Solutions to CSEC Maths P2 JUNE 2012

Question 1a

Calculate the exact value of

$$\frac{3\frac{1}{5}-\frac{2}{3}}{2\frac{4}{5}}$$

Numerator: $3\ \frac{1}{5} - \frac{2}{3}$

$= \frac{16}{5} - \frac{2}{3}$

$= \frac{48-10}{15}$

$= \frac{38}{15}$

Denominator: $2\ \frac{4}{5} = \frac{14}{5}$

$\frac{Numerator}{Denominator}$ $= \frac{38}{15} \div \frac{14}{5}$

$= \frac{38}{15} \times \frac{5}{14}$

$= \frac{19}{21}$

Question 1b

Data Given: The table below shows the cost price, selling price and profit or loss as a Percentage of the cost price.

Cost Price	Selling Price	Percentage Profit or Loss
$55.00	$44.00	_____
______	$100.00	25% profit

Required to Copy and Complete the table below inserting the missing values.

Cost Price	Selling Price	Percentage Profit or Loss
$55.00	$44.00	20% loss

\$80.00	\$100.00	25% profit

<u>For the 1st item:</u>

Since the selling price is less than the cost price for the first item,

$\Longrightarrow Loss = Cost\ Price - Selling\ Price$

$= \$55 - \$44$

$= \$11.00$

$\therefore Pecentage\ Loss = \frac{Loss}{Cost\ Price} \times 100$

$= \frac{11}{55} \times 100$

$= 20\%$

<u>For the 2nd item:</u>

$A\ 25\%\ profit\ implies\ that\ the\ selling\ price\ is\ 125\%\ of\ the\ cost\ price.$

$\therefore The\ cost\ price = \frac{100}{125} \times 100$

$= \$80.00$

Question 1c part (i)

Data Given: The table below shows some rates of exchange

US \$1.00 = EC \$2.70
TT \$1.00 = EC \$0.40

Required to calculate the value of EC \$1 in TT \$

$EC\ \$0.40 = TT\ S1.00$

$EC\ \$1.00 = \frac{TT\ \$1.00}{40} \times 100$

$EC\ \$1.00 = TT\ \$2.50$

Question 1c part (ii)

Required to calculate the value of US \$80 in EC \$

$US\ \$1.00 = EC\ \$2.70$

$US\ \$80.00 = EC\ \$2.60\ \times 80$

$= EC\ \$216.00$

Question 1c part (iii)

Required to calculate the value of TT \$648 in US \$

$TT\ \$1.00 = EC\ \$0.40$

$TT\ \$648 = EC\ \$0.40 \times 648$

$EC\ \$259.20$

Now,

$EC\ \$2.70 = US\ \$1.00$

$EC\ \$1.00 = \frac{US\ \$1.00}{EC\ 2.70}$

$EC\ \$259.20 = \frac{US\ \$1.00}{EC\ 2.70} \times EC\ \$259.20$

$= \$96.00$

$\therefore TT\ \$648.00 = US\ \$96.00$

Question 2a part (i)

Required to Factorize

$$2x^3y + 6x^2y^2$$

Step 1: We factor out the common factor in each term

$2x^2y$ is a common factor in each term

Thus, $2x^2y(x) + 2x^2y(3y)$

Step 2: Factorize to get $2x^2y(x + 3y)$

Question 2a part (ii)

Required to Factorize

$$9x^2 - 4$$

Step 1: We re-write the expression as the difference of two squares:

$(3x)^2 - (2)^2$

Step 2: We factorize to get:

$(3x - 2)(3x + 2)$

Question 2a part (iii)

Required to Factorize

$$4x^2 + 8xy - xy - 2y^2$$

Step 1: Group like terms

$$4x(x+2y) - y(x+2y)$$

Step 2: Factorize to get:

$$(x+2y)(4x-y)$$

Question 2b

Required to Solve

$$\frac{2x-3}{3} + \frac{5-x}{2} = 3$$

Step 1: Find the LCM

$$\frac{2(2x-3)+3(5-x)}{6} = 3$$

$$\frac{4x-6+15-3x}{6} = 3$$

Step 2: Simplify

$$\frac{x+9}{6} = 3$$

Step 3: Make x the subject of the formula

$$x + 9 = 3(6)$$

$$x + 9 = 18$$

$$x = 18 - 9$$

$$x = 9$$

Question 2c

Given Data: We are given two equations:

$$3x - 2y = 10$$

$$2x + 5y = 13$$

Required to solve both equations simultaneously

Using the Elimination Method

Step 1: Let $3x - 2y = 10$ be Equation 1

Let $2x + 5y = 13$ be Equation 2

Step 2: Multiply Equation 1 by 2 to get Equation 3

$$2(3x - 2y) = 2(10)$$

$$6x - 4y = 20 \qquad \text{[Equation 3]}$$

Step 3: Multiply Equation 2 by 3 to get Equation 4

$3(2x + 5y) = 3(13)$

$6x + 15y = 39$ [Equation 4]

Step 4: Equation 4 – Equation 3

$6x + 15y = 39 \quad -$

$\underline{6x - 4y = 20}$

$\underline{\quad 19y = 19}$

Step 5: Make y the subject of the formula to find the value of y

$y = \frac{19}{19}$

$y = 1$

Step 6: Substitute $y = 1$ into Equation 1 to find the value of x

$3x - 2y = 10$

$3x - 2(1) = 10$

$3x - 2 = 10$

$3x = 10 + 2$

$3x = 12$

$x = \frac{12}{3}$

$x = 4$

Thus $x = 4$ and $y = 1$

Question 3a part (i)

Data Given: In a survey of 36 students,

30 play tennis x play volleyball only

$9x$ play both tennis and volleyball

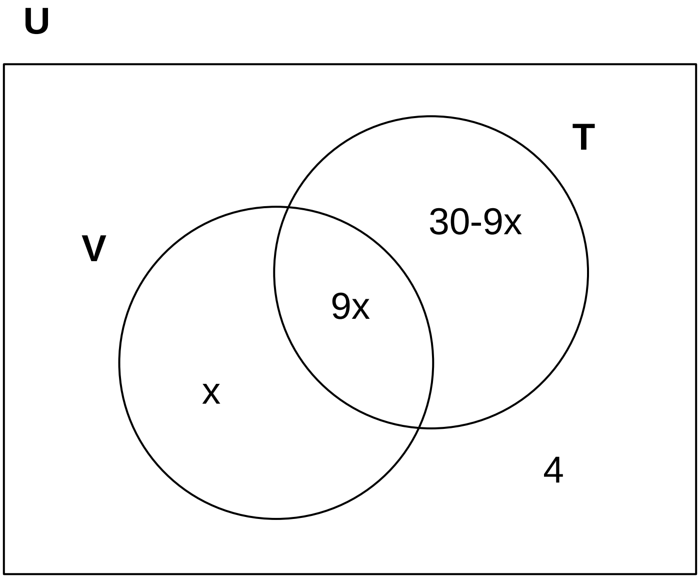
4 do not play either tennis or volleyball

$U = \{\ Students\ in\ survey\}$

$V = \{Students\ who\ play\ volleyball\}$

$T = \{Students\ who\ play\ tennis\}$

Required to Copy and Complete the Venn Diagram showing the number of students in subsets marked y and z



Question 3a part (ii)(a)

Required to write an expression in x for the total number of students in the survey

The number of students who play volleyball only $= x$

The number of students who play tennis only $= 30 - x$

The number of students who play both tennis and volleyball $= 9x$

The number of students who do not play tennis nor volleyball $= 4$

Total number of students in the survey $= x + (30 - 9x) + 9x + 4$

$= x + 30 - 9x + 9x + 4$

$= x + 30 + 4$

$= x + 34$

Question 3a part (ii)(b)

Required to write an equation in x for the total number of students in the survey and to solve for x

Total number of students $= 36$

Total number of students $= x + 34$

Thus $x + 34 = 36$

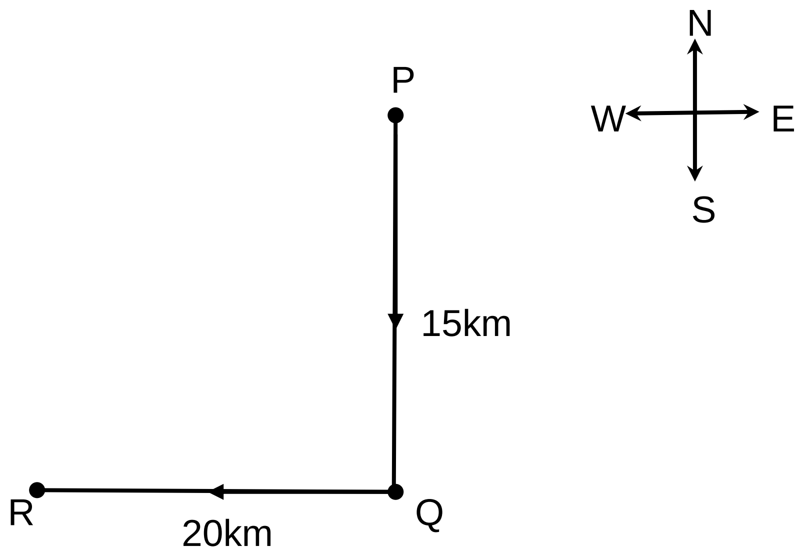
$x = 36 - 34$

$x = 2$

Question 3b part (i)

Data Given: Diagram showing the journey of a ship which started at port P , sailed $15km$ due South to port Q and then further $20km$ due West to port R

Required to Copy and label the diagram to show points Q and R and distances $20km$ and $15km$



Question 3b part (ii)

Required to the shortest distance of the ship from the port to where the journey started

Using Pythagoras' Theorem

$PR^2 = PQ^2 + RQ^2$

$PR = \sqrt{(15)^2 + (20)^2}$

$= 25km$

Question 3b part (iii)

Required to Calculate the measure of angle QPR, giving answer to the nearest degree

$sinQ\hat{P}R = \frac{opp}{hyp}$

$sinQ\hat{P}R = \frac{20}{25}$

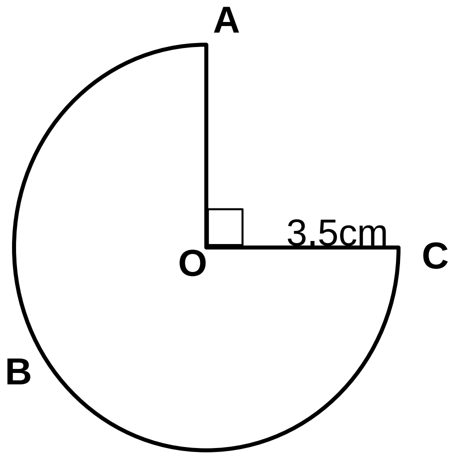
$\therefore Q\hat{P}R = \left(\frac{20}{25}\right)$

$= 53.1°$

53° [to the nearest degree]

Question 4a part(i)

Data Given: Diagram showing the cross-section of a prism



Required to calculate the length of the arc ABC

Length of the arc $ABC = \frac{270°}{360°} \times Circumference\ of\ a\ complete\ circle$

$= \frac{3}{4} \times 2\pi \times r$

$= \frac{3}{4} \times 2 \times \frac{22}{7} \times 3.5$

$= 16.5cm$

Question 4a part(ii)

Required to calculate the perimeter of the sector $OABC$

Perimeter of $OABC = Length\ of\ AO + Arc\ Length\ ABC + Length\ of\ Radius\ CO$

$= 16.5 + 3.5 + 3.5$

$= 23.5cm$

Question 4a part (iii)

Required to calculate the area of the sector $OABC$

$Area\ of\ sector\ OABC = \frac{270°}{360°} \times Area\ of\ a\ complete\ circle$

$= \frac{3}{4} \times \frac{22}{7} \times (3.5)^2$

$= 28.875$

$= 28.88cm^2$ [to 2 decimal places]

Question 4b part (i)

Data Given: A prism is 20cm long and made of tin

$1cm^3$ of tin has a mass of $7.3g$

Required to calculate the volume of the prism

$$Volume\ of\ the\ Prism = Cross - sectional\ Area\ \times Height$$

$$= 28.875 \times 20$$

$$= 577.5cm^3$$

Question 4b part (ii)

Required to calculate the mass of the prism, to the nearest kg

$1cm^3 of\ tin\ weighs\ 7.3g$

$577.5cm^3 of\ tin = 7.3 \times 577.5g$

$= 4215.75g$

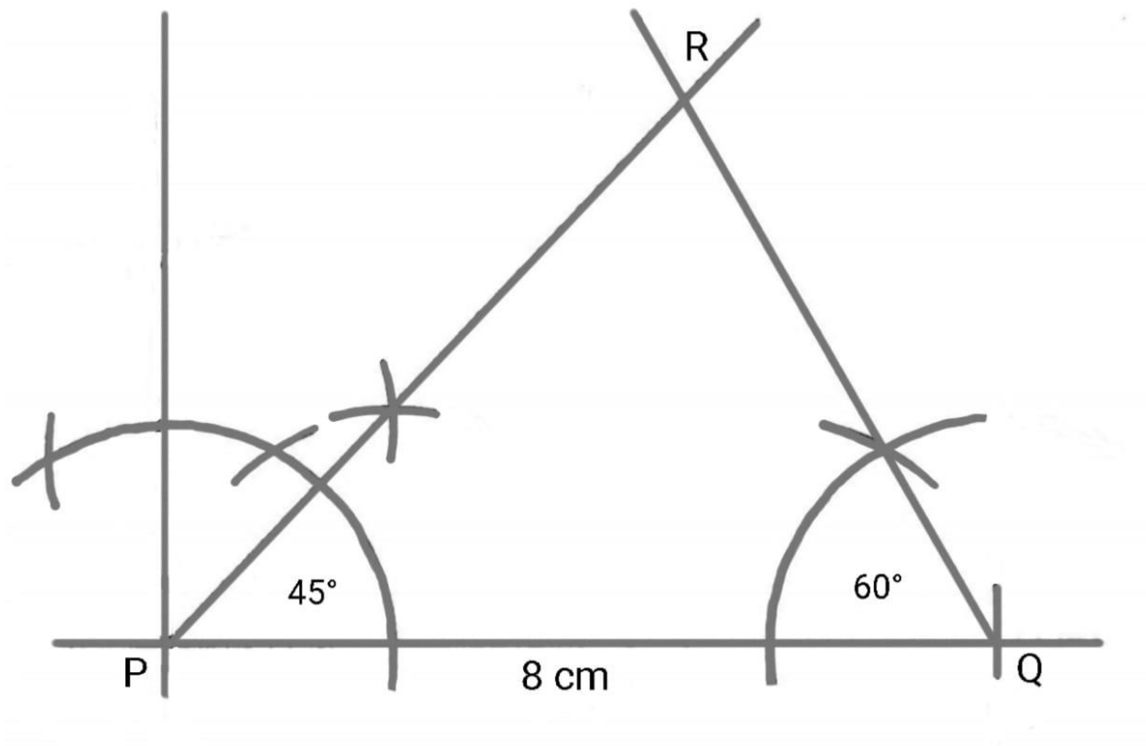
$= \frac{4215.75}{1000}$

$= 4.2kg$

=4 kg [to the nearest kg]

Question 5a part(i)

Required to Construct triangle PQR with $PQ = 8cm$, $\angle PQR = 60°$ and $\angle QPR = 45°$



Question 5a part (ii)

Required to measure and state the length of RQ

Using a Ruler to measure the length of RQ

The length of $RQ = 6.0cm$]

Question 5b part (i)

Data Given: The line l passes through the points $S(6,6)$ and $T(0,-2)$

Required to find the gradient of the line

Let $(x_1, y_1) = (0,-2)$ and $(x_2, y_2) = (6,6)$

$Gradient = \frac{(y_2-y_1)}{(x_2-x_1)}$

$= \frac{6-(-2)}{6-0}$

$= \frac{8}{6}$

$= \frac{4}{3}$

Question 5b part (ii)

Required to find the equation of the line l

The line l is of the form $y = mx + c$

Since $(0,-2)$ lies on the line l, then the y-intercept is -2 and the gradient $\mathrm{m} = \frac{4}{3}$

Substituting $m = \frac{4}{3}$ and $c = -2$ into $y = mx + c$ to get $y = \frac{4}{3}x - 2$

Thus the equation of the line $l =$ $y = \frac{4}{3}x - 2$

Question 5b part (iii)

Required to find the midpoint of the line segment TS

$Midpoint = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \frac{6+0}{2}, \frac{6+(-2)}{2}$

$= (3,2)$

Question 5b part (iv)

Required to find the length of the line segment TS

$$The\ length\ of\ TS = \sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(6-0)^2 + (6-(-2))^2}$$

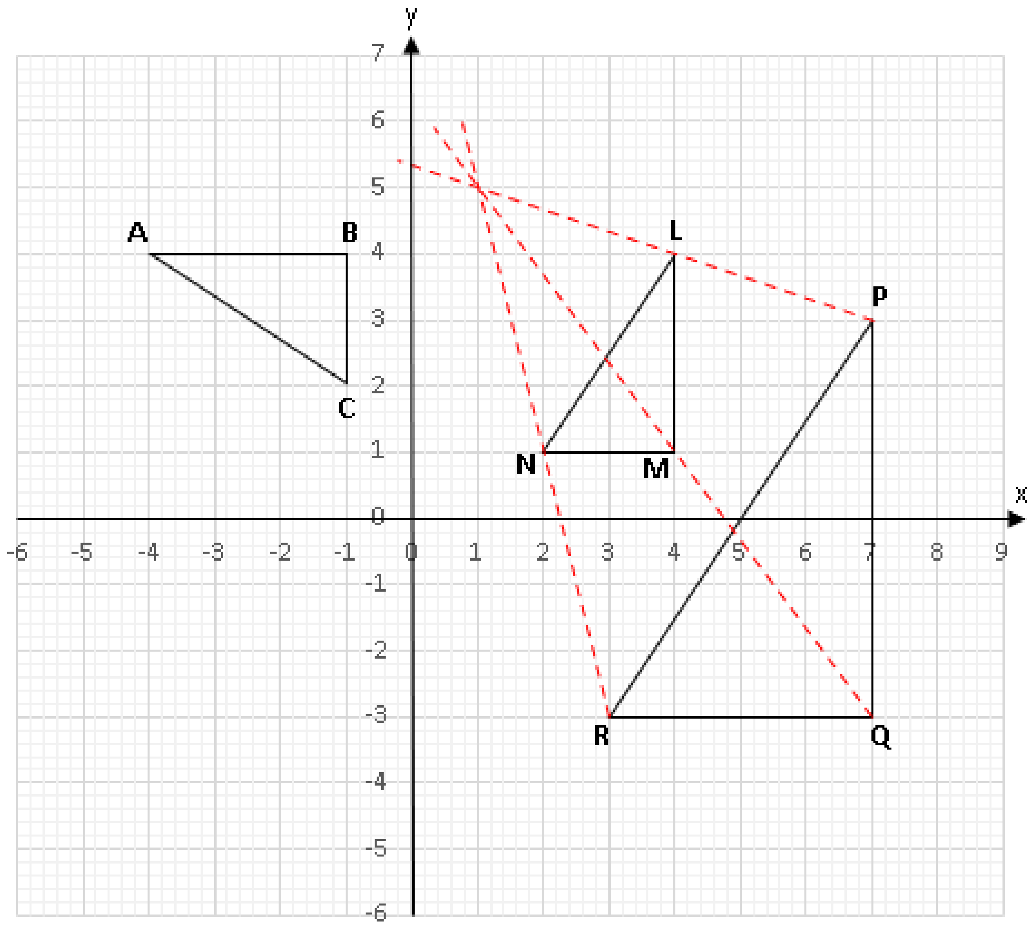
$$= \sqrt{6^2 + 8^2}$$

$$= 10\ units$$

Question 6a

Data Given: Graph showing triangle LMN and its image PQR after an enlargement

Required to Locate the centre of enlargement



The centre of enlargement was found by finding the point of intersection between the end points of triangle LNM triangle PRQ.

The centre of enlargement was found to be (1,5)

Question 6b

Required to State the scale factor and the center of enlargement

$Scalar\ Factor = \frac{Length\ of\ PQ}{Length\ of\ LM}$

$= \frac{6}{3}$

$= 2$

Question 6c

Required to Determine the value of $\frac{Area\ of\ PQR}{Area\ of\ LMN}$

Since $\frac{RQ}{NM} = \frac{2}{1}$

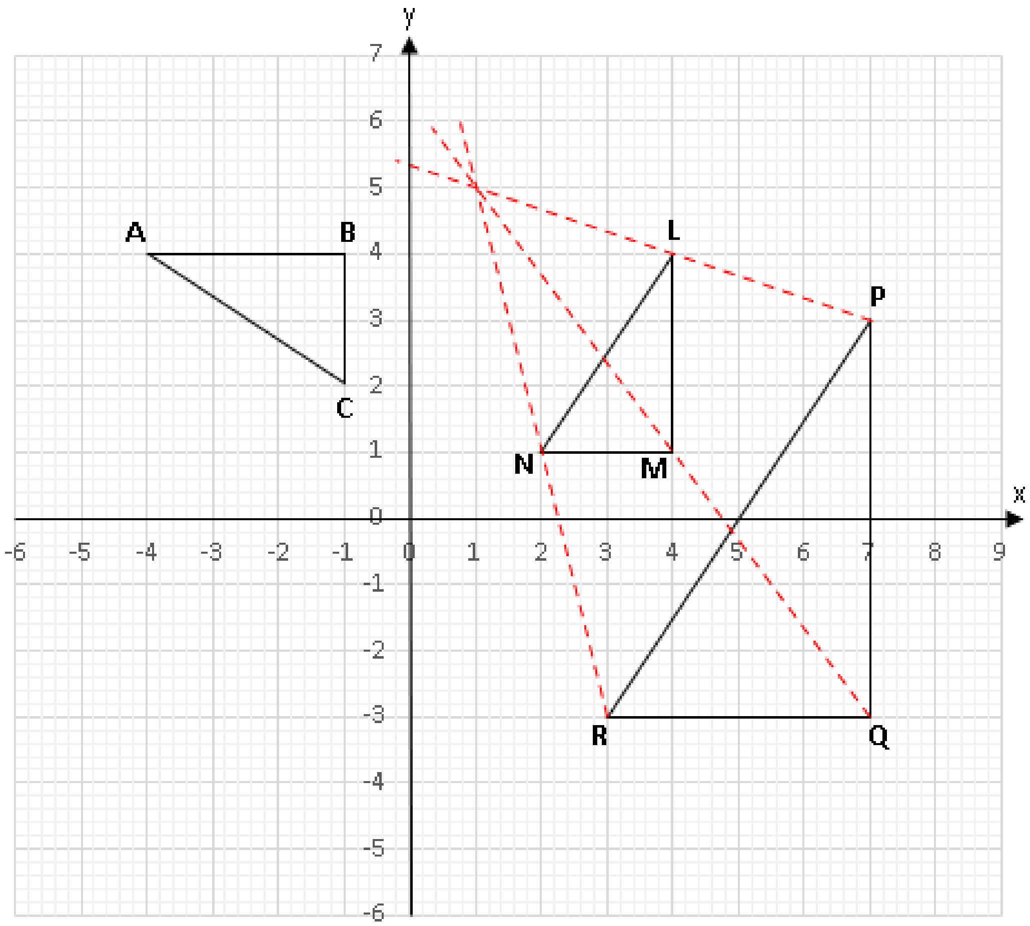
Then $\frac{Area\ of\ PQR}{Area\ of\ LMN} = \frac{(2)^2}{(1)^2}$

$= \frac{4}{1}$

$= 4$

Question 6d

Required to Draw and Label Triangle ABC



Question 6e

Describe fully the transformation which maps triangle LMN onto triangle ABC

Triangle LMN undergo a rotation of 90° in an anti-clockwise direction about the origin

(0.0)

Question 7a

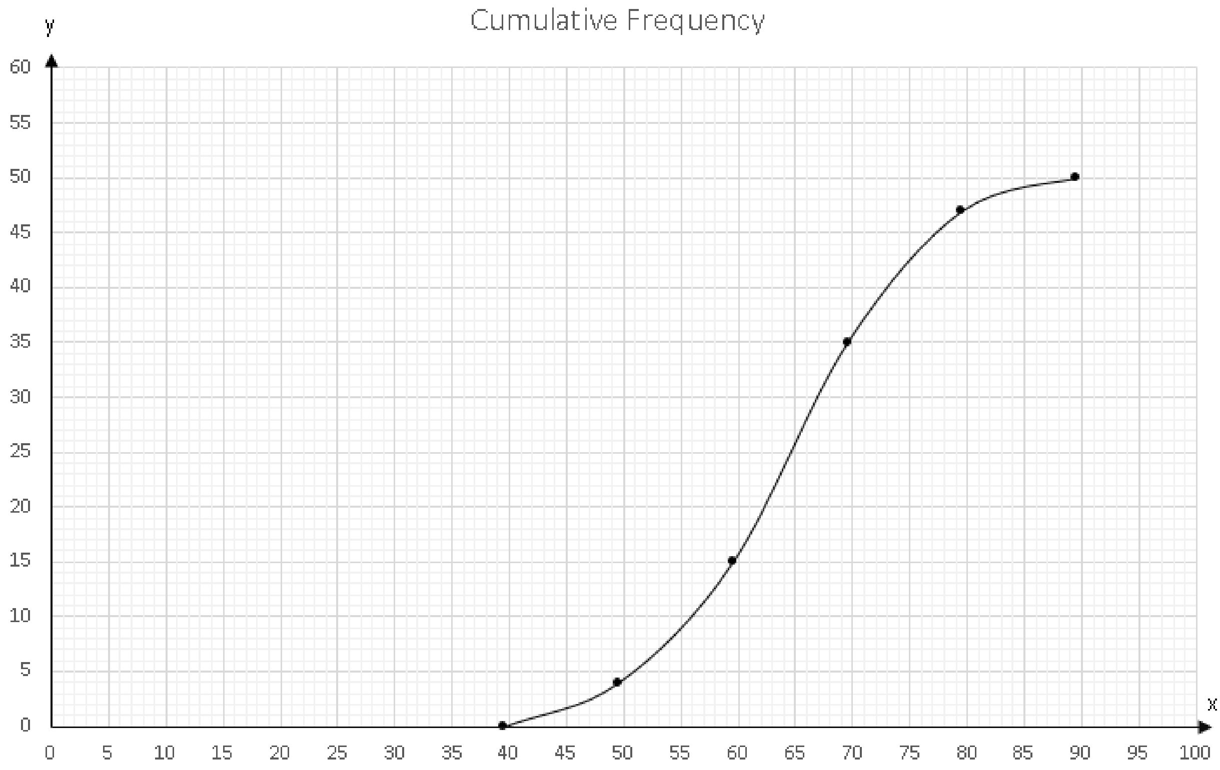
Data Given: Table showing the ages of persons who visited the clinic during the week

Required to Copy and Complete the table to show cumulative frequency

Age, x LCL – UCL	**LCB $\leq x \leq$ UCB**	**Number of Persons**	**Cumulative Frequency**	**Points to be Plotted**
				(39.5,0)
40 – 49	$39.5 \leq x \leq 49.5$	4	4	(49.5,4)
50 – 59	$49.5 \leq x \leq 59.5$	11	15	(59.5,15)
60 – 69	$59.5 \leq x \leq 69.5$	20	35	(69.5,35)
70 – 79	$69.5 \leq x \leq 79.5$	12	47	(79.5,47)
80 – 89	$79.5 \leq x \leq 89.5$	3	50	(89.5,50)

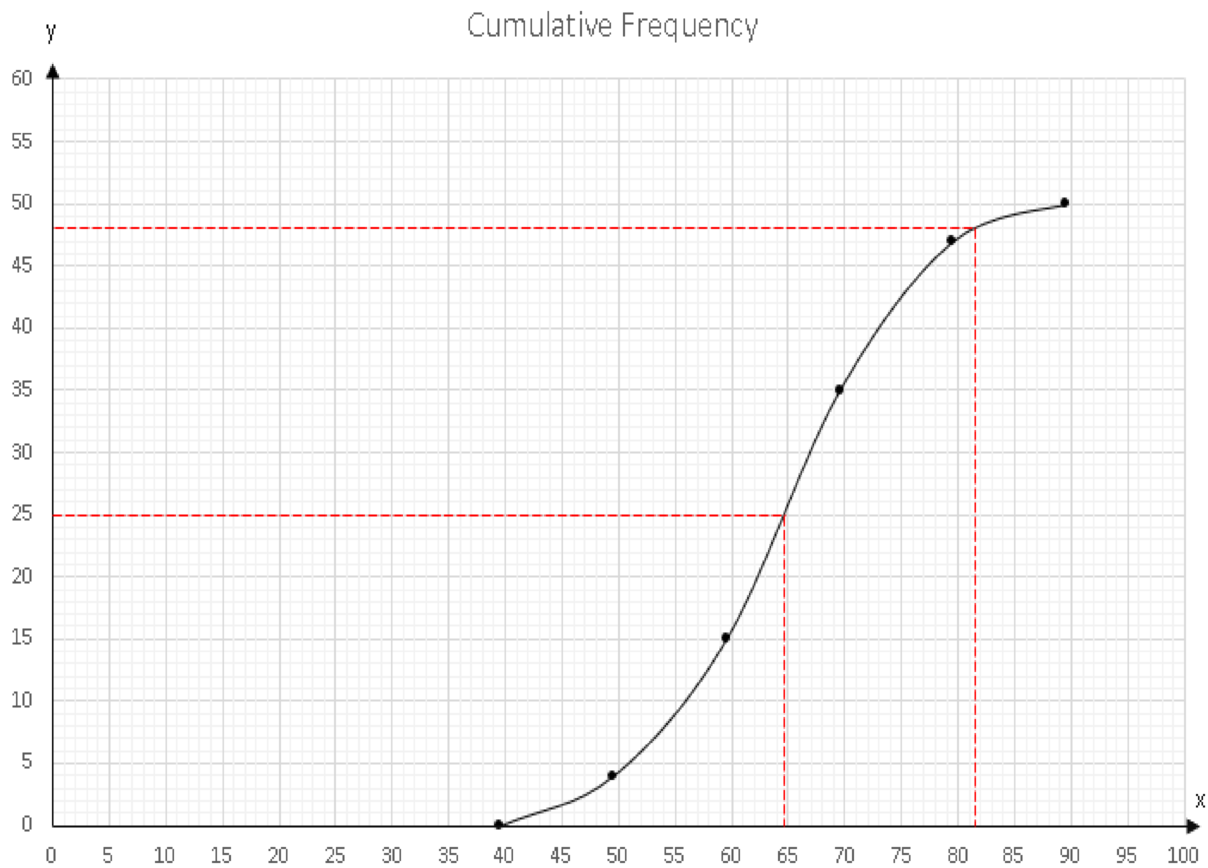
Question 7b

Required to Draw the Cumulative Frequency curve to represent the data



Question 7c part (i)

Required to Estimate the median age for the data



The Median Age is calculated by finding the x value (years) when the y value is $\frac{1}{2}$ of 50 on the graph

Thus, the Median Age is found to be 64.75 when y = 25

Question 7c part (i)

Required to Estimate the probability that a person who visited the clinic was 75 years or younger

$$P(A\ persons\ is\ 75\ years\ or\ younger)$$

$$= Number\ of\ persons\ 75\ years\ or\ younger\ \div\ Total\ Number\ of\ Persons$$

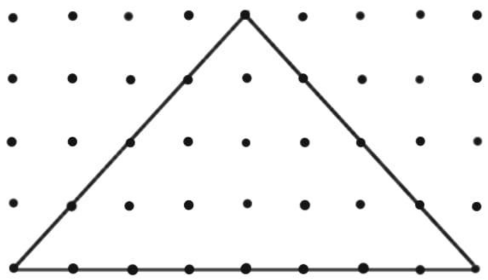
$$= 42.5\ \div 50$$

$$= \frac{17}{20}$$

Question 8a

Given Data: Diagram showing three figures in a sequence of figures

Required to draw the fourth figure in the sequence of figures



Question 8b

Required to copy and complete the table given

Figure	Area of Triangle	Number of Pins on Base
1	1	$(2 \times 1) + 1 = 3$
2	4	$(2 \times 2) + 1 = 5$
3	9	$(2 \times 3) + 1 = 7$
4	$4^2 = 16$	$(2 \times 4) + 1 = 9$
$\sqrt{100} = 10$	100	$(2 \times 10) + 1 = 21$
20	$20^2 = 400$	$(2 \times 20) + 1 = 41$
n	n^2	$2n + 1$

Question 9a part (i)

Required to Solve $y = 8 - x$ and $2x^2 + xy = -16$

Let $y = 8 - x$ [Equation 1]

$2x^2 + xy = -16$ [Equation 2]

Using the Method of Substitution

Step 1: Substitute Equation 1 into Equation 2

$$2x^2 + x(8 - x) = -16$$

$$2x^2 + 8x - x^2 = -16$$

$x^2 + 8x = -16$

$x^2 + 8x + 16 = 0$

$(x + 4)(x + 4) = 0$

$x = -4$

Step 2: Substitute $x = 4$ into Equation 1

$y = 8 - (-4)$

$= 8 + 4$

$= 12$

Thus $x = -4$ and $y = 12$

Question 9a part (ii)

Required to State: With reason or not, $y = 8 - x$ is a tangent to the curve with equation

$$2x^2 + xy = -16$$

Solve the Equation $2x^2 + xy = -16$ to obtain the root $x = -4$

Since the roots are real and equal, the straight line does not intersect the curve but rather it touches the curve at the point $(-4, 12)$

Question 9b part (i)

Data Given: x roses and y orchids in each bouquet with the following constraints

The number of orchids must be at least half the number of roses

There must be at least 2 roses

There must be no more than 12 flowers in the bouquet

Required to write the three inequalities for the constraints given

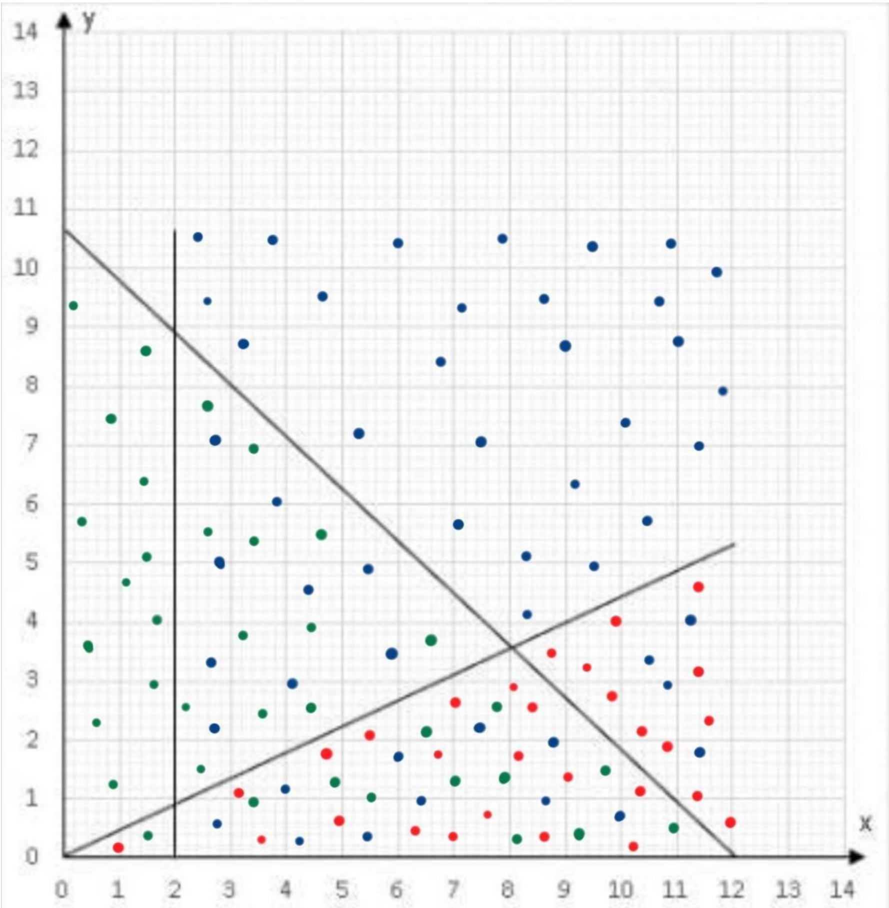
$y \geq \frac{1}{2}x$

$x \geq 2$

$x + y \leq 12$

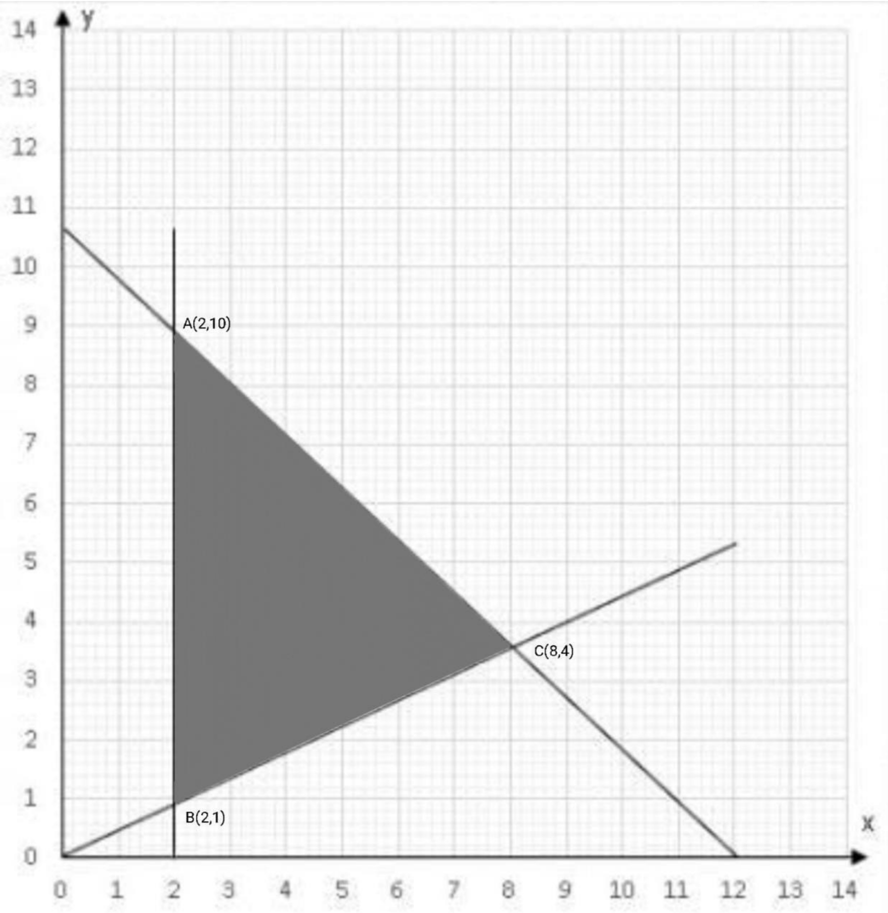
Question 9b part (ii)

Required to shade the region that satisfies all three inequalities



Question 9b part (iii)

Required to State the co-ordinates of the points which represent the vertices of the region showing the solution set



$\triangle ABC$ is the region which is shaded.

The co-ordinates for $\triangle ABC$ is $A = (2{,}10), B = (2{,}1)$ and $C = (8{,}4)$

Question 9b part (iv)

Data Given: A profit of \$3 is made on each rose and \$4 on each orchid

Required to Determine the maximum possible profit on the sale of a bouquet

Step 1: Find an expression for the Total Profit made

Total profit made on x roses at \$3 each and on y orchids at \$4 each $= (x \times 3) + (y \times 4)$

Step 2: Find an equation for the Total Profit

$P = 3x + 4y$

Step 3: Find the Total Profit made at each of the three points.

At point $A = (2{,}10)$ $\quad P = 3(2) + 4(10)$

$P = \$46.00$

A point $B = (2{,}1)$ $\quad P = 3(2) + 4(1)$

$P = \$10.00$

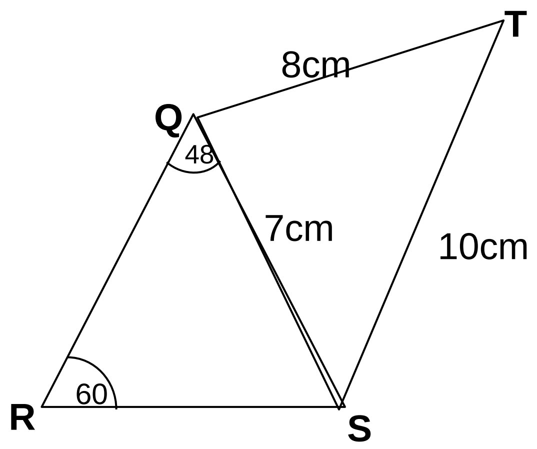
A point $C = (8,4)$ $P = 3(8) + 4(4)$

$P = \$40.00$

The Maximum Profit occurs at point A since the Total Profit made at point A is $46

Question 10a part(i)

Data Given: Diagram showing quadrilateral $QRST$



Required to Calculate the length of RS

Using the Sine Rule

$\frac{QS}{sinR} = \frac{RS}{sinQ}$

$\frac{7}{sin60°} = \frac{RS}{sin48°}$

$7sin48° = RSsin60°$

$RS = \frac{7sin48°}{sin60°}$

$= 6.006$

$= 6.01$ [to 2 decimal places]

Question 10a part(ii)

Required to Calculate the measure of $\angle QTS$

Using the Cosine Rule

$QS^2 = QT^2 + TS^2 - 2(QT)(TS)cos\hat{T}$

$7^2 = 8^2 + 10^2 - 2(8)(10)cos\hat{T}$

$cos\hat{T} = \frac{(7^2-8^2-10^2)}{(-2(8)(10)}$

$= \frac{-115}{-160}$

$\hat{T} = (0.7187)$

$= 44.02$

$= 44.0°$ [to the nearest 0.1°]

Question 10b part(i)(a)

Required to Calculate the measure of angle OUZ

$Z\hat{O}U = 180° - 70°$

$= 110°$

In triangle $ZOU, OZ = OU$ [since they are radii of the same circle]

$\Rightarrow$ triangle ZOU is isosceles

$Thus, O\hat{U}Z = \frac{180° - 110°}{2}$

$O\hat{U}Z = 35°$

Question 10b part(i)(b)

Required to Calculate the measure of angle UVY

$U\hat{Y}V = \frac{70°}{2}$

$= 35°$

[The angle that is subtended by a chord at the circumference of a circle is half of that angle that the chord subtends at the center of the circle, and standing on the same arc]

$O\hat{U}V \text{ or } Y\hat{U}V = 90°$

[The angle made by the tangent UW to a circle and a radius OU , at the point of contact, O is a right angle.

$U\hat{V}Y = 180° - (90° + 35°)$

$= 55°$

[The sum of the interior angles of a triangle = 180°]

Question 10b part(i)(c)

Required to Calculate the measure of angle UWO

$W\hat{O}U = 70°$

$O\hat{U}W = 90°$

[The sum of the three interior angles of a triangle = 180°]

$U\hat{W}O = 180° - (90° + 70°)$

$= 20°$

Question 10b part(ii)(a)

Required to Name the triangle which is congruent to triangle ZOU

$OZ = OY$ and $OU = OX$

$Z\hat{O}U = Y\hat{O}X$

[Vertically opposite angles are equal when two straight lines intersect]

Therefore, $\triangle ZOU$ is congruent to $\triangle YOX$

Question 10b part(ii)(b)

Required to Name the triangle which is congruent to triangle YXU

Consider $\triangle YXU$ and $\triangle ZUX$

$YU = ZX$

$$Y\hat{X}U = Z\hat{U}X = 90°$$

UX is a common side to both triangles and therefore triangle YXU and triangle ZUX have the same hypotenuse and share a common side.

Thus, Triangle YXU is congruent to Triangle ZUX

Question 11a part(i)(a)

Given Data: $OA = \ (6\ 2\ ), OB = (3\ 4\ )\ and\ OC = (12\ \ -2\ )$

Required to Express the vector BA in the form $(x\ y\ )$

Using the Vector Triangle Law

$BA = BO + OA$

$= -(3\ 4) + (6\ 2)$

$= (3\ \ -2)$

Question 11a part(i)(b)

Required to Express the vector BC in the form $(x\ y)$

Using the Vector Triangle Law

$BC = BO + OC$

$= -(3\ 4) + (12\ \ -2)$

$= (9\ \ -6)$

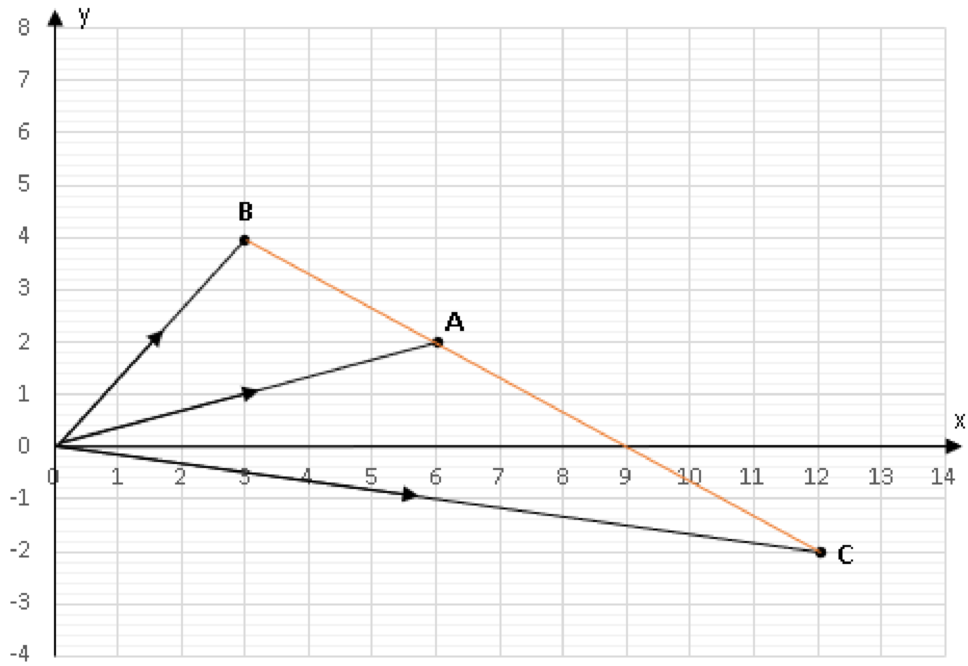
Question 11a part(ii)

Required to State one geometrical relationship between BA and BC

$|BC| = 3|BA|$

The vector BA is a scalar multiple of the vector BC

Question 11a part(iii)



Question 11b part(i)

Data Given: $\begin{pmatrix} a & -4 \\ 1 & b \end{pmatrix}\begin{pmatrix} 2 & -4 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

Required to Calculate the value of a and b

Step 1: Ensure that the dimensions of the matrices allows matrix multiplication

In this case, it does, as all three matrices are 2×2 matrices

Thus, the result will be of the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

Step 2: Multiply the matrices

$\begin{pmatrix} (2 \times a) + (-4 \times 1) & (-4 \times a) + (-4 \times -3) \\ (1 \times 2) + (b \times 1) & (1 \times -4) + (b \times -3) \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

$\begin{pmatrix} 2a - 4 & 12 - 4a \\ b + 2 & -4 - 3b \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

Step 3: Equating entries to find the value of a and b

$2a - 4 = 2$	$12 - 4a = 0$
$2a = 6$	$12 = 4a$
$a = \frac{6}{2}$	$\frac{12}{4} = a$
$a = 3$	$3 = a$

$b + 2 = 0$

$b = -2$

$-4 - 3b = 2$

$-4 - 2 = 3b$

$-6 = 3b$

$-\frac{6}{3} = b$

$-2 = b$

Thus, $a = 3$ and $b = -2$

Question 11b part(ii)

Required to Find the inverse of $(2\ -4\ 1\ -3\ )$

Let $A = (2\ -4\ 1\ -3\ )$

$(3\ -4\ 1\ -2\ )(2\ -4\ 1\ -3\ ) = (2\ 0\ 0\ 2\ )$

$(3\ -4\ 1\ -2\ )(2\ -4\ 1\ -3\ ) = 2(1\ 0\ 0\ 1\ )$

This is of the form $AA^{-1} = I$

$\frac{1}{2}(3\ -4\ 1\ -2\ )(2\ -4\ 1\ -3\ ) = (1\ 0\ 0\ 1\ )$

$\left(\frac{3}{2}\ -\frac{4}{2}\ \frac{1}{2}\ -\frac{2}{2}\right)(2\ -4\ 1\ -3\ ) = (1\ 0\ 0\ 1\ )$

Therefore, $A^{-1} = \left(\frac{3}{2}\ -2\ \frac{1}{2}\ -1\ \right)$

Question 11b part(iii)

Required to Solve for x and y in $(2\ -4\ 1\ -3\ ) = (x\ y\ ) = (12\ 7\ )$

Step 1: Multiply the matrix equation by A^{-1}

$\left(\frac{3}{2}\ -2\ \frac{1}{2}\ -1\ \right)(2\ -4\ 1\ -3\ )(x\ y\ ) = \left(\frac{3}{2}\ -2\ \frac{1}{2}\ -1\ \right)(12\ 7\ )$

$(x\ y\ ) = \left(\frac{3}{2}\ -2\ \frac{1}{2}\ -1\ \right)(12\ 7\ )$

$(x\ y\ ) = \left(\left(\frac{3}{2} \times 12\right) + (-2 \times 7)\ \left(\frac{1}{2} \times 12\right) + (-1 \times 7)\ \right)$

$(x\ y\ ) = (4\ -1\ )$

$x = 4$ and $y = -1$