Solutions to CSEC Maths P2 June 2011

Question 1a part (i)

Calculate the exact value of

$$\frac{2\frac{1}{4} + 1\frac{1}{8}}{4\frac{1}{2}}$$

$$= \frac{\frac{9}{4} + \frac{9}{8}}{\frac{9}{2}}$$

$$= \frac{\frac{2(9) + 1(9)}{8}}{\frac{9}{2}}$$

$$= \frac{\frac{18 + 9}{9}}{\frac{9}{2}}$$

$$= \frac{\frac{27}{8}}{\frac{9}{2}}$$

$$= \frac{\frac{27}{8} \div \frac{9}{2}}{\frac{9}{2}}$$

$$= \frac{27}{8} \div \frac{9}{2}$$

### Question 1a part (ii)

Calculate the exact value of

$$3.96 \times 0.25 - \sqrt{0.0256}$$
  
= 0.99 - 0.16  
= 0.83

# Question 1b

### Data Given

Pamela's shopping bill as shown below.

Items	Quantity	Unit Price \$	Total Cost \$
Rice	$6\frac{1}{2}$ kg	2.40	W
Potatoes	4 bags	X	52.80
Milk	<i>Y</i> cartons	2.35	14.10
Sub Total			82.50
Z % VAT			9.90
TOTAL			92.40

 $\label{eq:calculate} \textbf{Calculate} \ \textbf{the values of W, X, Y and Z}.$ 

$$W = Quantity of rice \times Unit Price$$
$$W = 6.5 \times \$2.40$$
$$= \$15.60$$
$$X = \frac{Total \ cost \ of \ potatoes}{Quantity \ of \ potatoes}$$
$$X = \frac{\$52.80}{4 \ bags}$$
$$= \$13.20$$
$$Y = \frac{Total \ cost \ of \ milk}{Unit \ Price \ of \ milk}$$
$$Y = \frac{\$14.10}{\$2.35}$$
$$= 6$$
$$Z = \frac{VAT}{Sub \ Total} \times 100$$
$$Z = \frac{\$9.90}{\$82.50} \times 100$$
$$= 12$$

Question 2a

Simplify  $\frac{x-2}{3} + \frac{x+1}{4}$ 

$$\frac{x-2}{3} + \frac{x+1}{4}$$
$$= \frac{4(x-2) + 3(x+1)}{12}$$
$$= \frac{4x-8+3x+3}{12}$$
$$= \frac{7x-5}{12}$$
 as a fraction in its lowest terms

Question 2b

Data Given

$$a * b = (a + b)^2 - 2ab$$

**Calculate** the value of 3 \* 4

$$3 * 4 = (3 + 4)^{2} - 2(3)(4)$$
$$= 49 - 24$$
$$= 25$$

#### Question 2c

(i) Factorise  $xy^3 + x^2y$ 

$$xy^{3} + x^{2}y = xy \cdot y^{2} + xy \cdot x$$
$$= xy (y^{2} + x)$$

(ii) Factorise 2mh - 2nh - 3mk + 3nk

$$2mh - 2nh - 3mk + 3nk = 2h(m - n) - 3k(m - n)$$
$$= (2h - 3k)(m - n)$$

#### Question 2d

#### Data Given

y varies directly to x

x	2	5	b
у	12	а	48

Calculate the values of a and b

 $y \propto x$ y = kx

Using x = 2 and y = 12

$$12 = k(2)$$
  
 $k = 6$ 

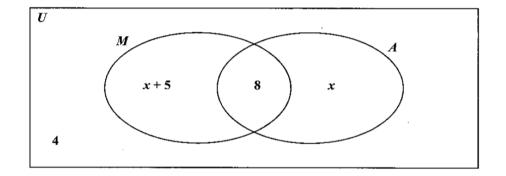
a = 6(5)a = 30

48 = 6(b)b = 8

 $\therefore$  a = 30 and b = 8.

#### Question 3a

#### Data Given



(i) **Find** the number of students who do not study either Art or Music.

$$(MUA)^{,} = 4$$

4 students study neither Art nor Music

(ii) **Calculate** the value of x

$$35 = 4 + (x + 5) + 8 + x$$
$$35 = 2x + 17$$
$$2x = 18$$
$$x = 9$$

(iii) **State** the number of students who study music only

Number of students who study music only = x + 5

= 9 + 5

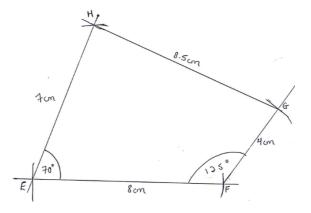
= 14 *students* 

Question 3b part (i)

### Data Given

EF = 8cm  $E\hat{F}G = 125^{\circ}$  FG = 4cm  $H\hat{E}F = 70^{\circ}$  EH = 7 cm

**Draw** quadrilateral *EFGH* 



Question 3b part (ii)

Measure and state the length of GH

Length of GH = 8.5 cm

#### Question 4a part (i)

**Solve** 5 - 2x < 9

$$5 - 2x < 9$$
$$5 < 9 + 2x$$
$$5 - 9 < 2x$$
$$-4 < 2x$$
$$2x > -4$$
$$x > -2$$

Question 4a part (ii)

**Determine** the smallest value of x that satisfies 5 - 2x < 9

For 
$$5-2x < 9$$
  
 $x > -2$   
 $\therefore x_{min} = -1$ 

### Question 4b part (i)

Data Given

$$\pi = 3.14$$

(a) **Calculate** the length of each side of the square

Area of Square = 
$$S \times S$$
  
 $121 = S^2$   
 $S = \sqrt{121}$ 

 $S = 11 \, cm$ 

(b) Calculate the perimeter of the square

Perimeter of the Square =  $4 \times S$ =  $4 \times 11$ = 44 cm

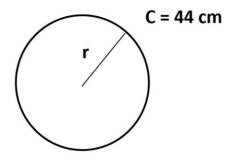
Question 4b part (ii)

### Data Given

The piece of wire used to form the square is used to form a circle.

(a) **Calculate** the radius of the circle

*Perimeter of the square = Circumference of the circle = 44cm* 



*Circumference of the circle* =  $2\pi r$ 

$$44 = 2\pi r$$
$$r = \frac{44}{2\pi}$$
$$r = \frac{44}{2\left(\frac{22}{7}\right)}$$
$$r = 7 \ cm$$

(a) **Calculate** the area of the circle

Area of the circle  $= \pi r^2$ 

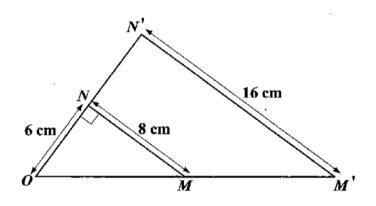
$$= \pi \times 7^{2}$$
$$= \frac{22}{7} \times 49$$
$$= 154 \ cm^{2}$$

Question 5a

#### Data Given

Triangle OMN and its image OM'N' under an enlargement with centre, O, and scale factor, k.

 $O\widehat{M}N = 90^{\circ}$ 



(i) **Calculate** the value of k

Length of NM for triangle ONM = 8 cmLength of N'M' for triangle ON'M' = 16 cm

 $k = \frac{\text{Length of } N'M' \text{ for triangle } ON'M'}{\text{Length of } NM \text{ for triangle } ONM}$  $k = \frac{16}{8}$ k = 2

(ii) **Calculate** the length of OM

Using Pythagoras' Theorem

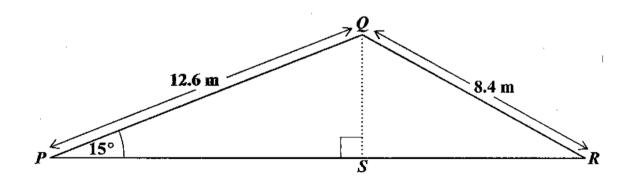
$$OM^{2} = ON^{2} + NM^{2}$$
$$OM^{2} = 6^{2} + 8^{2}$$
$$OM^{2} = 100$$
$$OM = \sqrt{100}$$
$$OM = 10 \ cm$$

(iii) **Calculate** the length of OM'

Length of 
$$0M' = k \times length of OM$$
  
= 2 × 10  
= 20 cm

### Question 5b

#### Data Given



(i) **Calculate** the length of QS

Considering triangle PQS

$$sin sin 15^{\circ} = \frac{opp}{hyp}$$

$$sin sin 15^{\circ} = \frac{QS}{PQ}$$

$$QS = PQ \times sin sin 15^{\circ}$$

$$= 12.6 \times sin sin 15^{\circ}$$

$$= 3.26 m$$

### (ii) **Calculate** $R\hat{Q}S$

Considering triangle RQS

$$\cos \cos R\hat{Q}S = \frac{adj}{hyp}$$
$$\cos \cos R\hat{Q}S = \frac{QS}{QR}$$
$$\cos \cos R\hat{Q}S = \frac{3.26}{8.4}$$
$$R\hat{Q}S = \left(\frac{3.26}{8.4}\right)$$
$$= 67.2^{\circ}$$

(iii) **Calculate** the area of PQR

Area of  $PQR = \frac{1}{2} (QP) (QR) sin sin PQR$ 

$$QP = 12.6 m$$
  
 $QR = 8.4 m$ 

$$P\hat{Q}R = P\hat{Q}S + R\hat{Q}S$$
$$P\hat{Q}S = 180^{\circ} - (15^{\circ} + 90^{\circ})$$
$$P\hat{Q}S = 180^{\circ} - 105^{\circ}$$
$$= 75^{\circ}$$

$$P\hat{Q}R = 75^{\circ} + 67.2^{\circ} = 142.2^{\circ}$$

Area of 
$$PQR = \frac{1}{2} (QP) (QR) \sin \sin P\hat{Q}R$$

Area of 
$$PQR = \frac{1}{2}$$
 (12.6) (8.4) sin sin 142.2°  
= 32.4 m<sup>2</sup>

Question 6a part (i)

Data Given

$$f(x) = 6x + 8 and g(x) = \frac{x-2}{3}$$

Calculate  $g\left(\frac{1}{2}\right)$ 

$$g(x) = \frac{x-2}{3}$$
$$g\left(\frac{1}{2}\right) = \frac{\frac{1}{2}-2}{3}$$
$$= \frac{-\frac{3}{2}}{3}$$
$$= -\frac{1}{2}$$

Question 6a part (ii)

**Write** an expression in terms of *x* for gf(x)

Replacing *x* in g(x) with f(x)

$$gf(x) = \frac{6x+8-2}{3}$$
$$= \frac{6x+6}{3}$$
$$= 2x+2$$

Question 6a part (iii)

**Find** the inverse function  $f^{-1}(x)$ 

```
Let y = f(x)

y = 6x + 8

Switch x and y

x = 6y + 8

6y = x - 8

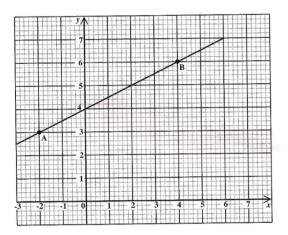
y = \frac{x - 8}{6}
```

$$\therefore f^{-1}(x) = \frac{x-8}{6}$$

### Question 6b

### Data Given

Line segment passing through two points, A and B.



(i) **Determine** the coordinates of A and B

Reading off the graph;

A = (-2, 3) and B = (4, 6)

(ii) **Determine** the gradient of the line segment AB

$$gradient, m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$A = (x_1, y_1) = (-2, 3)$$

$$B = (x_2, y_2) = (4, 6)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{6 - 3}{4 - (-2)}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

#### (iii) **Determine** the equation of the line passing through A and B

Using Point B (4, 6)

$$y - y_1 = m (x - x_1)$$
$$y - 6 = \frac{1}{2} (x - 4)$$
$$y - 6 = \frac{1}{2} x - 2$$
$$y = \frac{1}{2} x - 2 + 6$$
$$y = \frac{1}{2} x + 4$$
$$2y = x + 8$$

### Question 7a

Data Given: Table showing the distribution of the masses of 100 packages

**Copy and complete** the table to show the cumulative frequency of the distribution

Mass (kg)	No. of Packages	Cumulative Frequency
1-10	12	12
11-20	28	40
21-30	30	70
31-40	22	92
41-50	8	100

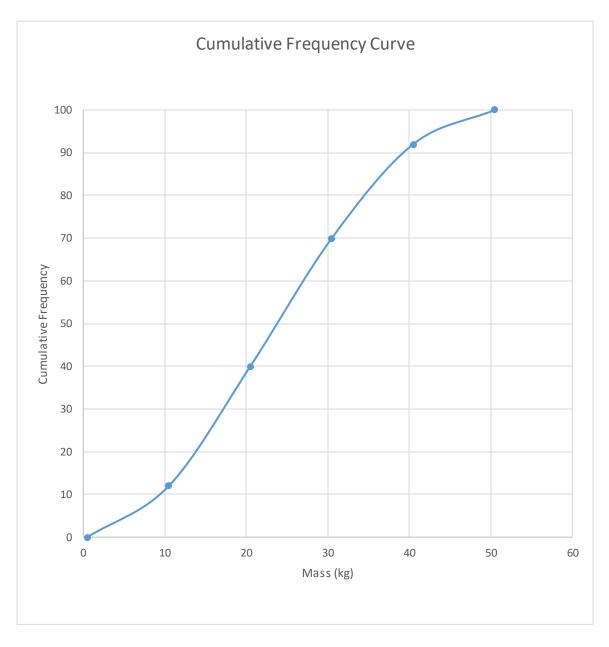
For

21-30 kg;	Cumulative Frequency = 40 + 30 = 70
31-40 kg;	Cumulative Frequency $= 70 + 22 = 92$
41-50 kg;	Cumulative Frequency = $92 + 8 = 100$

### Question 7b

Draw the cumulative frequency curve

Mass (kg)	Lower Class	Upper Class	No. of Packages	Cumulative
	Boundary	Boundary		Frequency
-	-	-	0	0
1-10	0.5	10.5	12	12
11-20	10.5	20.5	28	40
21-30	20.5	30.5	30	70
31-40	30.5	40.5	22	92
41-50	40.5	50.5	8	100



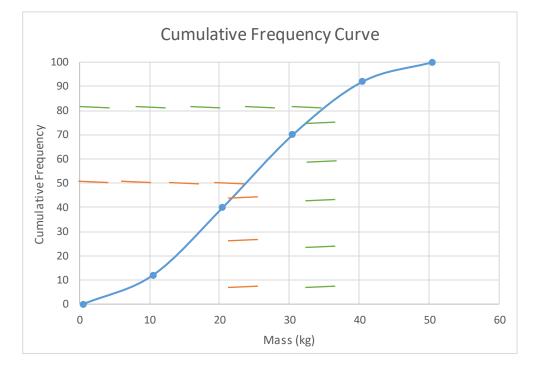
### Points to be plotted= (Upper Class Boundary, Cumulative Frequency)

Where

y - axis; 1 cm = 10 packages x - axis; 2 cm = 10 kg

#### Question 7c

#### (i) **Estimate** from the graph the median mass of the packages



 $\frac{Cumulative Frequency}{2} = 100$   $\frac{Cumulative Frequency}{2} = 50$ 

Based on the graph above; when cumulative frequency = 50; Median mass of the packages = 24 kg

(ii) **Estimate** from the graph the probability that a randomly chosen package has a mass less than 35 kg

$$P(mass < 35 kg) = \frac{Number of packages with mass < 35kg}{Total number of packages}$$

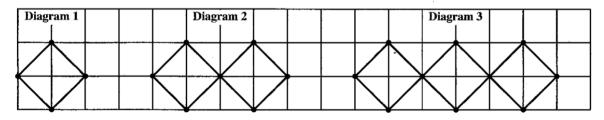
Based on the graph above; when mass of package = 35 kg;

#### Cumulative frequency = 81

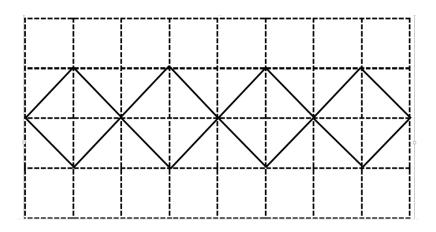
$$P(mass < 35 \ kg) = \frac{81}{100}$$

Question 8a

Data Given



Draw the fourth diagram



#### Question 8b

(i) **Determine** the number of sticks in the sixth diagram

Based on the data given each successive diagram increases by 4 sticks.

No. of sticks in 4<sup>th</sup> diagram = 16 sticks

No. of sticks in  $5^{\text{th}}$  diagram = 16 + 4 = 20 sticks

Therefore, no. of sticks in 6th diagram = 20 + 4 = 24 sticks

(ii) **Determine** the number of thumb tacks in the seventh diagram

Based on the data given each successive diagram increases by 3 thumb tacks.

No. of thumb tacks in  $4^{\text{th}}$  diagram = 13 thumb tacks No. of thumb tacks in  $5^{\text{th}}$  diagram = 13 + 3 = 16 thumb tacks No. of thumb tacks in  $6^{\text{th}}$  diagram = 16 + 3 = 19 thumb tacks

Therefore, no. of thumb tacks in 7th diagram = 19 + 3 = 22 thumb tacks

Question 8c

(i)

(ii)

Complete the table by filling in (i) and (ii)

No. of Sticks s	Rule Connecting t and s	No. of Thumb Tacks t
4	$1 + \left(\frac{3}{4} \times 4\right)$	4
8	$1 + \left(\frac{3}{4} \times 8\right)$	7
12	$1 + \left(\frac{3}{4} \times 12\right)$	10
52	$1 + \left(\frac{3}{4} \times 52\right)$	40
72	$1 + \left(\frac{3}{4} \times 72\right)$	55

(i) Rule connecting t and  $s = 1 + \left(\frac{3}{4} \times 52\right) = 1 + (3 \times 13) = 40$  thumb tacks

(ii) Number of thumb tacks = 
$$1 + \left(\frac{3}{4} \times s\right)$$
  
 $55 = 1 + \left(\frac{3}{4} \times s\right)$   
 $\frac{3}{4} \times s = 54$   
 $s = \frac{54}{\frac{3}{4}}$   
 $= 72$ 

### Question 8d

Write the rule connecting t and s to demonstrate the relationship between them

$$t = 1 + \left(\frac{3}{4} \times s\right)$$

#### Question 9a

Solve the pair of simultaneous equations

 $y = x^{2} - x + 3 \dots \dots Equation 1$  $y = 6 - 3x \dots \dots Equation 2$ 

Solving simultaneously

Equating Equation 1 and Equation 2

 $6 - 3x = x^2 - x + 3$ 

 $x^2 + 2x - 3 = 0$ 

Factorizing the Quadratic

$$x^{2} + 3x - x - 3 = 0$$
  

$$x(x + 3) - 1(x + 3) = 0$$
  

$$(x + 3) (x - 1) = 0$$
  

$$x = -3 \text{ or } x = 1$$

When 
$$x = -3$$
;  $y = 6 - 3x$   
 $= 6 - 3(-3)$   
 $= 6 - 3(-3)$   
 $= 6 + 9$   
 $= 15$   
When  $x = 1$ ;  $y = 6 - 3x$   
 $= 6 - 3(1)$   
 $= 6 - 3$   
 $= 3$ 

x	-3	1
у	15	3

Question 9b part (i)

**Express** the function  $f(x) = 4x^2 - 8x - 2$ , in the form  $a(x + h)^2 + k$ 

$$4x^{2} - 8x - 2 \equiv a(x+h)^{2} + k$$
$$\equiv [a (x+h)(x+h)] + k$$
$$\equiv [a (x^{2} + 2hx + h^{2})] + k$$
$$\equiv ax^{2} + 2ahx + ah^{2} + k$$

Equating coefficients

 $x^2$ ; a = 4

$$x; 2ah = -8$$
$$2(4)h = -8$$
$$8h = -8$$
$$h = -1$$

constant;  $ah^{2} + k = -2$ (4)(-1)<sup>2</sup> + k = -24 + k = -2k = -6

$$\therefore 4x^2 - 8x - 2 \equiv 4(x - 1)^2 - 6$$

in the form  $a(x+h)^2 + k$  where a = 4, h = -1 and k = -6.

Question 9b part (ii)

**State** the maximum value of  $4x^2 - 8x - 2$ 

$$4x^{2} - 8x - 2 \equiv 4(x - 1)^{2} - 6$$
  
Maximum value of  $4x^{2} - 8x - 2 = 4(0) - 6$   
 $= -6$ 

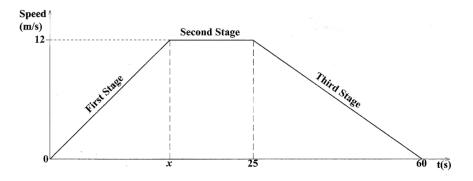
Question 9b part (iii)

**State** the value of *x* for which f(x) is a minimum

f(x) is a minimum when  $4(x - 1)^2 = 0$  and x = 1.

### Question 9c

Data Given



In the 1<sup>st</sup> Stage, the speed of the car increases from 0 m/s to 12 m/s in x seconds accelerating at 0.6  $m/s^2$ .

#### (i) **Calculate** the value of x

Gradient of 1st Stage = Acceleration in 1st Stage

$$\frac{y_2 - y_1}{x_2 - x_1} = 0.6$$

Where Points being used are (0,0) and (x, 12).

$$\frac{12-0}{x-0} = 0.6$$
$$\frac{12}{x} = 0.6$$
$$x = \frac{12}{0.6}$$
$$x = 20 s$$

(ii) **Explain** the gradient of the graph in the 2<sup>nd</sup> Stage with respect to the car's actions.

In the  $2^{nd}$  stage, the car is moving at a constant speed of 12 m/s so the gradient is fixed as a change in acceleration only would affect the gradient.

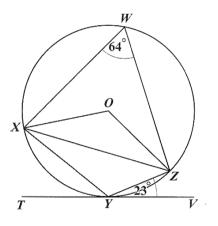
(iii) **Calculate** the distance travelled by the car in the 3<sup>rd</sup> Stage

Since area under a speed-time graph gives distance, we consider the 3<sup>rd</sup> stage as a separate triangle.

$$Area = \frac{B \times H}{2}$$
$$= \frac{(60 - 25) \times 12}{2}$$
$$= 210 m$$

## Question 10a

#### Data Given



(i) **Calculate** < XYZ

XYZO is a quadrilateral so the opposite angles are supplementary and the total sum of angles in a triangle is 180°.

$$X\hat{Y}Z = 180^{\circ} - 64^{\circ} = 116^{\circ}$$

(ii) **Calculate** < YXZ

For a tangent, at its point of contact with the circle the angle created would be equivalent to the angle in the alternate segment.

$$Y\hat{X}Z = Z\hat{Y}V = 23^{\circ}$$

(iii) **Calculate** < *OXZ* 

Triangle OXZ is an isosceles triangle since the radius of a circle is constant;

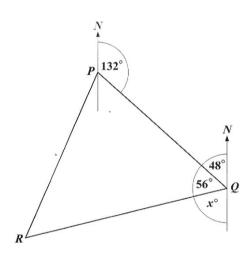
Length of 
$$OX = Length of OZ = radius of the circle$$

Therefore the base angles would be equivalent.

$$O\hat{X}Z = \frac{180^{\circ} - 128^{\circ}}{2} \quad since the total sum of angles in a triangle = 180^{\circ}$$
$$= \frac{52^{\circ}}{2}$$
$$= 26^{\circ}$$

Question 10b part (i)

Data Given



#### **Calculate** the value of *x*

# Since the total sum of angles for a straight line = $180^{\circ}$ $x = 180^{\circ} - (56^{\circ} + 48^{\circ})$ $= 76^{\circ}$

Question 10b part (ii)

#### Data Given

PQ = 220 km

QR = 360 km

Calculate the distance RP

Applying cosine rule  $a^2 = b^2 + c^2 - 2bc \cos \cos A$ 

Where

a = RP, b = PQ, c = QR and  $A = P\hat{Q}R$ 

$$a^{2} = b^{2} + c^{2} - 2bc \cos \cos A$$

$$RP^{2} = PQ^{2} + QR^{2} - 2(PQ)(QR) \cos \cos (P\hat{Q}R)$$

$$RP^{2} = (220)^{2} + (360)^{2} - 2(220)(360) \cos \cos (56^{\circ})$$

$$RP^{2} = 48400 + 129600 - (158400)(\cos \cos 56^{\circ})$$

$$RP = \sqrt{[48400 + 129600 - (158400)(\cos \cos 56^{\circ})]}$$

$$RP = 299 \, km$$

Question 10b part (iii)

Calculate the bearing of R from P

Applying sine rule to triangle PQR

$$\frac{a}{\sin \sin A} = \frac{b}{\sin \sin B}$$

Where

$$a = RQ$$
,  $A = R\hat{P}Q$ ,  $b = RP$  and  $B = P\hat{Q}R$ 

$$\frac{a}{\sin \sin A} = \frac{b}{\sin \sin B}$$

$$\frac{RQ}{\sin \sin (R\hat{P}Q)} = \frac{RP}{\sin \sin (P\hat{Q}R)}$$

$$\frac{360}{\sin \sin (R\hat{P}Q)} = \frac{299}{\sin \sin (56^{\circ})}$$

$$(R\hat{P}Q) = \frac{360}{\frac{299}{\sin \sin (56^{\circ})}}$$

$$R\hat{P}Q = \left(\frac{360 \times \sin \sin 56^{\circ}}{299}\right)$$

$$R\hat{P}Q = 86.5^{\circ}$$

Bearing of J from L (in a clockwise direction) =  $132^{\circ} + R\hat{P}Q$ =  $132^{\circ} + 86.5^{\circ}$  = 218.5°

Question 11a

**Determine** the inverse of (3524)

Let A = (3524)

$$A^{-1} = det \ det \ A \times A$$
$$det \ det \ A = (4)(3) - (5)(2)$$
$$= 12 - 10$$
$$= 2$$
$$Now \ A^{-1} = \frac{1}{2}(4 - 5 - 2 - 3)$$
$$= \left(2\frac{-5}{2} - 1 - \frac{3}{2}\right)$$

Question 11b part (i)

#### Data Given

Under a matrix transformation M = (0 a b 0), the points R and T were mapped onto R' and T':

$$R(7,2) \rightarrow R'(2,-7)$$

$$T(-5,4) \to T'(4,5)$$

**Determine** the values of *a* and *b* 

$$M \times R = R'$$
  
(0 a b 0) × (7 2) = ( 2 - 7)  
(0 + 2a 7b + 0) = ( 2 - 7)

. .

Equating terms gives

```
2a = 2
         a = 1
      7b = -7
       b = -1
\therefore M = (01 - 10)
```

Question 11b part (ii)

Describe the geometric transformation, M

The transformation matrix, M = (01 - 10) rotates the matrix it is applied to in a clockwise manner about the origin 0 at an angle of 90°.

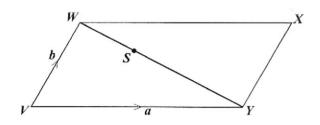
Question 11c part (i)

Data Given

WXYV is a parallelogram where;

 $\overrightarrow{VY} = a$  and  $\overrightarrow{VW} = b$ 

*Point*, *S*; WS:SY = 1:2



(a) Write  $\overrightarrow{WY}$  in terms of a and b

$$\overrightarrow{WY} = \overrightarrow{WV} + \overrightarrow{VY}$$
$$= (-b) + a$$

(b) Write  $\overrightarrow{WS}$  in terms of a and b

$$\overrightarrow{WS} = \frac{1}{3} \overrightarrow{WY}$$
$$= \frac{1}{3}(a - b)$$
$$= \frac{1}{3}a - \frac{1}{3}b$$

(c) Write  $\overrightarrow{SX}$  in terms of a and b

Applying the vector triangle law

$$\overrightarrow{SX} = \overrightarrow{SW} + \overrightarrow{WX} \quad \text{where } \overrightarrow{SW} = -\overrightarrow{WS}$$
$$= -\frac{1}{3}a + \frac{1}{3}b + a$$
$$= \frac{2}{3}a + \frac{1}{3}b$$

Question 11c part (ii)

Data Given

R is the midpoint of VW.

**Prove** R, S and X are collinear.

Applying the vector triangle law

$$\overrightarrow{RX} = \overrightarrow{RW} + \overrightarrow{WX}$$
$$= \frac{1}{2}b + a$$

Using  $\overrightarrow{RX}$  and  $\overrightarrow{SX}$ ; in order for the points to be collinear

$$\overrightarrow{SX} = k \left( \overrightarrow{RX} \right) \text{ where } k \text{ is a scalar multiple}$$
$$\overrightarrow{RX} = \left( a + \frac{1}{2}b \right)$$
$$\overrightarrow{SX} = \left( \frac{2}{3}a + \frac{1}{3}b \right) = \frac{2}{3} \left( a + \frac{1}{2}b \right) \text{ where } k = \frac{2}{3}$$

The vectors  $\overrightarrow{RX}$  and  $\overrightarrow{SX}$  are parallel since  $\overrightarrow{SX}$  is a scalar multiple of  $\overrightarrow{RX}$  and X is a common point for the both vectors; therefore R, S and X are collinear.