$ , of the tank is 1.50 m.

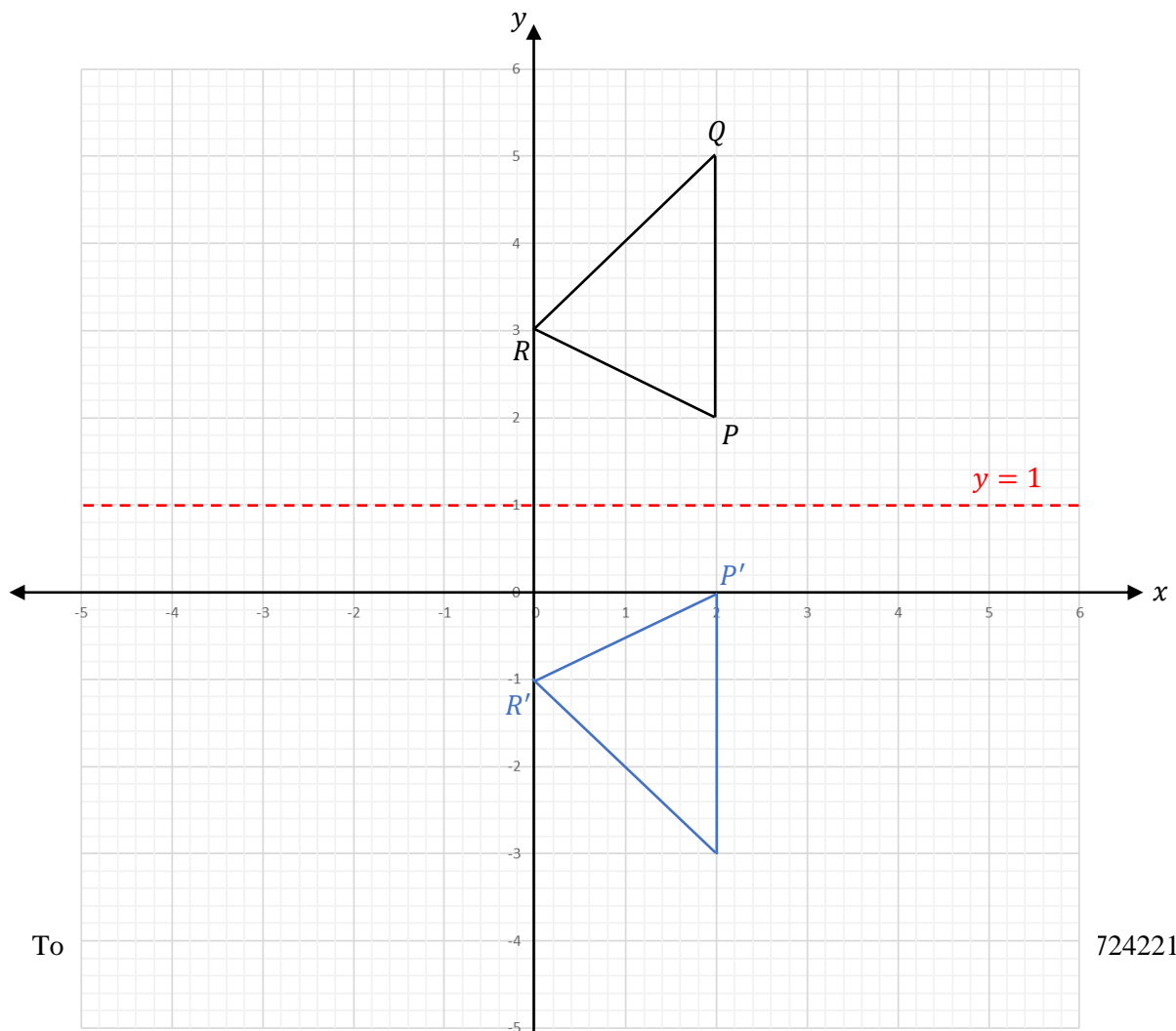
Question 6(a)

Required to state the coordinates of  $R$ .

The coordinates of  $R$  are  $(0, 3)$ .

Question 6(b)(i)

Required to draw  $\Delta P'Q'R'$ , a reflection of  $\Delta PQR$  in the line  $y = 1$ .

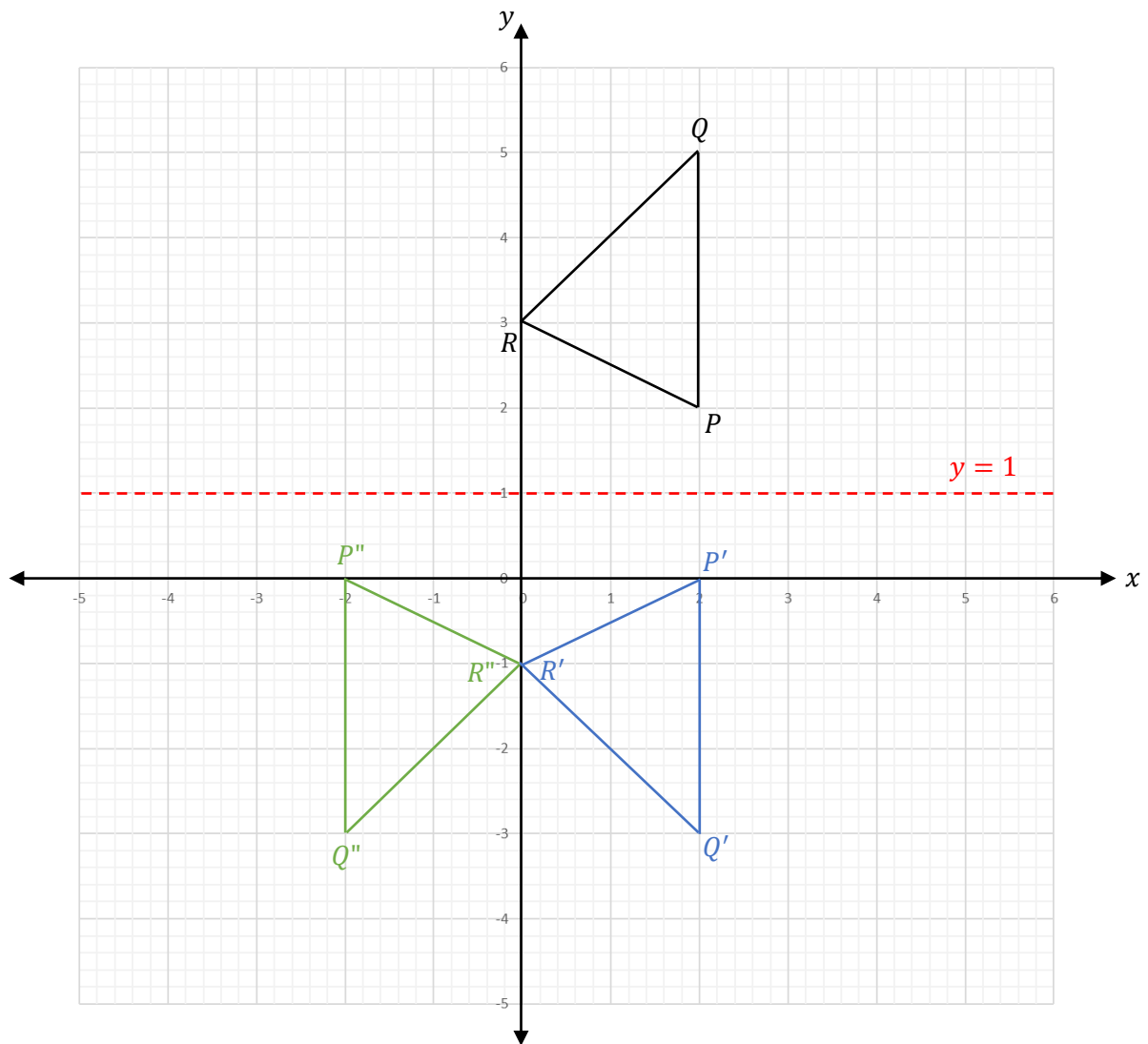




$Q'$

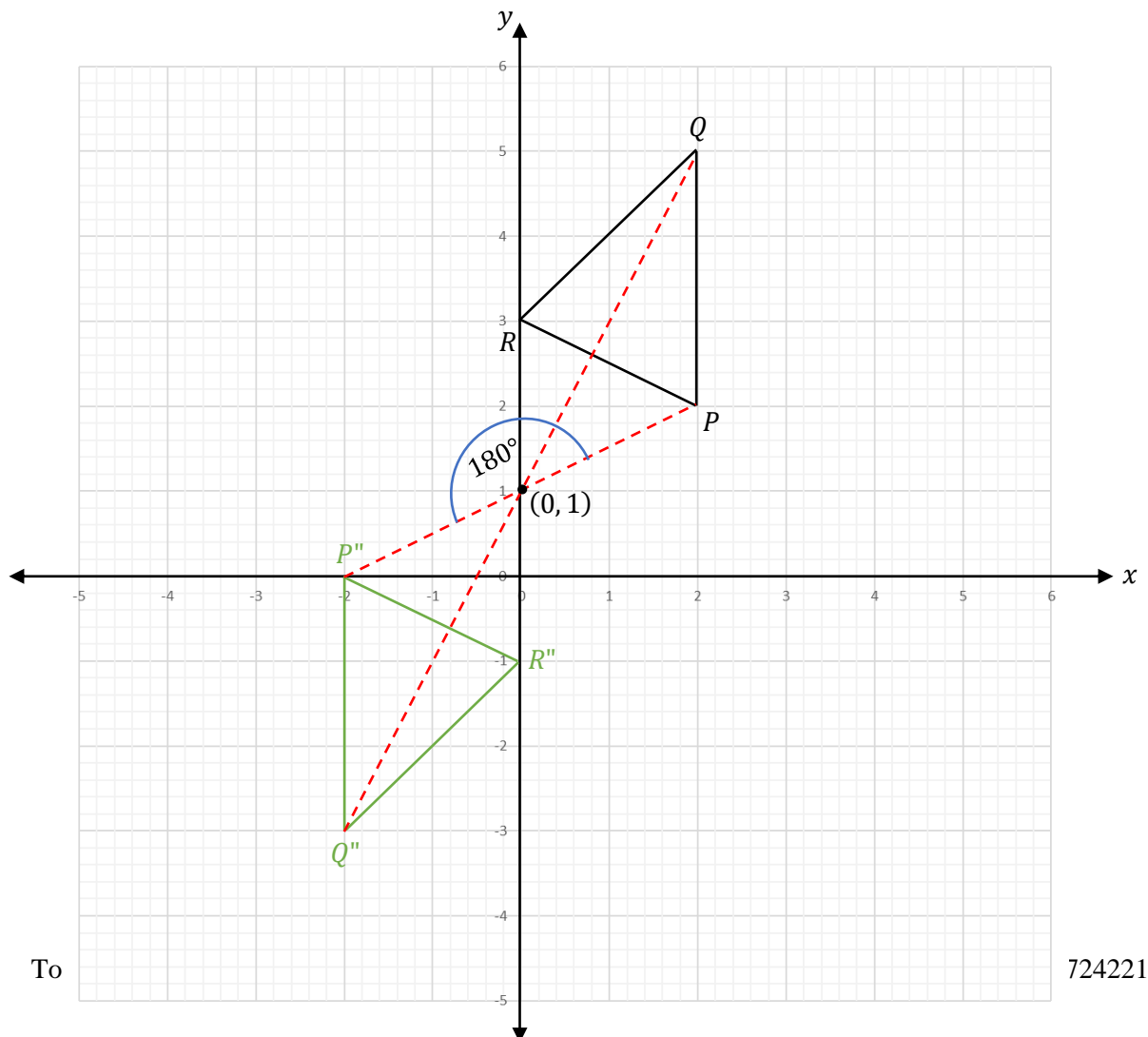
Question 6(b)(ii)

Required to draw  $\Delta P''Q''R''$ , a reflection of  $\Delta P'Q'R'$  in the line  $x = 0$ .



Question 6(c)

Required to describe, fully, the transformation that maps  $\Delta P''Q''R''$  onto  $\Delta PQR$ .

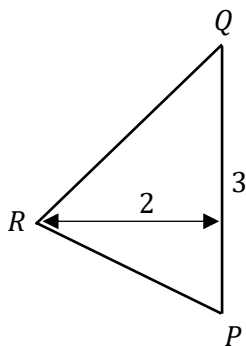


The transformation that maps  $\Delta P''Q''R''$  onto  $\Delta PQR$  is a rotation of  $180^\circ$  about the point  $(0, 1)$ .

#### Question 6(d)

Required to calculate the area of the image made with triangle  $PQR$  undergoes an enlargement of scale factor 2.

Consider  $\Delta PQR$ :



$$\begin{aligned} \text{Area of } \Delta PQR &= \frac{b \times h}{2} \\ &= \frac{3 \times 2}{2} \\ &= 3 \text{ square units} \end{aligned}$$

$$\text{Area of enlarged triangle} = 3 \times (\text{scale factor})^2$$

$$\begin{aligned} &= 3 \times (2)^2 \\ &= 3 \times 4 \\ &= 12 \text{ square units} \end{aligned}$$

### Question 7(a)(i)

Required to determine the range of the marks obtained by 10 students in a test.

The highest mark is 45.

The lowest mark is 26.

Range = Highest mark - Lowest mark

$$= 45 - 26$$

$$= 19$$

### Question 7(a)(ii)

Required to determine the median of the marks obtained by 10 students in a test.

Arranging the data in ascending order gives:

~~26~~ ~~29~~ ~~31~~ ~~35~~ 37 38 ~~38~~ ~~38~~ ~~42~~ ~~45~~

$$\text{Median} = \frac{37+38}{2}$$

$$= 37.5$$

## Question 7(a)(iii)

Required to determine the interquartile range of the marks obtained by 10 students in a test.

Arranging the data in ascending order gives:

26    29    31    35    37    38    38    38    42    45

The lower quartile,  $Q_1 = 31$ .

The upper quartile,  $Q_3 = 38$ .

$$\begin{aligned} \text{Interquartile Range} &= Q_3 - Q_1 \\ &= 38 - 31 \\ &= 7 \end{aligned}$$

## Question 7(a)(iv)

Required to determine the probability that a student chosen at random scores less than half the total marks in the test.

The total marks in the test = 60.

$$\begin{aligned} \text{Half of the total marks} &= \frac{60}{2} \\ &= 30 \end{aligned}$$

*P(student scores less than half the total)*

$$= \frac{\text{Number of students who scored less than 30}}{\text{Total number of students}}$$

$$= \frac{2}{10}$$

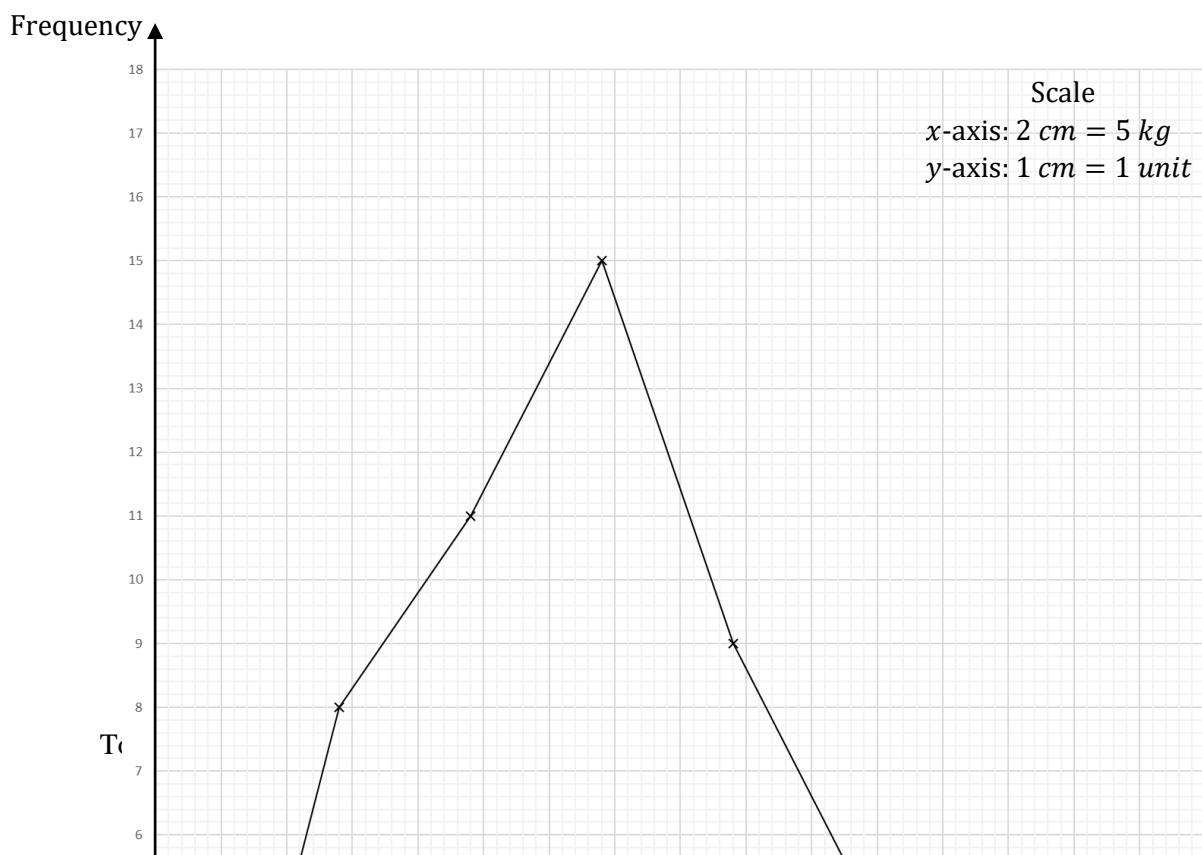
$$= \frac{1}{5} \quad \text{or} \quad 0.2 \quad \text{or} \quad 20\%$$

### Question 7(b)

Required to draw a frequency polygon to represent the information in the table.

Mass ( <i>kg</i> )	Midpoint	Frequency
60 – 64	62	8
65 – 69	67	11
70 – 74	72	15
75 – 79	77	9
80 – 84	82	5
85 – 89	87	2

Title: Frequency polygon showing the masses, in *kg*, of 50 adults prior to the start of a fitness program.

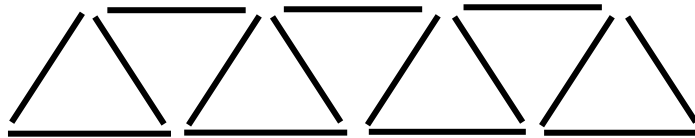


Question 8(a)

Mass (kg)

Required to draw Figure 4 of the sequence.

Figure 4 of the sequence is shown below.



Question 8(b)

Required to complete the rows (i), (ii), (iii) and (iv).

Figure	Number of Toothpicks in Pattern	Perimeter of Figure
1	3	$0 + 1 + 2 = 3$
2	7	$1 + 2 + 2 = 5$
3	11	$2 + 3 + 2 = 7$
(i) 4	<u>15</u>	<u><math>3 + 4 + 2 = 9</math></u>
⋮		

(ii)

(iii)

	<u>20</u>	<u>79</u>	$19 + 20 + 2 = 41$
	⋮		
	<u>32</u>	127	<u><math>31 + 32 + 2 = 65</math></u>
	⋮		
(iv)	$n$	<u><math>4n - 1</math></u>	<u><math>(n - 1) + n + 2 = 2n + 1</math></u>

(i) For Figure 4, number of toothpicks =  $4(4) - 1$

$$= 16 - 1$$

$$= 15$$

(ii) For Figure 4, number of toothpicks =  $4(20) - 1$

$$= 80 - 1$$

$$= 79$$

(iii)  $4n - 1 = 127$

$$4n = 127 + 1$$

$$4n = 128$$

$$n = \frac{128}{4}$$

$$n = 32$$

(iv) For Figure  $n$ , number of toothpicks =  $4n - 1$



**Question 9(a)(i)**

Required to show, by calculation, that the EXACT roots of the quadratic equation

$$x^2 + 2x - 5 = 0 \text{ are } -1 \pm \sqrt{6}.$$

$x^2 + 2x - 5 = 0$  is of the form  $ax^2 + bx + c = 0$  where  $a = 1$ ,  $b = 2$  and  $c = -5$ .

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-5)}}{2(1)} \\&= \frac{-2 \pm \sqrt{4 + 20}}{2} \\&= \frac{-2 \pm \sqrt{24}}{2} \\&= \frac{-2 \pm \sqrt{4 \times 6}}{2} \\&= \frac{-2 \pm \sqrt{4} \sqrt{6}}{2} \\&= \frac{-2 \pm 2\sqrt{6}}{2} \\&= \frac{2(-1 \pm \sqrt{6})}{2} \\&= -1 \pm \sqrt{6}\end{aligned}$$

Question 9(a)(ii)

Required to solve the simultaneous equations:  $2 + x = y$

$$xy = 5$$

$$2 + x = y \quad \rightarrow \text{Equation 1}$$

$$xy = 5 \quad \rightarrow \text{Equation 2}$$

Substituting Equation 1 into Equation 2 gives:

$$x(2 + x) = 5$$

$$2x + x^2 = 5$$

$$x^2 + 2x - 5 = 0$$

The exact roots of the quadratic equation  $x^2 + 2x - 5 = 0$  are  $-1 \pm \sqrt{6}$ .

Hence,  $x = -1 - \sqrt{6}$  and  $x = -1 + \sqrt{6}$ .

When  $x = -1 - \sqrt{6}$ ,

$$y = 2 + (-1 - \sqrt{6})$$

$$= 2 - 1 - \sqrt{6}$$

$$= 1 - \sqrt{6}$$

When  $x = -1 + \sqrt{6}$ ,

$$y = 2 + (-1 + \sqrt{6})$$

$$= 2 - 1 + \sqrt{6}$$

$$= 1 + \sqrt{6}$$

$$\therefore x = -1 - \sqrt{6} \text{ and } y = 1 - \sqrt{6} \quad \text{or} \quad x = -1 + \sqrt{6} \text{ and } y = 1 + \sqrt{6}.$$

### Question 9(b)(i)

Required to complete the table for the function  $y = 2^x$ .

When  $x = -1$ ,

$$y = 2^{-1}$$

$$y = \frac{1}{2}$$

When  $x = 2$ ,

$$y = 2^2$$

$$y = 4$$

When  $x = 1$ ,

$$y = 2^1$$

$$y = 2$$

When  $x = 4$ ,

$$y = 2^4$$

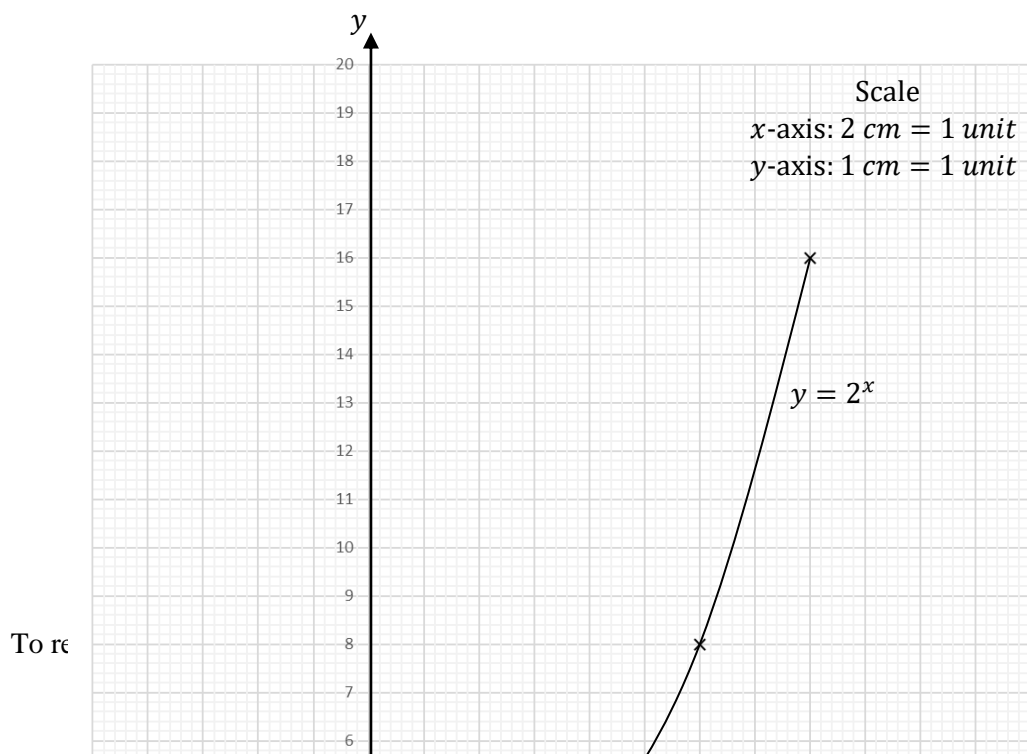
$$y = 16$$

The completed table is shown below:

$x$	-1	0	1	2	3	4
$y$	$\frac{1}{2}$	1	2	4	8	16

### Question 9(b)(ii)

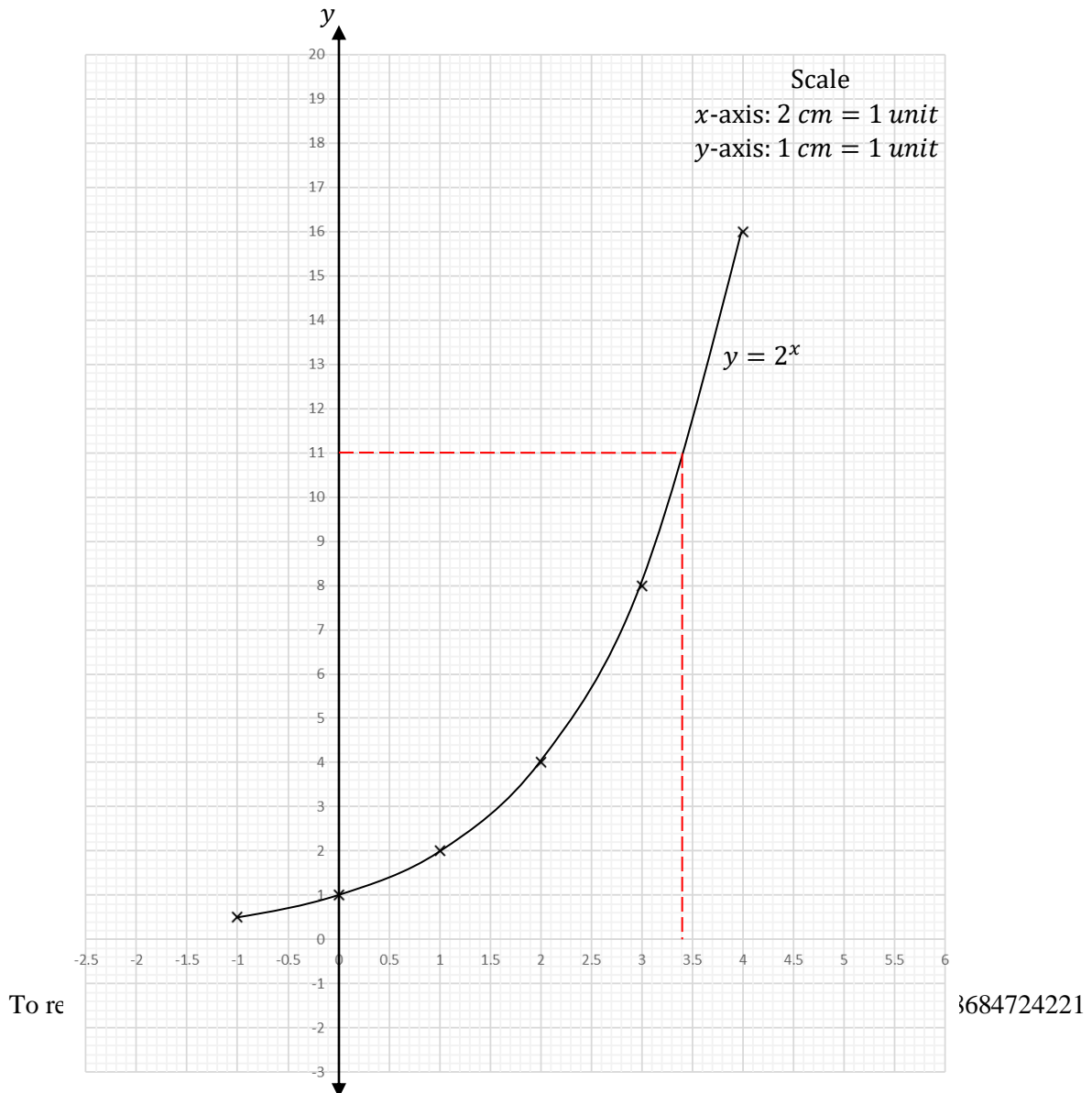
Required to draw the graph of  $y = 2^x$ .





Question 9(b)(iii)

Required to determine the value of  $x$  for which  $2^x = 11$  by drawing appropriate lines on the graph.





$\therefore$  When  $2^x = 11$ ,  $x = 3.4$ .

#### Question 10(a)(i)

Required to calculate the measure of  $\widehat{ACD}$ , giving reasons for each step of your answer.

The angle  $\widehat{AOD}$  subtended by a chord ( $AD$ ) at the center of a circle,  $O$ , is twice the angle that the chord subtends at the circumference, standing on the same arc.

Hence,

$$\begin{aligned} \widehat{ACD} &= \frac{1}{2} \widehat{AOD} \\ &= \frac{1}{2} (114^\circ) \\ &= 57^\circ \end{aligned}$$

#### Question 10(a)(ii)

Required to calculate the measure of  $\widehat{AED}$ , giving reasons for each step of your answer.

The angle between a tangent ( $EA$  and  $ED$ ) to a circle and a radius ( $OA$  and  $OD$ ) at the point of contact ( $A$  and  $D$ ) is a right angle.

Hence,  $\widehat{OAE} = \widehat{ODE}$

$$= 90^\circ$$

Consider the quadrilateral  $AODE$ .

The sum of the interior angles of a quadrilateral is  $360^\circ$ .

$$\begin{aligned} \hat{AED} &= 360^\circ - (90^\circ + 114^\circ + 90^\circ) \\ &= 360^\circ - 294^\circ \\ &= 66^\circ \end{aligned}$$

#### Question 10(a)(iii)

Required to calculate the measure of  $\hat{OAC}$ , giving reasons for each step of your answer.

The angle between a tangent ( $DG$ ) to a circle and a chord ( $DC$ ) at the point of contact ( $D$ ) is equal to the angle ( $\hat{DAC}$ ) in the alternate segment.

Since  $OA$  and  $OD$  are both radii of the same circle, then  $OA = OD$  and  $\triangle OAD$  is isosceles.

$$\begin{aligned} \hat{OAD} &= \hat{ODA} \\ &= \frac{180^\circ - 114^\circ}{2} \\ &= \frac{66^\circ}{2} \\ &= 33^\circ \end{aligned}$$

The base angles of an isosceles triangle are equal and the sum of the interior angles in a triangle is  $180^\circ$ .

Hence,

$$\hat{OAC} = 33^\circ - 18^\circ$$

$$= 15^\circ$$

### Question 10(a)(iv)

Required to calculate the measure of  $\widehat{ABC}$ , giving reasons for each step of your answer.

The sum of the interior angles in a triangle add up to  $180^\circ$ .

$$\widehat{ADC} = 180^\circ - (18^\circ + 57^\circ)$$

$$= 180^\circ - 75^\circ$$

$$= 105^\circ$$

The opposite angles of a cyclic quadrilateral are supplementary.

Hence,

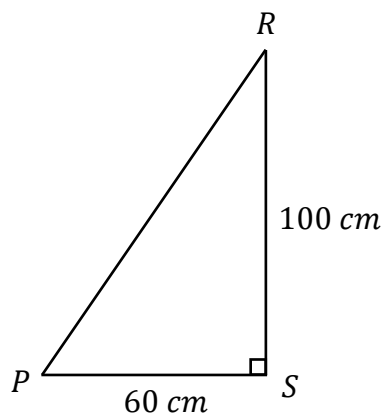
$$\widehat{ABC} = 180^\circ - 105^\circ$$

$$= 75^\circ$$

### Question 10(b)(i)

Required to calculate the length of the wire  $RP$ .

Consider  $\triangle PRS$ :



Using Pythagoras' Theorem:

$$RP^2 = PS^2 + SR^2$$

$$= (60)^2 + (100)^2$$

$$= 3600 + 10\,000$$

$$= 13\,600$$

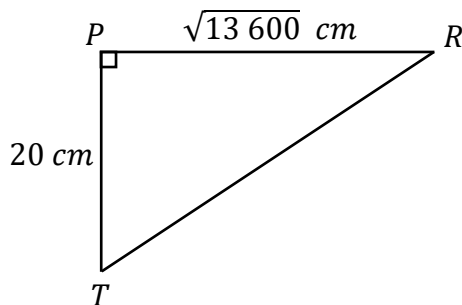
$$RP = \sqrt{13\,600}$$

$$= 116.6 \text{ cm} \quad (\text{to 1 decimal place})$$

Question 10(b)(ii)

Required to calculate the length of the wire  $RT$ .

Consider  $\triangle PRT$ :



Using Pythagoras' Theorem:

$$RT^2 = PT^2 + PR^2$$

$$= (20)^2 + (\sqrt{13\,600})^2$$

$$= 400 + 13\,600$$

$$= 14\,000$$

$$RT = \sqrt{14\,000}$$

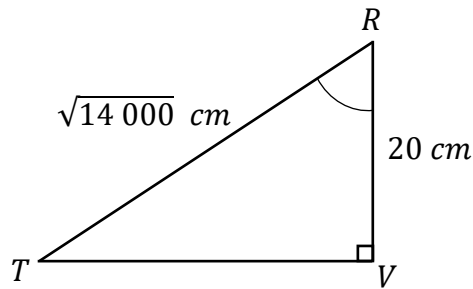


= 118.3 cm (to 1 decimal place)

Question 10(b)(iii)

Required to calculate the angle  $TRV$ .

Consider  $\Delta TRV$ :



$$\cos T\hat{R}V = \frac{VR}{RT}$$

$$= \frac{20}{\sqrt{14\,000}}$$

$$= 0.169$$

$$T\hat{R}V = \cos^{-1}(0.169)$$

$$= 80.3^\circ \quad (\text{to 1 decimal place})$$

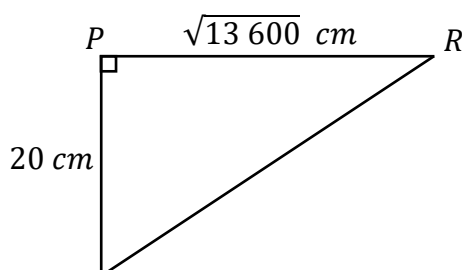
Question 10(b)(iv)

Required to complete the following statements:

The size of the angle through which the wire moves from  $RP$  to  $RT$  is .....

An angle which is the same in size as angle  $RTV$  is .....

Consider  $\Delta PRT$ :



$T$

From  $RP$  to  $RT$ , the wire moves through  $P\hat{R}T$ .

$$\begin{aligned}\tan P\hat{R}T &= \frac{PT}{PR} \\ &= \frac{20}{\sqrt{13\ 600}} \\ &= 0.171\end{aligned}$$

$$\begin{aligned}P\hat{R}T &= \tan^{-1}(0.171) \\ &= 9.7^\circ \quad (\text{to 1 decimal place})\end{aligned}$$

$R\hat{T}V$  has the same size as  $P\hat{T}V$  (or  $S\hat{U}W$  or  $Q\hat{W}U$ ).

The completed statements are as follows:

The size of the angle through which the wire moves from  $RP$  to  $RT$  is .....  $9.7^\circ$  .....

An angle which is the same in size as angle  $RTV$  is ..... angle  $PRT$  .....

Question 11(a)(i)

Required to determine the vector  $\overrightarrow{OQ}$ .

$$\overrightarrow{OP} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\overrightarrow{PQ} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

So, we have,

$$\begin{aligned} \overrightarrow{OQ} &= \overrightarrow{OP} + \overrightarrow{PQ} \\ &= \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} 3 + (-1) \\ 4 + 2 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ 6 \end{pmatrix} \end{aligned}$$

Question 11(a)(ii)

Required to show that  $\overrightarrow{OQ}$  is parallel to  $\overrightarrow{RS}$ , giving a reason for your answer.

$$\begin{aligned}\overrightarrow{RS} &= \begin{pmatrix} 1 \\ 3 \end{pmatrix} \\ &= \frac{1}{2} \begin{pmatrix} 2 \\ 6 \end{pmatrix} \\ &= \frac{1}{2} \overrightarrow{OQ}\end{aligned}$$

Since  $\overrightarrow{RS}$  is a scalar multiple of  $\overrightarrow{OQ}$ , then  $\overrightarrow{OQ}$  and  $\overrightarrow{RS}$  are parallel.

#### Question 11(b)(i)

Required to express  $\overrightarrow{XZ}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$$\begin{aligned}\overrightarrow{XZ} &= \overrightarrow{XY} + \overrightarrow{YZ} \\ \overrightarrow{XZ} &= \mathbf{a} + \mathbf{b}\end{aligned}$$

#### Question 11(b)(ii)

Required to express  $\overrightarrow{MY}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .

$M$  is the midpoint of  $XZ$ .

$$\begin{aligned}\overrightarrow{XM} &= \frac{1}{2} \overrightarrow{XZ} \\ &= \frac{1}{2} (\mathbf{a} + \mathbf{b})\end{aligned}$$

$$\therefore \overrightarrow{MX} = -\frac{1}{2} (\mathbf{a} + \mathbf{b})$$

$$\begin{aligned}
 \overrightarrow{MY} &= \overrightarrow{MX} + \overrightarrow{XY} \\
 &= -\frac{1}{2}(\mathbf{a} + \mathbf{b}) + \mathbf{a} \\
 &= -\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} + \mathbf{a} \\
 &= \frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b} \\
 &= \frac{1}{2}(\mathbf{a} - \mathbf{b})
 \end{aligned}$$

### Question 11(c)(i)

Required to determine the inverse of  $A$ .

$$A = \begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix}$$

$$\det A = ad - bc$$

$$= (-1)(2) - (0)(3)$$

$$= -2 - 0$$

$$= -2$$

$$\text{adj}(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ -3 & -1 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{\det A} \times \text{adj}(A)$$

$$= \frac{1}{-2} \begin{pmatrix} 2 & 0 \\ -3 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{-2} & \frac{0}{-2} \\ \frac{-3}{-2} & \frac{-1}{-2} \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ \frac{3}{2} & \frac{1}{2} \end{pmatrix}$$

### Question 11(c)(ii)

Required to show that  $A^{-1}A = I$ , the identity matrix.

The identity matrix is  $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ .

Now,

$$A^{-1} = \frac{1}{-2} \begin{pmatrix} 2 & 0 \\ -3 & -1 \end{pmatrix}$$

So, we have,

$$\begin{aligned} A^{-1}A &= \frac{1}{-2} \begin{pmatrix} 2 & 0 \\ -3 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix} \\ &= \frac{1}{-2} \begin{pmatrix} (2 \times -1) + (0 \times 3) & (2 \times 0) + (0 \times 2) \\ (-3 \times -1) + (-1 \times 3) & (-3 \times 0) + (-1 \times 2) \end{pmatrix} \\ &= \frac{1}{-2} \begin{pmatrix} -2 + 0 & 0 + 0 \\ 3 + (-3) & 0 + (-2) \end{pmatrix} \\ &= \frac{1}{-2} \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} \\ &= \begin{pmatrix} \frac{-2}{-2} & \frac{0}{-2} \\ \frac{0}{-2} & \frac{-2}{-2} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

Hence,  $A^{-1}A = I$ .

### Question 11(c)(iii)

Required to determine the matrix  $A^2$ .

$$\begin{aligned}
A^2 &= A \times A \\
&= \begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 3 & 2 \end{pmatrix} \\
&= \begin{pmatrix} (-1 \times -1) + (0 \times 3) & (-1 \times 0) + (0 \times 2) \\ (3 \times -1) + (2 \times 3) & (3 \times 0) + (2 \times 2) \end{pmatrix} \\
&= \begin{pmatrix} 1 + 0 & 0 + 0 \\ -3 + 6 & 0 + 4 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ 3 & 4 \end{pmatrix}
\end{aligned}$$

$$\therefore A^2 = \begin{pmatrix} 1 & 0 \\ 3 & 4 \end{pmatrix}$$

#### Question 11(c)(iv)(a)

Required to explain why the matrix product  $AB$  is not possible.

The number of columns of  $A$  which is 2 is not equal to the number of rows of  $B$  which is 3. Hence, the product  $AB$  is not possible.

#### Question 11(c)(iv)(b)

Required to state the order of the matrix product  $BA$ .

The size of the matrix  $B$  is  $3 \times 2$ .

The size of the matrix  $A$  is  $2 \times 2$ .

Therefore, the size of the matrix product  $BA$  is  $3 \times 2$ .