Solutions to CSEC Maths P2 January 2020

Question 1(a)(i)

Required to calculate the exact value of $4\frac{1}{5} \times \frac{1}{3} - 1\frac{1}{4}$.

$$4\frac{1}{5} \times \frac{1}{3} - 1\frac{1}{4} = \left(\frac{21}{5} \times \frac{1}{3}\right) - \frac{5}{4}$$

$$= \frac{7}{5} - \frac{5}{4}$$

$$= \frac{7(4) - 5(5)}{20}$$

$$= \frac{28 - 25}{20}$$

$$= \frac{3}{20} \qquad \text{(in exact form)}$$

Question 1(a)(ii)

Required to calculate the exact value of $\frac{4.1 - 1.25^2}{0.005}$.

$$\frac{4.1 - 1.25^2}{0.005} = \frac{2.5375}{0.005}$$

$$= 507.5 \qquad \text{(in exact form)}$$

Question 1(b)(i)

Required to calculate the number of people in the stadium when 75% of the seats are occupied.

$$\text{Number of people} = 75\% \text{ of } 15\ 400$$

$$= \frac{75}{100} \times 15\ 400$$

$$= 11\ 550$$

$\therefore$ The number of people in the stadium when 75% of the seats are occupied is 11 550 persons.

Question 1(b)(ii)

Required to calculate the percentage increase in the number of seats.

$$\text{Increase in number of seats} = 20\,790 - 15\,400$$
$$= 5390$$

$$\text{Percentage Increase} = \frac{\textit{Increase in number of seats}}{\textit{Original number of seats}} \times 100$$
$$= \frac{5390}{15\,400} \times 100$$
$$= 35\%$$

$\therefore$ The percentage increase in the number of seats is 35%.

Question 1(c)

Required to show that the light flashes 43 200 times in one day.

$$1 \text{ day} = 24 \text{ hours}$$

$$1 \text{ hour} = 60 \text{ minutes}$$
$$24 \text{ hours} = 24 \times 60$$
$$= 1440 \text{ minutes}$$

$$1 \text{ minute} = 60 \text{ seconds}$$
$$1440 \text{ minutes} = 1440 \times 60$$
$$= 86\,400 \text{ seconds}$$

The light flashed five times in each 10-second interval.

In 86 400 seconds, the number of 10-second intervals $= \frac{86\ 400}{10}$

$= 8640$

Since there are 5 flashes in every 10 seconds,

Then, the number of times the light flashes in one day $= 8640 \times 5$

$= 43\ 200$

Question 2(a)(i)

Required to factorise completely the expression $5h^2 - 12hg$.

$5h^2 - 12hg$

$= h(5h - 12g)$

Question 2(a)(ii)

Required to factorise completely the expression $2x^2 - x - 6$.

$2x^2 - x - 6$

$= 2x^2 - 4x + 3x - 6$

$= 2x(x - 2) + 3(x - 2)$

$= (2x + 3)(x - 2)$

Question 2(b)

Required to solve the equation $r + 3 = 3(r - 5)$.

$r + 3 = 3(r - 5)$

$r + 3 = 3r - 15$

$3 + 15 = 3r - r$

$18 = 2r$

$\frac{18}{2} = r$

$9 = r$

$\therefore r = 9$

Question 2(c)

Required to make k the subject of the formula.

$2A = \pi k^2 + 3t$

$\pi k^2 = 2A - 3t$

$k^2 = \frac{2A-3t}{\pi}$

$k = \sqrt{\frac{2A-3t}{\pi}}$

Question 2(d)

Required to write two inequalities to represent the scenario.

The area of the entire plot of land $= 10$ hectares

The number of hectares of corn $= c$

The number of hectares of potatoes $= p$

Given that the total area planted cannot exceed the area of the plot, then the inequality is: $c + p \geq 10$

Given that the number of hectares of corn is at least twice the number of hectares of potatoes, the inequality is: $c \geq 2p$

Question 3(a)

Required to complete the statement.

The prism has

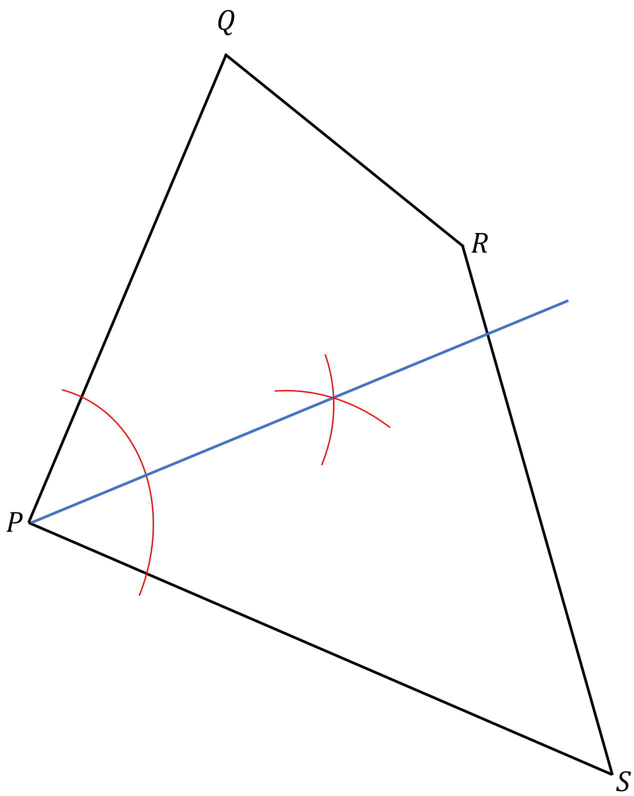
___8___ faces

___18___ edges and

___12___ vertices.

Question 3(b)(i)

Required to construct the line representing the fence.



Question 3(b)(ii)

Required to write down the length of the fence, in metres.

The line drawn is 7.4 cm by measurement.

Since 1 centimetre represents 10 metres, the length of the fence is

$7.4 \times 10 = 74$ metres

Question 3(c)(i)

Required to write down the mathematical name for the quadrilateral $PQRS$.

The quadrilateral $PQRS$ has only one pair of parallel sides.

Therefore, $PQRS$ is a trapezium.

Question 3(c)(ii)

Required to determine the values of a, b and k.

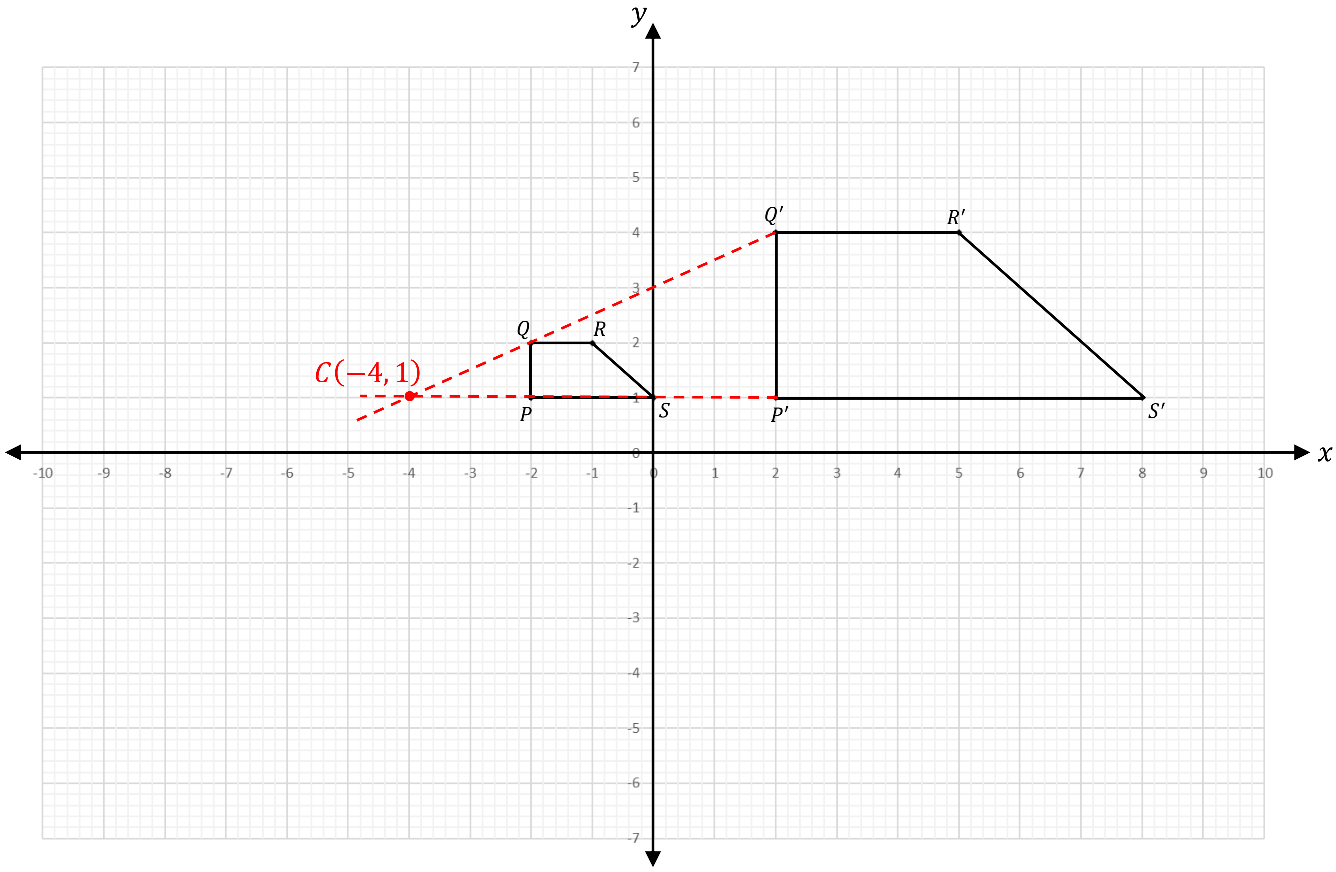
When joining the image points to their corresponding object points, the center of enlargement is found to be $C(-4, 1)$.

Hence, $a = -4$ and $b = 1$.

The scale factor $= \frac{P'Q'}{PQ}$

$$= \frac{3}{1}$$

$$= 3$$



Question 4(a)(i)

Find the value of $f(4) + f(-4)$.

$$f(4) = \frac{2(4)+7}{5}$$

$$= \frac{8+7}{5}$$

$$= \frac{15}{5}$$

$$= 3$$

$$f(-4) = \frac{2(-4)+7}{5}$$

$$= \frac{-8+7}{5}$$

$$= -\frac{1}{5}$$

Hence,

$$f(4) + f(-4) = 3 + \left(-\frac{1}{5}\right)$$

$$= 3 - \frac{1}{5}$$

$$= \frac{14}{5}$$

Question 4(a)(ii)(a)

Required to calculate the value of x for which $f(x) = 9$.

$f(x) = \frac{2x+7}{5}$

Since $f(x) = 9$, then

$\frac{2x+7}{5} = 9$

$2x + 7 = 9(5)$

$2x + 7 = 45$

$2x = 45 - 7$

$2x = 38$

$x = \frac{38}{2}$

$x = 19$

Question 4(a)(ii)(b)

Required to determine the value of $f^{-1}(9)$.

$f(x) = \frac{2x+7}{5}$

Let $y = f(x)$.

$y = \frac{2x+7}{5}$

Interchange variables x and y.

$x = \frac{2y+7}{5}$

Make y the subject of the formula.

$5x = 2y + 7$

$5x - 7 = 2y$

$y = \frac{5x-7}{2}$

$\therefore f^{-1}(x) = \frac{5x-7}{2}$

Question 4(b)(i)

Required to determine the equation of the line L_1.

The points are $(4, 0)$ and $(0, 2)$.

Gradient $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{2-0}{0-4}$

$= \frac{2}{-4}$

$= -\frac{1}{2}$

From point $(0, 2)$, it can be deduced that when $x = 0$, $y = 2$.

Hence, the y-intercept, $c = 2$.

The equation of the line L_1 is $y = -\frac{1}{2}x + 2$.

Question 4(b)(ii)

Required to determine the gradient of the line L_2.

Since L_1 and L_2 are perpendicular lines, the product of their gradients is -1.

Gradient of $L_2 = \frac{-1}{-\frac{1}{2}}$

$= 2$

Question 5(a)(i)

Required to estimate median.

The median will occur at the 50^{th} percentile.

Using the cumulative frequency curve, the median is estimated to be 3.2 grams.

Question 5(a)(ii)

Required to estimate upper quartile.

The upper quartile will occur at the 75^{th} percentile.

Using the cumulative frequency curve, the upper quartile is estimated to be 4.3 grams.

Question 5(a)(iii)

Required to estimate the semi-interquartile range.

The lower quartile will occur at the 25^{th} percentile.

Using the cumulative frequency curve, the lower quartile is estimated to be 2.3 grams.

$$\text{Semi-interquartile range} = \frac{Q_3 - Q_1}{2}$$

$$= \frac{4.3 - 2.3}{2}$$

$$= \frac{2}{2}$$

$= 1$

Question 5(a)(iv)

Required to determine the number of students whose estimate is 2.8 grams or less.

Using the cumulative frequency curve, a mass of 2.8 grams corresponds to 38.

Hence, 38 students had estimates that were less than 2.8 grams.

Question 5(b)(i)

Required to use the cumulative frequency curve given to complete the frequency table.

Mass of Seed, m (grams)	**Frequency**
$0 < m \leq 2$	20
$2 < m \leq 4$	48
$4 < m \leq 6$	25
$6 < m \leq 8$	6
$8 < m \leq 10$	1

Question 5(b)(ii)

Required to find the probability that the student estimated the mass to be greater than 6 grams.

$$P(mass > \ 6 \ grams) = \frac{Number\ of\ students\ who\ estimated\ the\ mass\ to\ be\ greater\ than\ 6g}{Total\ number\ of\ students}$$

$= \frac{7}{100}$ or 0.07 or 7%

Question 6(a)(i)

Required to show that the area of the rectangle is $2352\ cm^2$.

Diameter of circle $=$ 2 $\times$ Radius of circle

$= 2 \times 7$

$= 14\ cm$

Length of rectangle $= 4 \times 14$

$= 56\ cm$

Width of rectangle $= 3 \times 14$

$= 42\ cm$

Area of the rectangle $= l \times w$

$= 56 \times 42$

$= 2352\ cm^2$

Question 6(a)(ii)

Required to calculate the area of the shaded region.

Area of the 12 circles $= 12 \times \pi r^2$

$$= 12 \times \frac{22}{7} \times (7)^2$$

$$= 1848\ cm^2$$

Area of the shaded region = Area of the rectangle – Area of the 12 circles

$$= 2352 - 1848$$

$$= 504\ cm^2$$

Question 6(b)(i)

Required to state the type of triangle shown.

Since triangle MNP has two equal angles, then triangle MNP is isosceles.

Question 6(b)(ii)

Required to determine the value of angle PNM.

The sum of angles in a triangle add up to 180°.

Angle $PNM = 180° - 2(52°)$

$$= 180° - 104°$$

$$= 76°$$

Question 6(b)(iii)

Required to calculate the area of triangle MNP.

Area of triangle $MNP = \frac{1}{2}(MN)(NP)\sin M\widehat{N}P$

$= \frac{1}{2}(12.5)(12.5)\sin 76°$

$= 75.8\ cm^2$

Question 7(a)

Required to complete rows (i), (ii) and (iii)

	Figure	Number of Sticks (S)	Number of Dots (D)
	1	12	13
	2	24	25
	3	36	37
	4	48	49
(i)	5	60	61
	⋮	⋮	⋮
(ii)	13	156	157
	⋮	⋮	⋮
(iii)	n	$12n$	$12n + 1$

(i) For Figure 5,

Number of sticks, $S = 5 \times 12$

$= 60$ sticks

For Figure 5,

Number of dots $= 60 + 1$

$= 61$ dots

(ii) Since the number of dots $= 156$

Then, the Figure Number $= \frac{156}{12}$

$= 13$

For Figure 12,

Number of dots $= 156 + 1$

$= 157$ dots

(iii) For Figure n,

Number of sticks $= 12n$

Number of dots $= 12n + 1$

Question 7(b)(i)

Required to determine the total number of dots in Figure 7 and 8.

Number of dots in Figure 7 $= 12(7) + 1$

$= 84 + 1$

$= 85$

Number of dots in Figure 8 $= 12(8) + 1$

$= 96 + 1$

$= 97$

$\therefore$ Total number of dots in Figure 7 and 8 $= 85 + 97$

$$= 182$$

Question 7(b)(ii)

Required to determine the total number of dots in Figure n and Figure $(n + 1)$.

Number of dots in Figure $n = 12n + 1$

Number of dots in Figure $(n + 1) = 12(n + 1) + 1$

$\therefore$ Total number of dots in Figure n and Figure $(n + 1) = 12n + 1 + 12(n + 1) + 1$

$$= 12n + 1 + 12n + 12 + 1$$

$$= 24n + 14$$

Question 8(a)

Required to solve $y^2 + 2y + 11 = x$ and $x = 5 - 3y$ simultaneously.

$y^2 + 2y + 11 = x \quad \rightarrow$ Equation 1

$x = 5 - 3y \quad \rightarrow$ Equation 2

Equating Equation 1 and Equation 2 gives:

$$y^2 + 2y + 11 = 5 - 3y$$

$$y^2 + 2y + 3y + 11 - 5 = 0$$

$$y^2 + 5y + 6 = 0$$

$$y^2 + 3y + 2y + 6 = 0$$

$$y(y + 3) + 2(y + 3) = 0$$

$$(y + 2)(y + 3) = 0$$

Either $y + 2 = 0$ or $y + 3 = 0$

$y = -2$ $\qquad$ $y = -3$

When $y = -2$,

$$x = 5 - 3(-2)$$

$$= 5 - (-6)$$

$$= 5 + 6$$

When $y = -3$,

$$x = 5 - (-3)$$

$$= 5 - (-3)$$

$$= 5 + 3$$

$= 11$ $= 8$

$\therefore x = 11, y = -2$ and $x = 8, y = -3$.

Question 8(b)(i)

Required to express $f(x)$ in the form $a(x+h)^2 + k$.

$$f(x) = 4x^2 - 8x - 2$$
$$= 4(x^2 - 2x) - 2$$
$$= 4(x^2 - 2x + 1) - 2 - 4(1)$$
$$= 4(x-1)^2 - 2 - 4$$
$$= 4(x-1)^2 - 6$$

which is in the form $a(x+h)^2 + k$ where $a = 4, h = -1$ and $k = -6$.

Question 8(b)(ii)

Required to state the minimum value of $f(x)$.

Minimum value of $f(x) = k$

$= -6$

Question 8(c)

Required to determine the acceleration of the car during Stage 2 and Stage 3 of the car's journey.

Stage 2 of the journey is represented by a horizontal line.

This indicates that the car is travelling at a constant speed.

Since the gradient of a horizontal line is equal to 0, then the acceleration during Stage 2 is 0 ms^{-2}.

For the acceleration of the car during Stage 3, the points are $(32, 12)$ and $(40, 0)$.

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{12 - 0}{32 - 40}$$

$$= \frac{12}{-8}$$

$$= -\frac{3}{2}\ ms^{-2}$$

$\therefore$ The acceleration of the car is $-\frac{3}{2}\ ms^{-2}$ which indicates that the car is decelerating (slowing down).

Question 9(a)(i)

Required to write down the mathematical names of the straight lines BC and OA.

The straight line BC is a chord of the circle.

The straight line OA is a radius of the circle.

Question 9(a)(ii)(a)

Required to determine the value of the angle x, showing working where necessary and giving a reason to support the answer.

Consider ΔOAB.

Since OA and OB are both radii of the same circle, then $OA = OB$.

Hence, ΔOAB is isosceles.

The sum of angles in a triangle add up to 180°.

So, we have,

$x = 180° - (37° + 37°)$

$\quad = 180° - 74°$

$\quad = 104°$

Question 9(a)(ii)(b)

Required to determine the value of the angle y, showing working where necessary and giving a reason to support the answer.

Consider ΔBCD.

Since the angle in a semi-circle is a right angle, then Angle $BCD = 90°$.

The sum of angles in a triangle add up to $180°$.

So, we have,

$y = 180° - (90° + 49°)$

$= 180° - 139°$

$= 41°$

Question 9(b)(i)

Required to calculate the value of angle w.

Since angles in a straight line sum up to $180°$,

$x = 180° - 133°$

$= 47°$

Therefore,

$w = 180° - (56° + 47°)$

$= 180° - 103°$

$= 77°$

Question 9(b)(ii)

Required to determine the bearing of P from Q.

Bearing of P from $Q = 360° - 47°$

$= 313°$

Question 9(b)(iii)

Required to calculate the distance RP.

Consider the triangle PQR. Using the cosine rule, we have

$$RP^2 = PQ^2 + QR^2 - 2(PQ)(QR)\cos P\hat{Q}R$$

$$= (210)^2 + (290)^2 - 2(210)(290)\cos 56°$$

$$= 44\ 100 + 84\ 100 - (121\ 800)(0.559)$$

$$= 128\ 200 - 68\ 109.7$$

$$= 60\ 090.3$$

$$RP = \sqrt{60\ 090.3}$$

$$= 245.13\ km$$

Question 10(a)(i)

Required to determine the values of p and q.

The points are $R(2, -5)$ and $R'(5, 2)$.

We have,

$$\begin{pmatrix} 0 & p \\ q & 0 \end{pmatrix}\begin{pmatrix} 2 \\ -5 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} (0 \times 2) + (p \times -5) \\ (q \times 2) + (0 \times -5) \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 + (-5p) \\ 2q + 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} -5p \\ 2q \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

Comparing the equivalent matrices and equating corresponding entries gives:

$-5p = 5$ $\qquad\qquad$ $2q = 2$

$p = \frac{5}{-5}$ $\qquad\qquad$ $q = \frac{2}{2}$

$p = -1$ $\qquad\qquad$ $q = 1$

$\therefore p = -1$ and $q = 1$

Question 10(a)(ii)

Required to describe fully the transformation, M.

$$M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

This transformation is an anticlockwise rotation of 90° about the origin.

Question 10(b)(i)(a)

Required to write in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$ an expression for $\overrightarrow{QS}$.

$$\overrightarrow{QS} = \overrightarrow{QP} + \overrightarrow{PS}$$

$$= (-\boldsymbol{u}) + \boldsymbol{v}$$

$$= \boldsymbol{v} - \boldsymbol{u}$$

Question 10(b)(i)(b)

Required to write in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$ an expression for $\overrightarrow{QM}$.

Since $QM : MS = 1 : 2$, then

$$\overrightarrow{QM} = \frac{1}{3}\overrightarrow{QS}$$

$$= \frac{1}{3}(\boldsymbol{v} - \boldsymbol{u})$$

$$= \frac{1}{3}\boldsymbol{v} - \frac{1}{3}\boldsymbol{u}$$

Question 10(b)(ii)

Required to show that $\overrightarrow{MR} = \frac{1}{3}(\boldsymbol{u} + 2\boldsymbol{v})$.

$$\overrightarrow{MR} = \overrightarrow{MQ} + \overrightarrow{QR}$$

$$= -\overrightarrow{QM} + \overrightarrow{QR}$$

$$= -\left(\frac{1}{3}\boldsymbol{v} - \frac{1}{3}\boldsymbol{u}\right) + \boldsymbol{v}$$

$$= -\frac{1}{3}\boldsymbol{v} + \frac{1}{3}\boldsymbol{u} + \boldsymbol{v}$$

$$= \frac{1}{3}\boldsymbol{u} + \frac{\mathbf{2}}{\mathbf{3}}\boldsymbol{v}$$

$$= \frac{1}{3}(\boldsymbol{u} + 2\boldsymbol{v})$$

Question 10(b)(iii)

Required to prove that R, M and T are collinear.

We are given that T is the midpoint of PQ.

$$\overrightarrow{TR} = \overrightarrow{TQ} + \overrightarrow{QR}$$

$$= \frac{1}{2}\boldsymbol{u} + \boldsymbol{v}$$

$$= \frac{1}{2}(\boldsymbol{u} + 2\boldsymbol{v})$$

Now,

$$\overrightarrow{MR} = \frac{1}{3}(\boldsymbol{u} + 2\boldsymbol{v})$$

$$= \frac{2}{3}\left[\frac{1}{2}(\boldsymbol{u} + 2\boldsymbol{v})\right]$$

$$= \frac{2}{3}\overrightarrow{TR}$$

So, the vector $\overrightarrow{MR}$ is a scalar multiple of $\overrightarrow{TR}$.

Hence, $\overrightarrow{MR}$ is parallel to $\overrightarrow{TR}$.

Also, R is a common point.

Therefore, R, M and T are collinear.