Solutions to CSEC Maths P2 January 2018

Question 1(a)(i)

$$5\frac{1}{2} \div 3\frac{2}{3} + 1\frac{4}{5} = 3\frac{3}{10} \, (By\ Calculator)$$

Question 1(a)(ii)

$$165 \times 0.38^2 = 165 \times 0.38 \times 0.38$$
$$Using\ a\ calculator \rightarrow 23.826 \, (exact\ form)$$

Question 1(b)

Two Decimal Places : 23.83

3 Significant Figures: 23.8

Nearest whole number: 24

Question 1(c)(i)

Simple interest earned = Amount Received – Principal Amount

$$5810 - 5000 = \$810$$

Question 1(c)(ii)

$$SI = \frac{PRT}{100}$$
$$810 = \frac{5000 \times R \times 3}{100}$$
$$810 = 150R$$
$$R = \frac{810}{150} = 5.4\%$$

Question 1(c)(iii)

$$SI = \frac{PRT}{100}$$
$$5000 = \frac{5000 \times 5.4 \times T}{100}$$
$$100 = 5.4T$$
$$T = \frac{100}{5.4} \cong 18.52\ years$$

Question 2(a) (i)

$$1 * 2 = \sqrt{1+8} = \sqrt{9} = \pm 3$$

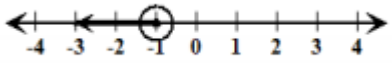
Question 2(a)(ii)

$$If\ the\ operation\ is\ commutative\ then\ 1 * 2 = 2 * 1$$
$$2 * 1 = \sqrt{6} \neq 3 \therefore a * b\ is\ not\ commutative$$

Question 2(b)(i)

$$3 - 2x > 5$$
$$-2x > 5 - 3$$
$$-2x > 2$$
$$2x < -2$$
$$x < -1$$

Question 2(b)(ii)



Question 2(c)(i)

Two adult tickets and three children tickets cost $43.00

One adult ticket and one ticket for a child cost $18.50

$$Let\ x\ be\ the\ cost\ of\ an\ adult\ ticket$$
$$Let\ y\ be\ the\ cost\ of\ a\ child\ ticket$$
$$2x + 3y = 43 \ldots (1)$$
$$x + y = 18.50 \ldots (2)$$
$$y = 18.50 - x \ldots (3)$$
$$Substitute\ (3)\ in\ (1)$$
$$2x + 3(18.5 - x) = 43$$
$$2x + 55.5 - 3x = 43$$
$$2x - 3x = 43 - 55.5$$
$$-x = -12.5$$
$$x = 12.5$$
$$One\ ticket\ costs\ \$12.50$$

Question 3(a)(i)

$$(P \cup R) = \{b, v, s, d, e, f, i, g\}$$

$$n(P \cup R) = 8$$

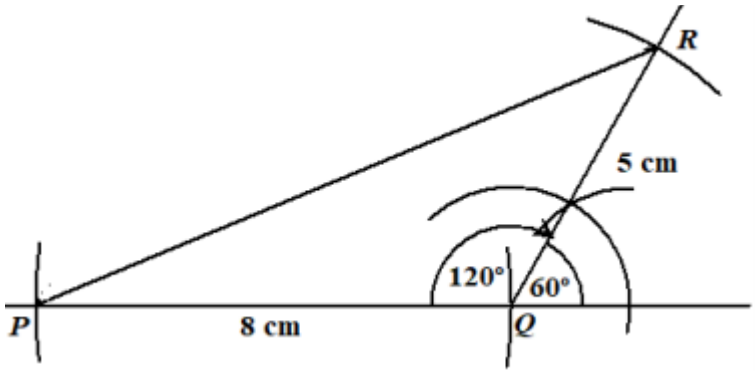
Question 3(a)(ii)(a)

$$M \cap P = \{b, d\}$$

Question 3(a)(ii)(b)

$$(M \cup R') = \{k, b, i, d, v, s, t, w\}$$

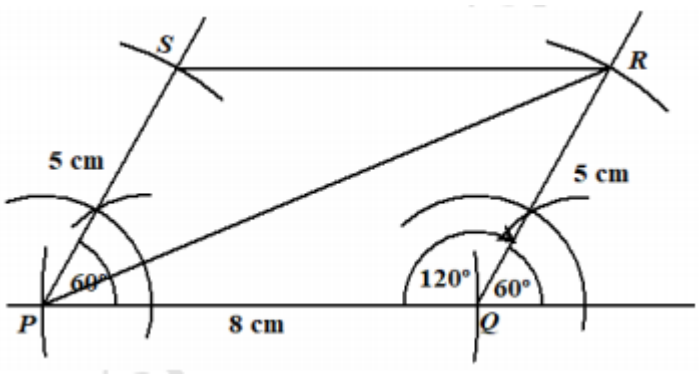
Question 3(b)(i)



Question 3(b)(ii)

PR = 11.35cm

Question 3(b)(iii)



Question 4(a)(i)

$$3x - 4y = 5$$
$$-4y = -3x + 5$$
$$y = -\frac{3x}{-4} + \frac{5}{-4}$$

$$y = \frac{3}{4}x - \frac{5}{4}$$

Question 4(a)(ii)

Gradient of line $= \frac{3}{4}$

Question 4(a)(iii)

$$3r - 4(2) = 5$$
$$3r = 5 + 8$$
$$3r = 13$$
$$r = \frac{13}{3} = 4\frac{1}{3}$$

Question 4(a)(iv)

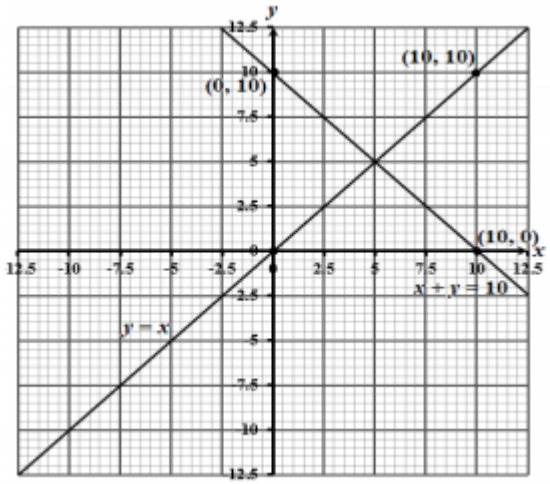
The product of the gradient of perpendicular lines is -1, therefore if the gradient of l is $-\frac{3}{4}$ then

Gradient of any line perpendicular to this is $= -\frac{4}{3}$

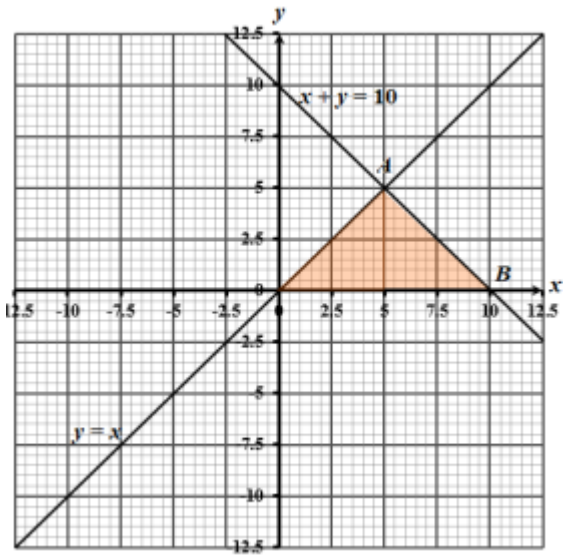
Equation of line:

$$y - y_1 = m(x - x_1)\ pt(6,0), m = -\frac{4}{3}$$
$$y - 0 = -\frac{4}{3}(x - 6)$$
$$y = -\frac{4}{3}x + 8$$

Question 4(b)(i)



Question 4(b)(ii)



Question 5(a)(i)

Hexagon

Question 5(a)(ii)

$$Perimeter = 5 \times 6 = 30cm$$

Question 5(a)(iii)

$$Sum\ of\ angles = (2n - 4) \times 90$$
$$2((6) - 4) \times 90 = 720$$
$$\therefore each\ interior\ angle = \frac{720}{6} = 120^o$$

Question 5(a)(iv)

$$Area\ of\ polygon = \frac{1}{2} side \times side \times sin\theta \times 6$$
$$= \frac{1}{2}(5)(5)(sin60)(6) = 64.95 \cong 65cm^2$$

Question 5(b)(i)

After 52 seconds the Volume of water poured into the tank is

$$75 \times 52 = 3900cm^3$$

$$\frac{2}{5}V = 3900$$
$$V = \frac{3900 \times 5}{2} = 9.75l$$

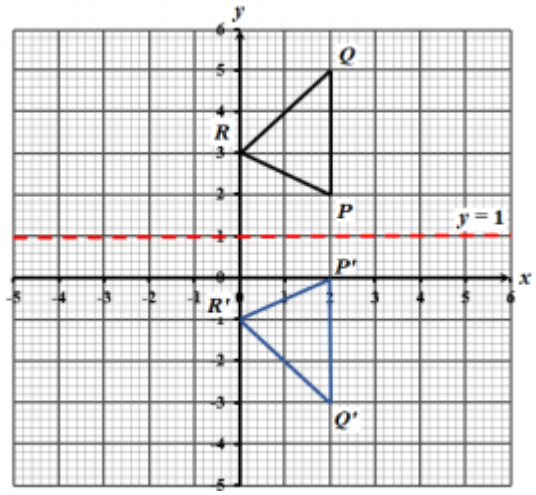
Question 5(b)(ii)

$$\frac{2}{5} \times h \times 64.95 = 3900$$
$$h = 1.50m$$

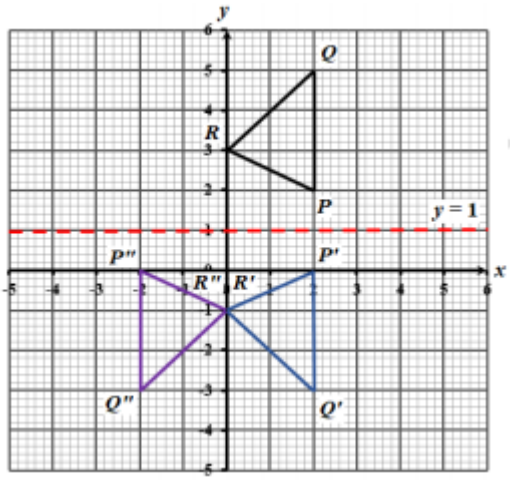
Question 6(a)

R(0,3)

Question 6(b)(i)



Question 6b(ii)



Question 6c

Rotation of 180° about point (0,1)

Question 6d

$$Area = \frac{3 \times 2}{2} = 3 \; square \; units$$
$$Triangle \; is \; enlarged \; by \; a \; factor \; of \; 2$$
$$\therefore Area = 3(2)^2$$

$= 12\ square\ units$

Question 7(a)(i)

Range = 45-26=19

Question 7(a)(ii)

$$Median = \frac{37 + 38}{2} = 37.5$$

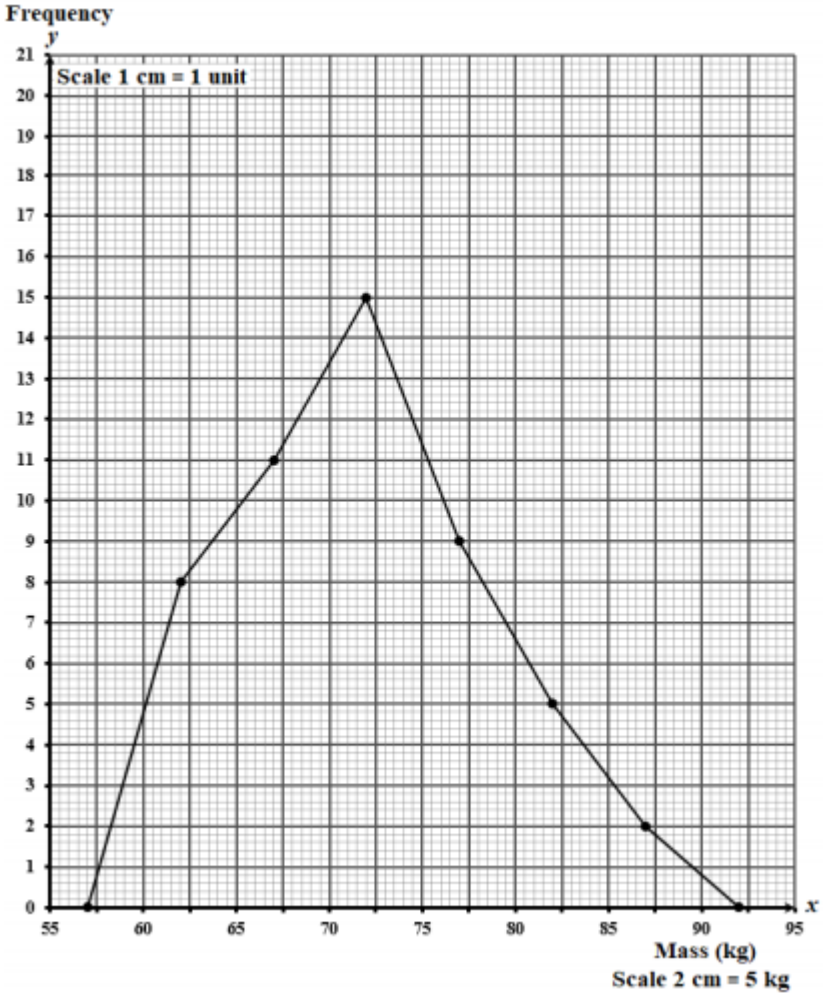
Question 7(a)(iii)

$$Q_3 = 38$$
$$Q_1 = 31$$
$$38 - 31 = 7$$

Question 7(a)(iv)

$$P(Student\ scores\ less\ than\ half\ the\ total) = \frac{2}{10} = \frac{1}{5}$$

Question 7(b)



Question 8(a)

Question 8(b)

	Figure	Number of Toothpicks in Pattern	Perimeter of Figure
	1	3	0+1+2=3
	2	7	1+2+2=5
	3	11	2+3+2=7
(i)	4	15	3+4+2=9
(ii)	20	79	19+20+2=41
(iii)	32	127	2(32)+1=65
(iv)	n	4n-1	2n+1

(i)

When n=4

$$t = 4n - 1 = 4(4) - 1 = 15$$
$$P = 2n + 1 = 2(4) + 1 = 9$$

(ii)

When P=41

$$2n + 1 = 41$$

(iiii) When t=127

$$4n - 1 = 127$$
$$4n = 127 + 1 = 128$$
$$n = 32$$
$$When\ n = 32$$
$$2(32) + 1 = 65$$

(iv)

$$t = 4n - 1$$
$$P = 2n + 1$$

Question 9(a)(i)

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$
$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-5)}}{2(1)} = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-5)}}{2(1)} = \frac{-2 \pm \sqrt{4 + 20}}{2}$$
$$x = \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 \pm \sqrt{4 \times 6}}{2} = \frac{-2 \pm \sqrt{4\sqrt{6}}}{2}$$
$$x = \frac{-2 \pm 2\sqrt{6}}{2} = \frac{-2 + 2\sqrt{6}}{2} or \frac{-2 - 2\sqrt{6}}{2}$$
$$x = -1 + \sqrt{6}\ or - 1 - \sqrt{6} = -1 \pm \sqrt{6}$$

Question 9(a)(ii)

$2 + x = y \ldots . (1)$
$xy = 5 \ldots \ldots \ldots (2)$
$Substitute\ y = 2 + x\ in\ (2)$
$x(2 + x) = 5$
$2x + x^2 = 5$
$x^2 - 5 = 0$
$x = -1 \pm \sqrt{6}$
$when\ x = -1 + \sqrt{6} \rightarrow y = 2 + \left(-1 + \sqrt{6}\right) = 2 - 1 + \sqrt{6} = \mathbf{1} + \sqrt{\mathbf{6}}$
$when\ x = -1 - \sqrt{6} \rightarrow y = 2 + \left(-1 - \sqrt{6}\right) = 2 - 1 - \sqrt{6} = \mathbf{1} - \sqrt{\mathbf{6}}$

Question 9(b)(i)

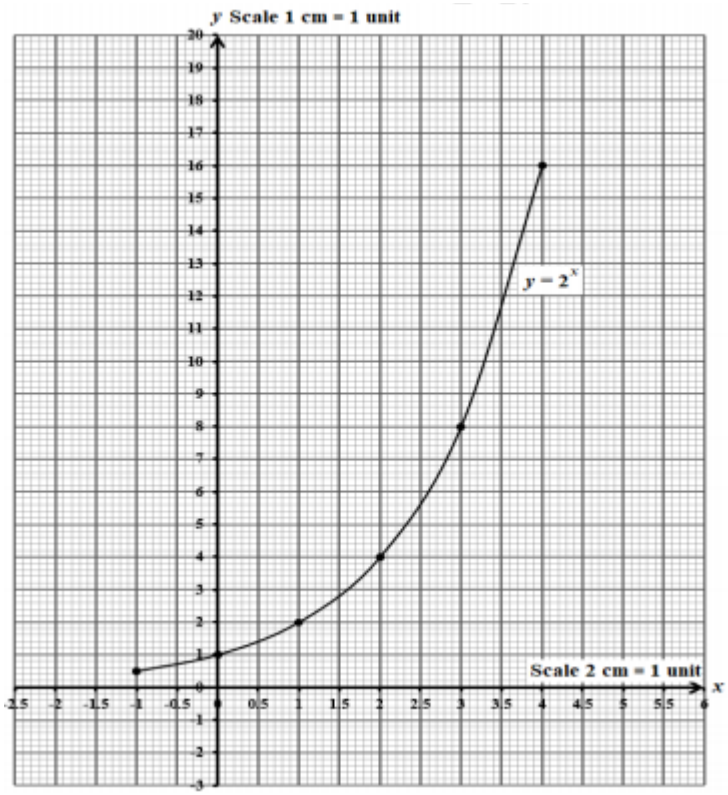
x	-1	0	1	2	3	4
y	½	1	2	4	8	16

When x=1, $y = 2^{-1} = \frac{1}{2^1} = \frac{1}{2}$

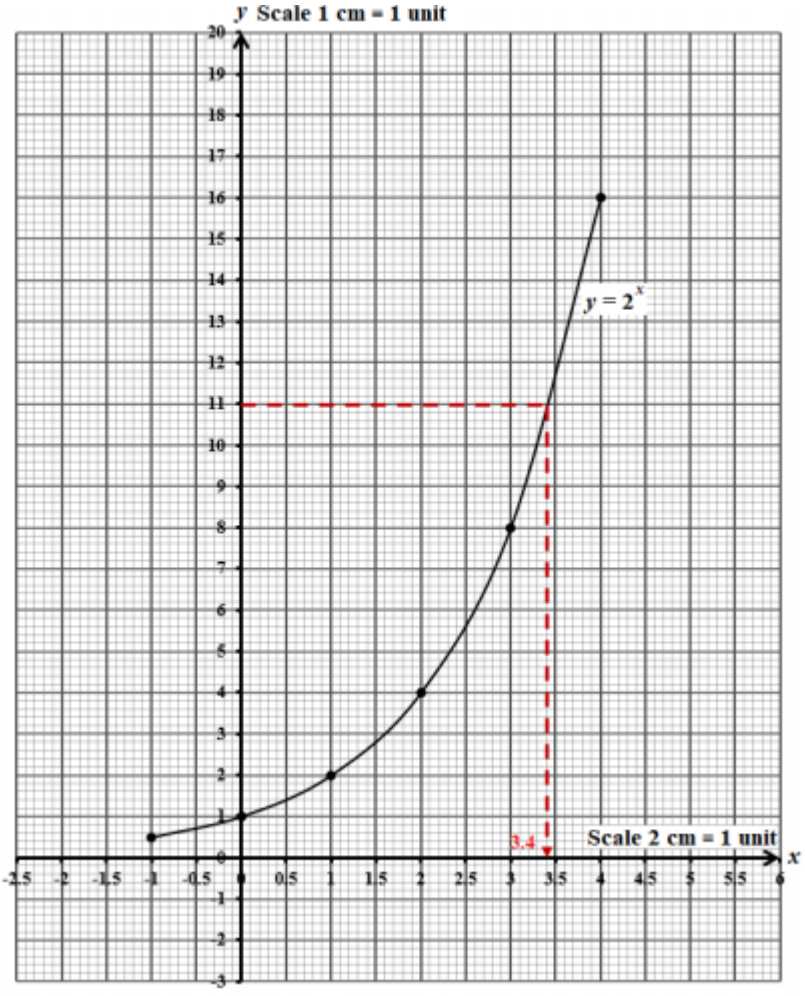
When x=1, $y = 2^1 = 2$

When x=2, $y = 2^2 = 4$
When x=4,$y = 2^4 = 16$

Question 9(b)(ii)

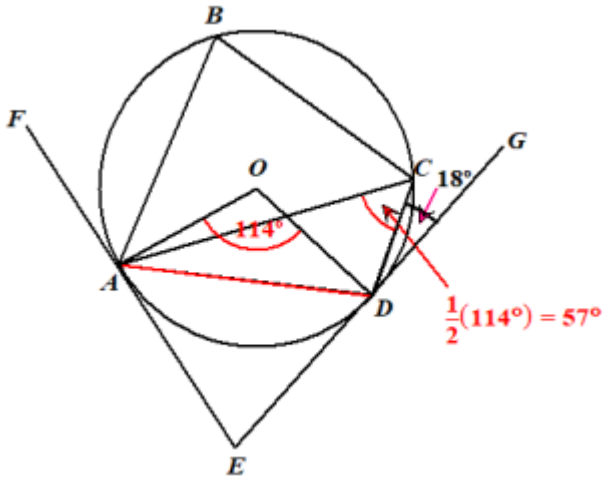


Question 9(b)(iii)



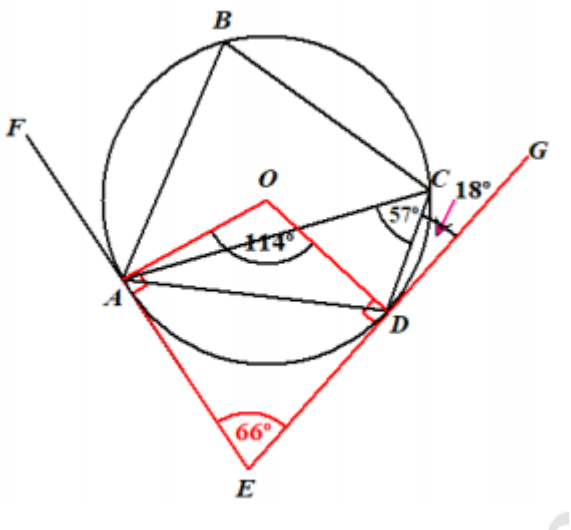
$x = 3.4$

Question 10(a)(i)



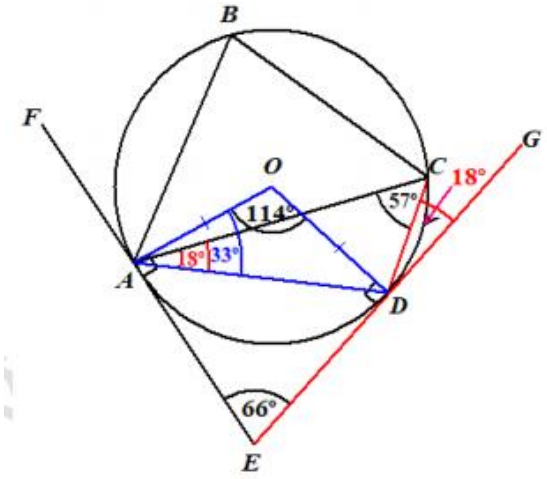
$< \text{ACD} = 57°$

Question 10(a)(ii)



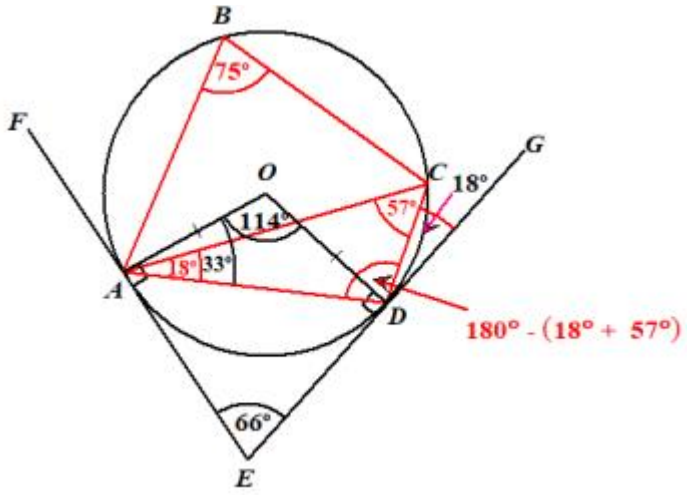
$< \text{AED} = 66°$

Question 10(a)(iii)



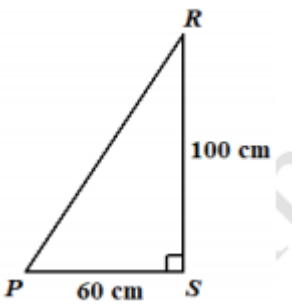
$< OAC = 18^{o}$

Question 10(a)(iv)



$< ABC = 75^{o}$

Question 10(b)(I)

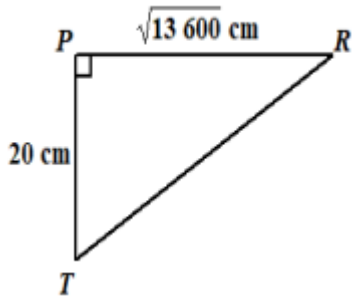


$RP^2 = 60^2 + 100^2$

$RP^2 = 13600$

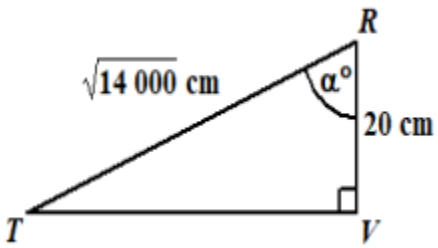
$$RP = \sqrt{13600}$$
$$RP = 116.61$$
$$RP = 116.6cm\ correct\ to\ 1dp$$

Question 10(b)(ii)



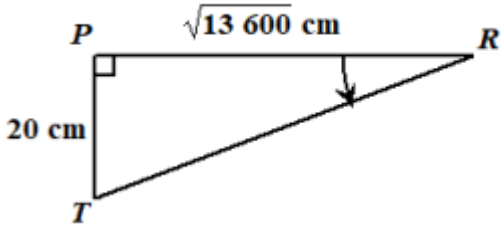
$$RT^2 = (20)^2 + \left(\sqrt{13600}\right)^2$$
$$RT^2 = \sqrt{14000}$$
$$RT\ \ 118.32 = 118.3cm\ correct\ to\ 1dp$$

Question 10(b)(iii)



$$cos\alpha = \frac{20}{\sqrt{14000}}$$
$$\alpha = 80.3^o$$

Question 10(b)(iv)



$$tan < PRT = \frac{30}{\sqrt{13600}}$$

$$< PRT = \tan^{-1}\left(\frac{20}{1183}\right) = 9.59^o = 9.6^o$$
$$< RTV = < PTV$$

The size of the angle through which the wire moves from RP to RT is 9.6^o. An angle which is the same in size as angle RTV is PRT.

Question 11(a)(i)

$$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} = \begin{pmatrix}3\\4\end{pmatrix} + \begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}2\\6\end{pmatrix}$$

Question 11(a)(ii)

$$\overrightarrow{RS} = \begin{pmatrix}1\\3\end{pmatrix} = \frac{1}{2}\begin{pmatrix}2\\6\end{pmatrix} = \frac{1}{2}\overrightarrow{OQ}$$
$$\overrightarrow{RS} \text{ is a scalar multiple of } \overrightarrow{OQ}, \text{ hence they are parallel}$$

Question 11(b)(i)

$$\overrightarrow{XZ} = \overrightarrow{XY} + \overrightarrow{YZ} = a + b$$

Question 11(b)(ii)

$$\overrightarrow{XM} = \frac{1}{2}\overrightarrow{XZ} = \frac{1}{2}(a+b)$$
$$\overrightarrow{MX} = -\left(\frac{1}{2}(a+b)\right)$$
$$\overrightarrow{MY} = \overrightarrow{MX} + \overrightarrow{XY} = -\frac{1}{2}(a+b) + a = -\frac{1}{2}a - \frac{1}{2}b + a = \frac{1}{2}a - \frac{1}{2}b = \frac{1}{2}(a-b)$$

Question 11(c)(i)

$$A = \begin{pmatrix}-1 & 0\\3 & 2\end{pmatrix}$$
$$\det A = (-1 \times 2) - (0 \times 3) = -2$$
$$A^{-1} = \frac{1}{-2}\begin{pmatrix}2 & 0\\-3 & -1\end{pmatrix} = \begin{pmatrix}-1 & 0\\\frac{3}{2} & \frac{1}{2}\end{pmatrix}$$

Question 11(c)(ii)

$$AA^{-1} = \begin{pmatrix}-1 & 0\\\frac{3}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}-1 & 0\\3 & 2\end{pmatrix} = \begin{pmatrix}(-1\times 1)+(0\times 3) & (-1\times 0)+(0\times 2)\\\left(\frac{3}{2}\times -1\right)+\left(\frac{1}{2}\times 3\right) & \left(\frac{3}{2}\times 0\right)+\left(\frac{1}{2}\times 2\right)\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

Question 11(c)(iii)

$$A^2 = \begin{pmatrix}-1 & 0\\3 & 2\end{pmatrix}\begin{pmatrix}-1 & 0\\3 & 2\end{pmatrix} = \begin{pmatrix}(-1\times -1)+(0\times 3) & (-1\times 0)+(0\times 2)\\(3\times -1)+(2\times 3) & (3\times 0)+(2\times 2)\end{pmatrix}$$
$$A^2 = \begin{pmatrix}1 & 0\\3 & 4\end{pmatrix}$$

Question 11(c)(iv)(a)

The number of columns of A is not equal to the number of rows of B.

Question11(c)(iv)(b)

The resulting matrix would be of form 3x2