Solutions to CSEC Maths P2 January 2019

Question 1(a)(i)

Required to evaluate $3.8 \times 10^2 + 1.7 \times 10^3$, giving your answer in standard form.

$$3.8 \times 10^2 + 1.7 \times 10^3 = (3.8 \times 10^2) + (1.7 \times 10^3)$$

$$= 380 + 1700$$

$$= 2080$$

$$= 2.08 \times 10^3 \qquad \text{(in standard form)}$$

Question 1(a)(ii)

Required to evaluate $\frac{\frac{1}{2} \times \frac{3}{5}}{3\frac{1}{2}}$, giving your answer as a fraction in its lowest terms.

$$\text{Numerator} = \frac{1}{2} \times \frac{3}{5}$$

$$= \frac{3}{10}$$

$$\text{Denominator} = 3\frac{1}{2}$$

$$= \frac{7}{2}$$

$$\therefore \text{Numerator} \div \text{Denominator} = \frac{3}{10} \div \frac{7}{2}$$

$$= \frac{3}{10} \times \frac{2}{7}$$

$$= \frac{3}{35}$$

Question 1(b)

Required to express the number 6 as a binary number.

$$\begin{array}{r|ll} 2 & 6 & \\ \hline 2 & 3 & r\,0 \\ \hline 2 & 1 & r\,1 \\ \hline & 0 & r\,1 \\ \hline \end{array}$$

$\therefore 6_{10} = 110_2$

Question 1(c)

Required to calculate how much the car is worth at the end of 2 years.

Initial cost of the car $= \$65\ 000$.

Percentage depreciation $= 8\%$

Value of car after depreciation $= (100 - 8)\%$

$= 92\%$

Value of the car after 1 year $= 92\%\ of\ \$65\ 000$

$= \frac{92}{100} \times 65\ 000$

$= \$59\ 800$

Value of the car after 2 years $= 92\%\ of\ \$59\ 800$

$= \frac{92}{100} \times 59\ 800$

$= \$55\ 016$

Question 1(d)

Required to determine the student's final mark as a percentage.

For Paper 01: Mark $= \frac{55}{100} \times 30$

$= 16.5$

For Paper 02: Mark $= \frac{60}{100} \times 50$

$= 30$

For Paper 03: Mark $= \frac{80}{100} \times 20$

$= 16$

Total Percentage Obtained $= \frac{16.5+30+16}{100} \times 100$

$= 62.5\%$

Question 2(a)(i)

Required to make x the subject of the formula.

$$y = \frac{x}{5} + 3p$$

$$\frac{x}{5} + 3p = y$$

$$\frac{x}{5} = y - 3p$$

$$x = 5(y - 3p)$$

Question 2(a)(ii)

Required to solve the following equation by factorization.

$$2x^2 - 9x = 0$$

$$x(2x - 9) = 0$$

Either $x = 0$ or $2x - 9 = 0$

$$2x = 9$$

$$x = \frac{9}{2}$$

Question 2(b)

Required to show that $l^2 - 3l - 378 = 0$.

Since the width is 3 metres less than the length, then the width is $(l - 3)$.

We are given that the area is $378\ m^2$.

So, we have,

$$\text{Area} = \text{length} \times \text{width}$$

$$378 = l \times (l - 3)$$

$$378 = l^2 - 3l$$

$$l^2 - 3l - 378 = 0$$

Question 2(c)(i)

Required to represent the information given as an equation in terms of F, e and k.

Force $\propto$ extension

$$F \propto e$$

$$\therefore F = ke$$

Question 2(c)(ii)

Required to find the value of x and y.

From the table, $F = 0.8$ when $e = 0.2$. So, we have,

$$0.8 = k(0.2)$$

$$k = \frac{0.8}{0.2}$$

$$k = 40$$

$$\therefore F = 40e$$

When $F = 25, e = x$. So, we have,

$$25 = 40x$$

$$x = \frac{25}{40}$$

$x = \frac{5}{8}$

When $F = y, e = 3.2$. So, we have,

$y = 40(3.2)$

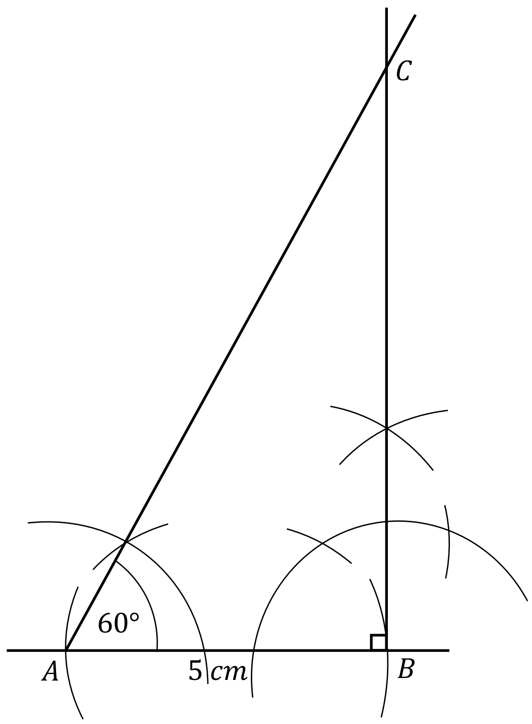
$y = 128$

$\therefore x = 0.625$ and $y = 128$

Question 3(a)

Required to construct the right-angled triangle ABC.

$AB = 5\ cm$, $\angle ABC = 90°$ and $\angle BAC = 60°$



Question 3(b)(i)(a)

Required to express c in terms of a and b.

Using Pythagoras' Theorem,

$c^2 = a^2 + b^2$

$c = \sqrt{a^2 + b^2}$

Question 3(b)(i)(b)

Required to write in terms of a, b and c, an expression for $\sin\theta + \cos\theta$.

$$\sin\theta = \frac{opp}{adj} = \frac{a}{c} \qquad \cos\theta = \frac{adj}{hyp} = \frac{b}{c}$$

$$\therefore \sin\theta + \cos\theta = \frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

Question 3(b)(ii)

Required to show that $(\sin\theta)^2 + (\cos\theta)^2 = 1$.

$$(\sin\theta)^2 + (\cos\theta)^2 = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$$

$$= \frac{a^2}{c^2} + \frac{b^2}{c^2}$$

$$= \frac{a^2+b^2}{c^2}$$

Since $c^2 = a^2 + b^2$, then we have

$$(\sin\theta)^2 + (\cos\theta)^2 = \frac{c^2}{c^2}$$

$$\therefore (\sin\theta)^2 + (\cos\theta)^2 = 1$$

Question 4(a)(i)

Required to determine the value of x for which the function is undefined.

The function is undefined when

$x - 5 = 0$

$x = 5$

Question 4(a)(ii)

Required to determine an expression for $h^{-1}(x)$.

$h(x) = \frac{2x+3}{5-x}$

Let $y = h(x)$.

$y = \frac{2x+3}{5-x}$

Interchange variables x and y.

$x = \frac{2y+3}{5-y}$

Make y the subject of the formula.

$x(5 - y) = 2y + 3$

$5x - xy = 2y + 3$

$2y + xy = 5x - 3$

$y(2 + x) = 5x - 3$

$y = \frac{5x-3}{2+x}$

Question 4(b)(i)

Required to determine the gradient of the line.

Two points on the line are $(0, 4)$ and $(5, 0)$.

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0-4}{5-0}$$

$$= -\frac{4}{5}$$

Question 4(b)(ii)

Required to determine the equation of the line.

Since the graph cuts the y-axis at $(0, 4)$, then the y-intercept $c = 4$.

Substituting $m = -\frac{4}{5}$ and $c = 4$ into the equation of the line gives:

$$y = mx + c$$

$$y = -\frac{4}{5}x + 4$$

Question 4(b)(iii)

Required to determine the equation of the perpendicular line that passes through P.

$$\text{The gradient of the perpendicular line} = \frac{-1}{-\frac{4}{5}}$$

$$= \frac{5}{4}$$

The equation of the perpendicular line is:

$$y - 0 = \frac{5}{4}(x - 5)$$

$y = \frac{5}{4}x - \frac{25}{4}$ or $4y = 5x - 25$

Question 5(a)(i)

Required to find the angle of the sector that will represent O-Fone.

$$\text{Angle of the sector to represent O-Fone} = \frac{Number\ of\ persons\ who\ use\ O{-}Fone}{Total\ number\ of\ persons\ in\ the\ survey} \times 360°$$

$$= \frac{10}{48} \times 360°$$

$$= 75°$$

Question 5(a)(ii)

Required to represent the information in the table on a clearly labelled pie chart.

$$\text{Angle of the sector to represent WireTech} = \frac{Number\ of\ persons\ who\ use\ \text{WireTech}}{Total\ number\ of\ persons\ in\ the\ survey} \times 360°$$

$$= \frac{20}{48} \times 360°$$

$$= 150°$$

$$\text{Angle of the sector to represent DigiLec} = \frac{Number\ of\ persons\ who\ use\ \text{DigiLec}}{Total\ number\ of\ persons\ in\ the\ survey} \times 360°$$

$$= \frac{12}{48} \times 360°$$

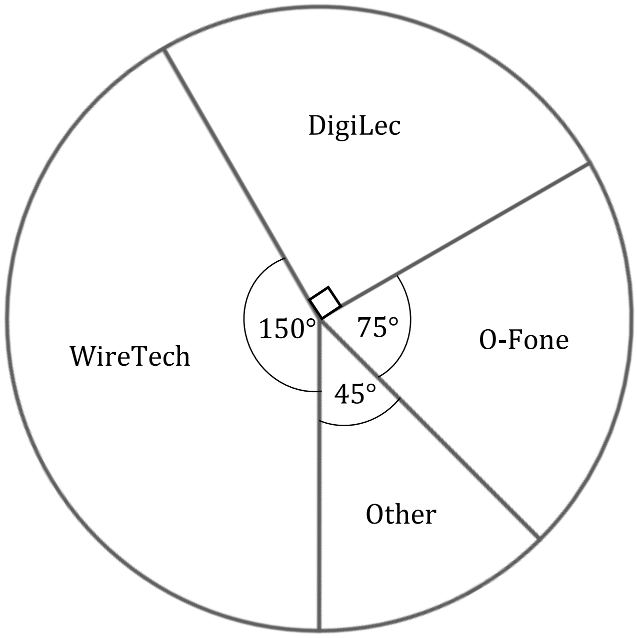
$$= 90°$$

$$\text{Angle of the sector to represent Other} = \frac{Number\ of\ persons\ who\ use\ \text{Other}}{Total\ number\ of\ persons\ in\ the\ survey} \times 360°$$

$$= \frac{6}{48} \times 360°$$

$$= 45°$$

Title: Pie Chart showing the mobile network used by 48 persons.



Question 5(b)(i)

Required to determine the probability that a girl chosen at random achieves a Grade 1.

$$P(girl\ achieves\ Grade\ 1) = \frac{Number\ of\ girls\ who\ achieved\ Grade\ 1}{Total\ number\ of\ girls}$$

$$= \frac{62}{250}$$

$$= \frac{31}{125}$$

Question 5(b)(ii)

Required to determine the percentage of boys who took the exam and achieved Grades I to III.

Number of boys who achieved Grades I to III $= 200 - (30 + 8)$

$$= 162$$

Percentage $= \frac{162}{200} \times 100$

$$= 81\%$$

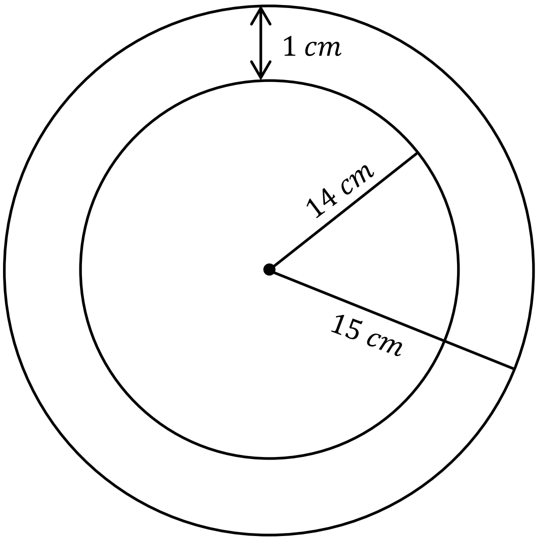
Question 5(b)(iii)

Required to compare the performance of the boys and the girls, considering the standard deviation in the table.

The standard deviation of the boy's performances is larger than the standard deviation of the girls' performances. Since standard deviation is a measure of the spread of the data it means that the marks of the boys were more 'spread out" or distributed over its range than the marks of the girls.

Question 6(a)

Required to draw a cross-sectional view of the container showing the measurements of the inner and outer radii.



Question 6(b)

Required to show that the capacity of the container is $73\ 304\ cm^3$.

$$\text{Volume} = \pi r^2 h$$

$$= \frac{22}{7} \times (14)^2 \times 119$$

$$= 73\ 304\ cm^3$$

Hence, the capacity of the container is $73\ 304\ cm^3$.

Question 6(c)

Required to determine the volume of the material used to make the container.

External volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times (15)^2 \times 120$$

$$= 84\ 857.14\ cm^3$$

Volume of the material used

= External volume of the container – Internal volume of the container

$= 84\ 857.14 - 73\ 304$

$= 11\ 553.14$ (to 2 decimal places)

Question 6(d)

Required to determine the mass, in kg, of the empty container.

Density $= \frac{Mass}{Volume}$

∴ Mass = Density × Volume

$= 2.2 \times 11\ 553.14$

$= 25\ 416.91\ g$

Now,

$1000\ g = 1\ kg$

$1\ g = \frac{1}{1000}$

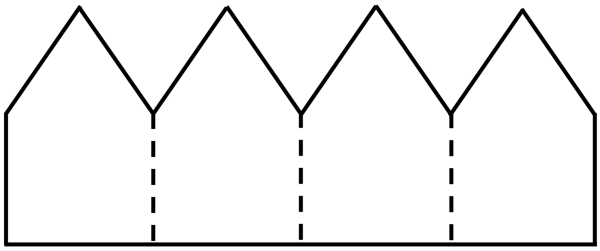
$25\ 416.91\ g = \frac{1}{1000} \times\ 25\ 416.91$

$= 25.4\ kg$ (to 2 decimal places)

Question 7(a)

Required to draw the 4th figure of th sequence.

Figure 4 of the sequence is shown below.



Question 7(b)

Required to complete the table given.

	Figure 1	Number of Outer Lines of Unit Length	Perimeter
	1	$1+2+2$	5
	2	$2+2+4$	8
	3	$3+2+6$	11
	⋮	⋮	⋮
(i)	6	$6+2+12$	20
	⋮	⋮	⋮
(ii)	21	$21+2+42$	65

(iii) ________ ________

$\vdots$	$\vdots$	$\vdots$
n	$n+2+2n$	$3n+2$

(i) For Figure 6,

$L = 6 + 2 + 2(6)$

$= 6 + 2 + 12$

For Figure 6,

Perimeter $= 6 + 2 + 12$

$= 20$

(ii) Since the perimeter $= 65$

Then, $3n + 2 = 65$

$3n = 65 - 2$

$3n = 63$

$n = \frac{63}{3}$

$n = 21$

For Figure 21,

$L = 21 + 2 + 2(21)$

$= 21 + 2 + 42$

(iii) For Figure n,

$L = n + 2 + 2n$

Perimeter $= 3n + 2$

Question 7(c)

Required to show that no figure can have a perimeter of 100 units.

If $P = 100$, then

$3n + 2 = 100$

$3n = 100 - 2$

$3n = 98$ (not divisible by 3)

$n = \frac{98}{3}$

However, n must be a positive integer and $\frac{98}{3}$ is not. So, $P \neq 100$ units.

Question 8(a)(i)

Required to complete the table for the function $f(x) = 3 + 2x - x^2$.

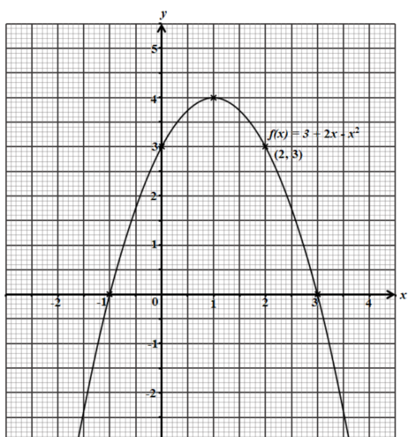
$$f(-2) = 3 + 2(-2) - (-2)^2$$
$$= 3 - 4 - 4$$
$$= -5$$

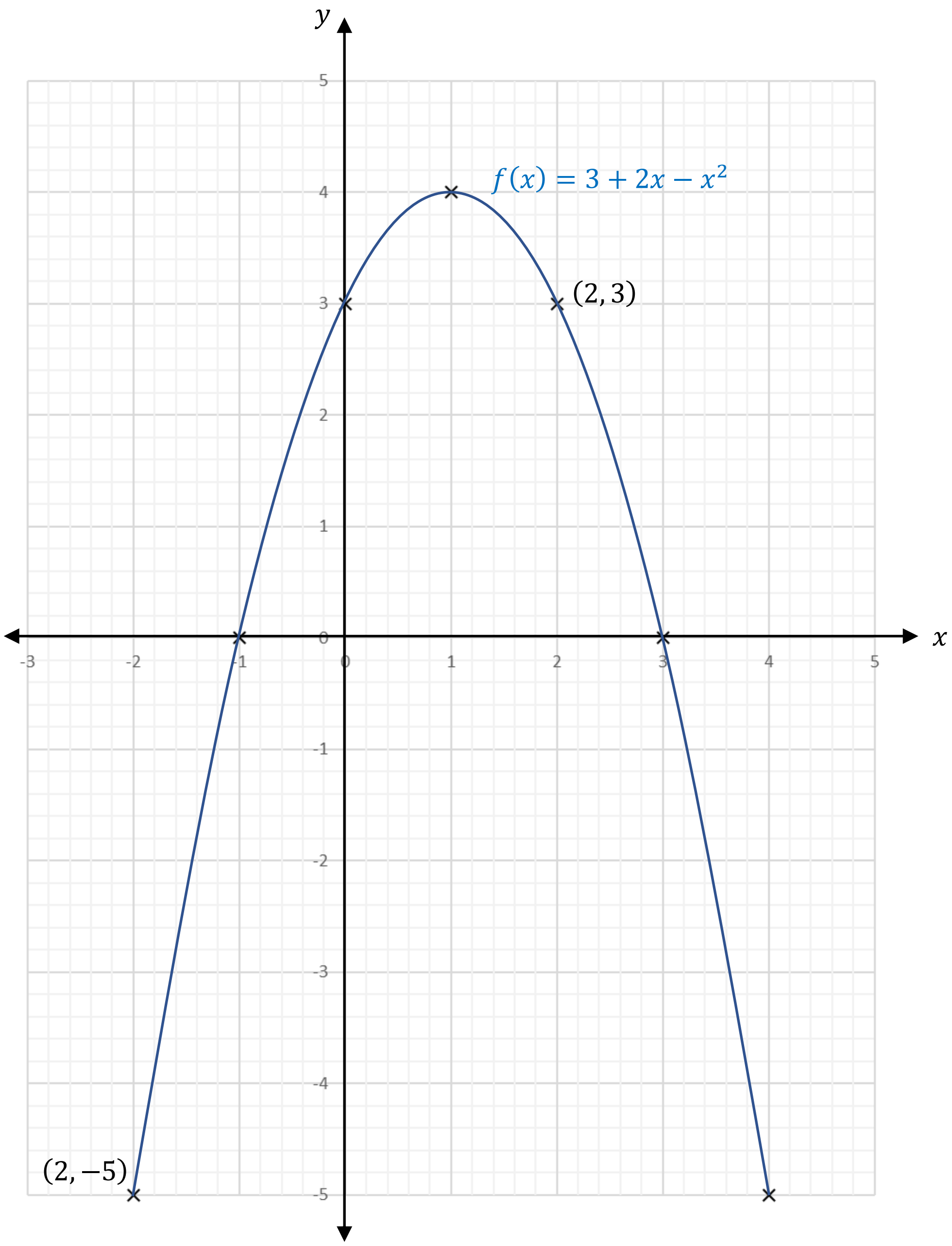
$$f(2) = 3 + 2(2) - (2)^2$$
$$= 3 + 4 - 4$$
$$= 3$$

x	-2	-1	0	1	2	3	4
$f(x)$	-5	0	3	4	3	0	-5

Question 8(a)(ii)

Required to complete the grid to show all the points in the table and hence, draw the function $f(x) = 3 + 2x - x^2$ for $-2 \leq x \leq 4$.





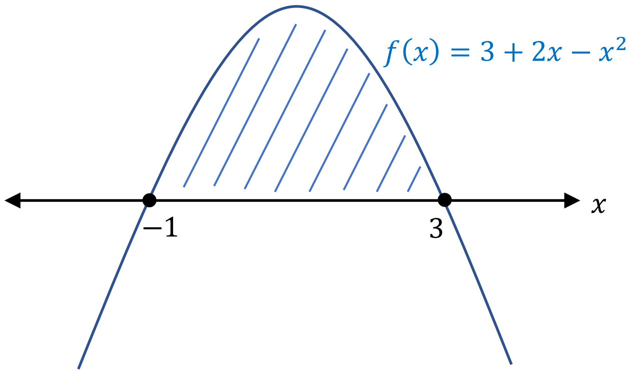
8(a)(iii)(a)

Required to determine the coordinates of the maximum point of $f(x)$.

From the graph, the maximum turning point of $f(x)$ is $(1, 4)$.

Question 8(a)(iii)(b)

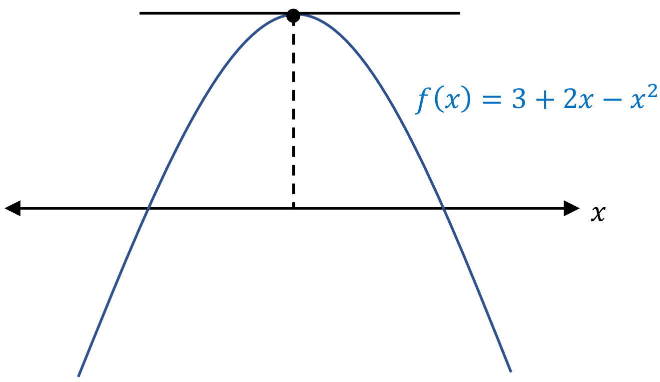
Required to determine the range of values of x for which $f(x) > 0$.



$f(x) > 0$ when $-1 < x < 3$.

Question 8(a)(iii)(c)

Required to determine the gradient of $f(x)$ at $x = 1$.



The tangent at $x = 1$ is a horizontal line whose gradient is zero.

Therefore, the gradient of $f(x)$ at $x = 1$ is 0.

Question 8(b)(i)

Required to write two inequalities to represent the information given.

The number of bottles of juice is at least 20.

The inequality is: $x \geq 20$

The number of cakes is no more than 15.

The inequality is: $y \leq 15$

Question 8(b)(ii)

Required to write an inequality for the required information.

The cost to make one bottle of juice is \$3.50.

The cost to make one cake is \$5.25.

Mr. Thomas spends \$163 each day to make the bottles of juice and the cakes.

The inequality is: $3.5x + 5.25y \leq 163$

Question 8(b)(iii)

Required to show that on any given day, it is not possible for Mr. Thomas to make 50 bottles of juice and 12 cakes.

50 bottles of juice at \$3.50 each $= 50 \times \$3.50$

$= \$175$

12 cakes at \$5.25 each $= 12 \times \$5.25$

$= \$63$

The total cost $= \$175 + \$63$

$= \$238$

Since \$238 is greater than \$163, it is not possible on his budget.

Question 9(a)(i)

Required to determine the value of the angle PTR, showing working where necessary and giving a reason to support the answer.

Consider the chord PR.

The angles subtended by a chord (PR) at the circumference of a circle $\left(P\hat{V}R \text{ and } P\hat{T}R\right)$ and standing on the same arc are equal.

Hence, $P\hat{T}R = 75°$.

Question 9(a)(ii)

Required to determine the value of the angle TPQ, showing working where necessary and giving a reason to support the answer.

Angles in a straight line add up to 180°.

$T\hat{R}Q = 180° - 60°$

$= 120°$

The opposite angles, $T\hat{P}Q$ and $T\hat{R}Q$, of a cyclic quadrilateral are supplementary.

$T\hat{P}Q = 180° - 120°$

$= 60°$

Question 9(a)(iii)

Required to determine the value of the angle POR, showing working where necessary and giving a reason to support the answer.

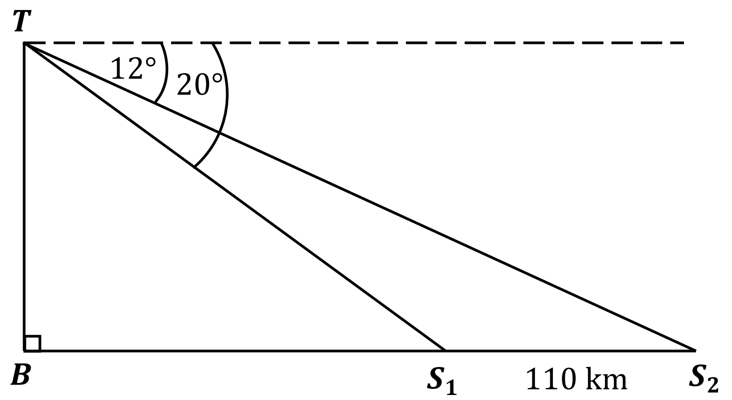
The angle subtended by a chord (PR) at the center of a circle $\left(P\hat{O}R\right)$ is twice the angle that the chord subtends at the circumference $\left(P\hat{T}R\right)$ standing on the same arc.

$$\therefore P\hat{O}R = 2(75°)$$
$$= 150°$$

Question 9(b)(i)

Required to complete the diagram below by inserting the angles of depression and the distance between the ships.

The completed diagram is shown below.



Question 9(b)(ii)(a)

Required to determine, to the nearest metre, the distance, TS_2, between the top of the lighthouse and Ship 2.

Consider ΔTS_1S_2.

Angle $S_1TS_2 = 20° - 12°$

$= 8°$

Angle $BTS_1 = 90° - 20°$

$= 70°$

Angle $TS_2B = 12°$ (alternate angles)

All angles in a triangle add up to $180°$.

Angle $TS_1S_2 = 180° - (8° + 12°)$

$= 180° - 20°$

$= 160°$

Using the sine rule,

$\frac{TS_2}{\sin 160°} = \frac{110}{\sin 8°}$

$TS_2 = \frac{110 \times \sin 160°}{\sin 8°}$

$= 270.3$

$= 270\ m$ (to the nearest metre)

Question 9(b)(ii)(b)

Required to determine the height of the lighthouse, TB.

Consider ΔTBS_2.

$\sin BS_2T = \frac{TB}{TS_2}$

$\sin 12° = \frac{TB}{270.3}$

$TB = \sin 12° \times 270.3$

$TB = 56\ m$ (to the nearest metre)

Question 10(a)(i)

Required to explain whether the matrix P is singular or non-singular.

We are given that $P = \begin{pmatrix} -1 & 2 \\ 0 & 5 \end{pmatrix}$.

$\det P = ad - bc$

$= (-1)(5) - (2)(0)$

$= -5 - 0$

$= -5$

Since the $\det P \neq 0$, then P is a non-singular matrix.

Question 10(a)(ii)

Required to determine the values of a and b.

$$PQ = R$$

$$\begin{pmatrix} -1 & 2 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 11 \\ 15 \end{pmatrix}$$

$$\begin{pmatrix} (-1 \times a) + (2 \times b) \\ (0 \times a) + (5 \times b) \end{pmatrix} = \begin{pmatrix} 11 \\ 15 \end{pmatrix}$$

$$\begin{pmatrix} -a + 2b \\ 5b \end{pmatrix} = \begin{pmatrix} 11 \\ 15 \end{pmatrix}$$

Comparing the equivalent matrices and equating corresponding entries, we get:

$5b = 15$

$b = \frac{15}{5}$

$b = 3$

Now,

$-a + 2b = 11$

$-a + 2(3) = 11$

$-a + 6 = 11$

$-a = 11 - 6$

$-a = 5$

$a = -5$

$\therefore a = -5$ and $b = 3$

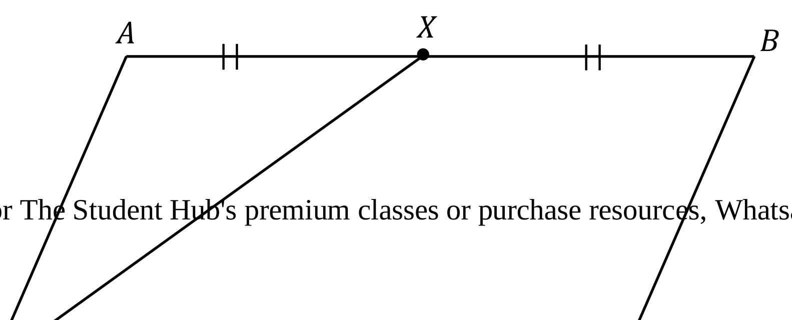
Question 10(a)(iii)

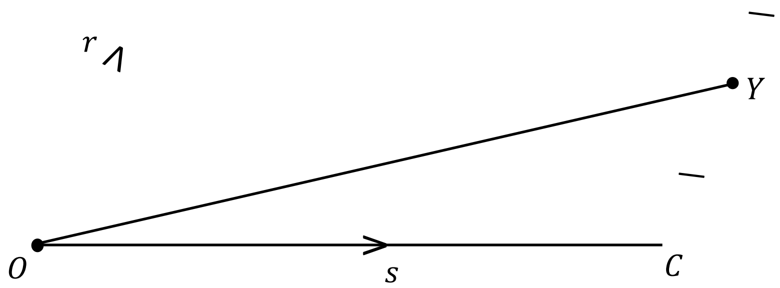
Required to state why the matrix QP is not possible.

The matrix multiplication QP is not possible because the number of columns in Q is not equal to the number of rows in P.

Question 10(b)(i)

Required to complete the diagram below to represent all the information given above.





Question 10(b)(ii)

Required to find the value of k.

Since $\overrightarrow{AB}$ and $\overrightarrow{OC}$ are opposite sides of the parallelogram, then

$\overrightarrow{AB} = \overrightarrow{OC}$

$\quad = s$

Since X is the midpoint of AB, then

$\overrightarrow{AX} = \frac{1}{2}\overrightarrow{AB}$

$\quad = \frac{1}{2}s$

Now,

$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$

$\quad = r + \frac{1}{2}s$

Since $\overrightarrow{CB}$ and $\overrightarrow{OA}$ are opposite sides of the parallelogram, then

$\overrightarrow{CB} = \overrightarrow{OA}$

$\quad = r$

Now,

$\overrightarrow{OY} = \overrightarrow{OC} + \overrightarrow{CY}$

$= s + \frac{1}{2}r$

$\therefore \overrightarrow{OX} + \overrightarrow{OY} = r + \frac{1}{2}s + s + \frac{1}{2}r$

$= \frac{3}{2}r + \frac{3}{2}s$

$= \frac{3}{2}(r + s)$ which is of the form $k(r + s)$, where $k = \frac{3}{2}$.