Solutions to 2018 June

#forthepeople

Question 1a part (i)	1 mark

Calculate the exact value of

$$73.18 - 5.23 \times 9.34$$

$$= 73.18 - (5.23 \times 9.34)$$

$$= 73.18 - 48.8482$$

$$= 24.3318$$

$$= 24.33 \text{ (to 2 decimal places)}$$

Question 1a part (ii)	1 mark

Calculate the exact value of

$$\frac{3.1^2}{6.17} + 1.12$$

$$=2.6775$$

$$=2.68 \text{ (to 2 d.p)}$$

Question 1b part (i)	1 mark

Data Given

W_j**=600+0.90**n

where 600 is the basic wage and n is the number of customers Jenny has served.

When $n = 230$, **What is** Jenny's Wage for the week?

$$=600+0.90(230)$$

$$=600+207$$

$$=\$807$$

Question 1b part (ii)	2 marks

Data Given
On a good week, Jenny earns at least $1000,

What is the least amount of customers required to Jenny to have a good week?

Therefore,

Using Formula,

$$W_j = 600 + 0.9n$$

$$600 + 0.9n = W_j$$

$$W_j \geq 1000$$

Hence,

$$600 + 0.9n \geq 1000$$

$$0.9n \geq 1000 - 600$$

$$0.9n \geq 400$$

$$n \geq \frac{400}{0.9}$$

$$n \geq 444.4$$

Since the number of customers is a whole number, we round this value to the next positive value.

Therefore, $n = 445$.

Question 1b part (iii)	1 mark

Data Given
Shawna receives $270 as a base wage plus $1.50 per customer. "m" is the symbol for no. of customers

Write formula for Shawna's weekly wage**.**

Shawna's Wage is denoted by

W_s**= 270 + 1.5**m

where 270 is the basic wage and m is the number of customers Shawna has served.

Question 1b part (iv)	1 mark

Data Given

In a particular week, Shawna and Jenny receive the same wage for serving the same amount of customers

Find the number of customers they each served

To do this, "m" and "n", the symbols representing the number of customers they each served are equal,

Therefore, $m = n$.

So,

Let them equal each other,

$$600+0.90n=270+1.50m$$

Since $m = n$,

$$600 - 270 = 1.5n - 0.9n$$

$$330 = 0.6n$$

$$550 = n$$

Question 2a part (i)	1 mark

Factorize

In form of the difference of two squares,

$$1 - 4h^2$$

$$= (1 - 2h)(1 + 2h)$$

Question 2a part (ii)	2 marks

Factorize

$$pq - q^2 - 3p + 3q = q(p - q) - 3(p + q)$$

$$= (p - q)(q - 3)$$

Question 2b part (i)	1 mark

Solve

$$\frac{3}{2}y = 12$$

$$(\times \frac{2}{3})$$

$$y = \frac{2}{3} \times 12$$

$$y = 8$$

Question 2b part (ii)	2 marks

Solve

$$2x^2 + 5x - 3 = 0$$

$$(2x - 1)(x + 3) = 0$$

$$2x - 1 = 0 \text{ and } x + 3 = 0$$

$$\text{or } x = -3$$

Question 2c part (i)	1 mark

Data Given:

Find F, when $m = 3, u = -1, v = 2 \; and \; t = 1$.

So,

$$F = \frac{3[2-(-1)]}{1}$$

$$= \frac{(3)(3)}{1}$$

$$= 9$$

Question 2c part (ii)	2 marks

Make v the subject of the formula

$$F = \frac{m(v-u)}{t}$$

$$Ft = mv - mu$$

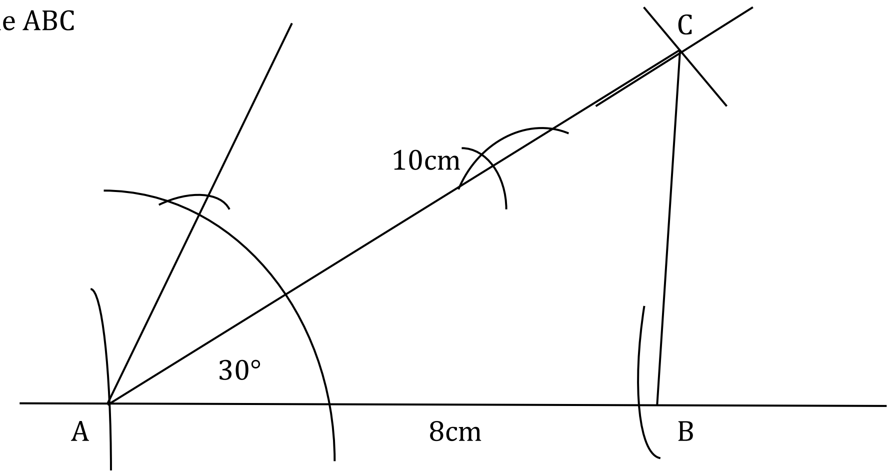
$$mv = Ft + mu$$

$$v = \frac{Ft+mu}{m}$$

Question 3a	4 marks

Data Given: *Triangle ABC, AB=8 cm, ‹BAC=30° and AC= 10 cm*

Draw Triangle ABC



Question 3b part i	4 marks

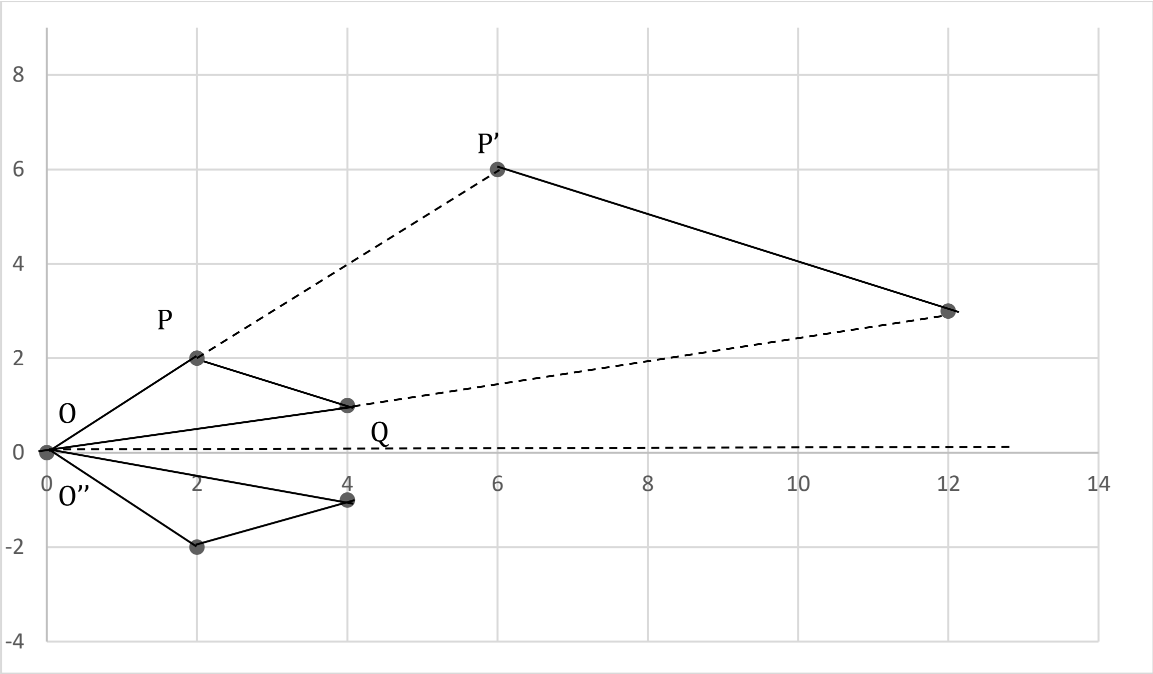
State coordinates of Q

$$Q = (4,1)$$

Question 3b parts ii and iii	4 marks

ii) Draw graph with enlargement of scale factor 3 whereby PQ is mapped unto $P'Q'$

iii) Draw reflection of OPQ about the line $y = 0$.



Question 4a part i	1 mark

Data Given: *Function, f, has domain, $A\{1,2,3\}$.*

Find $f(1)$

$$f(1) = \frac{1}{2}(1) - 3$$
$$= \frac{1}{2} - 3$$
$$= -2\frac{1}{2}$$

Question 4a part ii	1 mark

Find value of x for which $f(x) = -2$

Let $f(x) = 2$,

$$\frac{1}{2}x - 3 = -2$$
$$\frac{1}{2}x = -2 + 3$$
$$\frac{1}{2}x = 1$$
$$x = 2 \times 1$$
$$x = 2$$

Question 4a part iii	2 marks

List ordered pairs of the function

We already know two, we need to know $f(3)$.

$$f(3) = \frac{1}{2}(3) - 3$$
$$= -1\frac{1}{2}$$

So, ordered pairs are; $(1, -2\frac{1}{2})$, $(2, -2)$ and $(3, -1\frac{1}{2})$.

Question 4a part iv	2 marks

Show that $f(x)$ ‡ 5

If we let $f(x) = 5$,

$$\frac{1}{2}x - 3 = 5$$
$$\frac{1}{2}x = 8$$
$$x = 16$$

Since 16 is out of the domain, A, of the function which includes $\{1,2,3\}$, $f(x)$ *cannot be equal to 5.*

Question 4b part i a)	2 marks

Solve inequality

$$3x - 1 < 11$$
$$3x < 12$$
$$x < \frac{12}{3}$$
$$x < 4$$

Question 4b part i b)	2 marks

Solve inequality

$$2 \leq 3x - 1$$

$$2 + 1 \leq 3x$$

$$3 \leq 3x$$

$$1 \leq x$$

$$x \geq 1$$

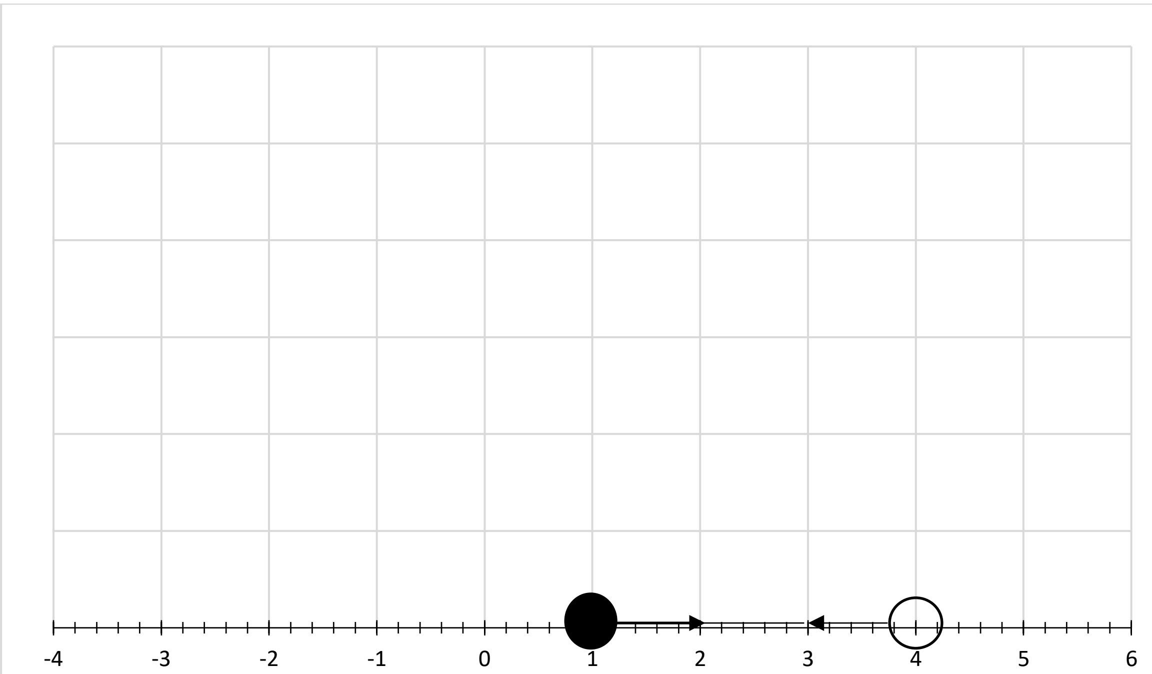
Question 4b part ii	2 marks

Represent on a number line

First inequality is represented by $x < 4$

Second inequality is represented by $x \geq 1$

On the same number line the inequality is drawn:



Question 5a part i	1 mark

Data Given: *Cricket occupies 45° of the pie chart, along with Tennis occupying* 94° *and with Football occupying the rest. Football represented by x*

Calculate value of x

All angles of a circle add up to 360°

So,

$$x = 360° - (94° + 45°)$$
$$= 221°$$

Question 5a part ii	1 mark

What percent of students chose cricket?

$$\text{Percentage} = \frac{94°}{360°} \times 100\%$$
$$= 26.11\ \% \text{ (to 2 d.p.)}$$

Question 5a part iii	2 mark

Data Given: *40 students chose Tennis.*

How many students are there in Total?

Percent of students who chose Tennis:

$$\text{Percentage} = \frac{45°}{360°} \times 100\%$$
$$= 12.5\ \%$$

If 12.5% of students equals 40 students, then

100% of students equals

$$\frac{40}{12.5} \times 100$$
$$= 320 \text{ students}$$

Therefore, Total Students is 320.

Question 5b part i	1 mark

Fill in the table, using the frequency polygon:

Number of matches(f)	5	7	3	3	4	2	1
Number of goals scored(x)	0	1	2	3	4	5	6

Question 5b part ii	1 mark

Find the Modal number of goals scored

The team scored 1 goal in 7 matches, this score occurs the most, having the highest frequency. Therefore, the modal number of goals scored by the team is 1.

Question 5b part iii	1 mark

Find the Median number of goals scored

25 matches were played in total. As median is the middle value, we determine the number of goals scored during the 13th match (since half of 25 is 12.5, rounding to 13). At the 13th match, the median score is 2.

Question 5b part iii	1 mark

Find the Mean number of goals scored

Number of matches(f)	5	7	3	3	4	2	1
Number of goals scored(x)	0	1	2	3	4	5	6
fx	0	7	6	9	16	10	6

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{0+7+6+9+16+10+6}{5+7+3+3+4+2+1}$$

$$= \frac{54}{25}$$

$$= 2.16 \text{ goals}$$

Question 6a part i 1 mark

Data Given: *Disc with square centre with side 6mm and a radius of 21mm.*

Find Circumference of the disc

$$\text{Circumference} = 2\pi r$$

$$= 2 \times \frac{22}{7} \times 21$$

$$= 132 \text{ mm}$$

Question 6a part ii 2 mark

Find area of cross section of disc, in

We first find the area of the whole disc, ignoring the square centre

$$\text{Area of whole disc} = \pi r^2$$

$$= \frac{22}{7} \times 21 \times 21$$

$$= 1386\ mm^2$$

Now, we find area of square centre,

$$\text{Area of square centre} = s \times s$$

$$= (6)^2$$

$$= 36\ mm^2$$

Subtracting the area of the square from the first area found, we get the area of the disc,

$$\text{Area of cross-section of disc} = 1386 - 36$$
$$= 1350\ mm^2$$

Question 6a part iii	3 marks

Data Given: *Discs made are 2mm in thickness.*

How many discs may be created from of available metal?

We first find the volume of one disc, now given the thickness,

$$\text{Volume of 1 disc} = \text{Cross-sectional area} \times \text{thickness}$$
$$= 1350 \times 2$$
$$= 2700\ mm^3$$

Converting to cm^3, given the information,

$$1000\ mm^3 = 1\ cm^3$$
$$1\ mm^3 = \frac{1}{1000}\ cm^3$$
$$2700\ mm^3 = \frac{2700}{1000}\ cm^3$$
$$= 2.7\ cm^3$$

Finding number of discs that may be made from ,

$$\text{Number of discs that can be made} = \frac{1000}{2.7}$$
$$= 370.4 \text{ discs (to 1 d.p.)}$$

Approximating to nearest whole number,

$\therefore$ Number of discs $= 370$ discs

Question 6b part i	1 mark

Data Given: *Globe is sphered physical representation of Earth. Using a string, equator represented by string of 160cm in length. Actual length of Earth's Equator is 40000km.*

Find what length of this string would represent 500 km

$$40\ 000 \text{ km is represented by } 160 \text{ cm}$$

$$\therefore 1 \text{ km will be represented by } \frac{160}{40\ 000} \text{ cm}$$

$$500 \text{ km will be represented by } \frac{160}{40\ 000} \times 500 = 2\text{cm}$$

So, 2cm represents 500 km on the length of string.

Question 6b part ii	2 marks

Data Given: *25cm is the length of string from Palmyra to Quintec.*

Find actual length from Palmyra to Quintec, PQ, in km.

$$160 \text{ cm represents } 40\ 000 \text{ km}$$

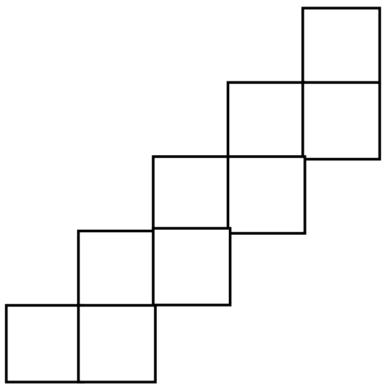
$$1 \text{ cm represents } \frac{40\ 000}{160} \text{ km}$$

$$25 \text{ cm represents } \frac{40\ 000}{160} \times 25 \text{ km} = 6250 \text{ km}$$

So, 25cm represents 6250 actual km.

Question 7a	2 marks

Draw Figure 4 of the sequence



Question 7b	2 marks

Data Given: A table is displayed with the figure number, number of squares in each figure and the perimeter of each figure.

Fill in the table given for rows (i),(ii) and (iii).

	Figure no.	Number of squares in Figure(S)	Perimeter of Figure(P)
	1	3	8
	2	5	12
	3	7	16
(i)	4	**9**	**20**

(iii)

(ii)

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21	43	**88**
n	$S = 2n + 1$	$P = 4n + 4$

(ii) When S =43,

$$n = \frac{S-1}{2}$$

$$n = \frac{43-1}{2}$$

$$n = 21$$

When $S = 43$,

$$P = 4n + 4$$

$$= 4(21) + 4$$

$$= 88$$

Question 7c 2 marks

Determine relationship between Perimeter of figure, P, and number of squares in figure, S.

$S = 2n + 1$

$P = 4n + 4$

Rearranging equation S to make n the subject of the formula gives,

$$n = \frac{S-1}{2}$$

Substituting this equation into equation P,

$$P = 4\left(\frac{S-1}{2}\right) + 4$$

$$P = 2(S - 1) + 4$$

$$P = 2S - 2 + 4$$

$$P = 2S - 2$$

Question 8a part i 2 marks

Data Given: *A graph is provided with some missing values. A table is also displayed with x and y values of the equation, some filled and some missing. The graph displayed is represented by the equation, .*

Fill in the table's missing values

x	0.1	0.2	0.5	1	1.5	2	2.2	2.5
y	10.3	5.6	**3.5**	4	5.2	**6.5**	7.1	7.9

When $x = 0.5$, substituting into equation y,

$$y = 3x + \frac{1}{x}$$

$$= 3(0.5) + \frac{1}{0.5}$$

$$= 1.5 + 2$$

$$= 3.5$$

When $x = 2$, substituting into equation y,

$$y = 3x + \frac{1}{x}$$

$= 3(2) + \frac{1}{2}$

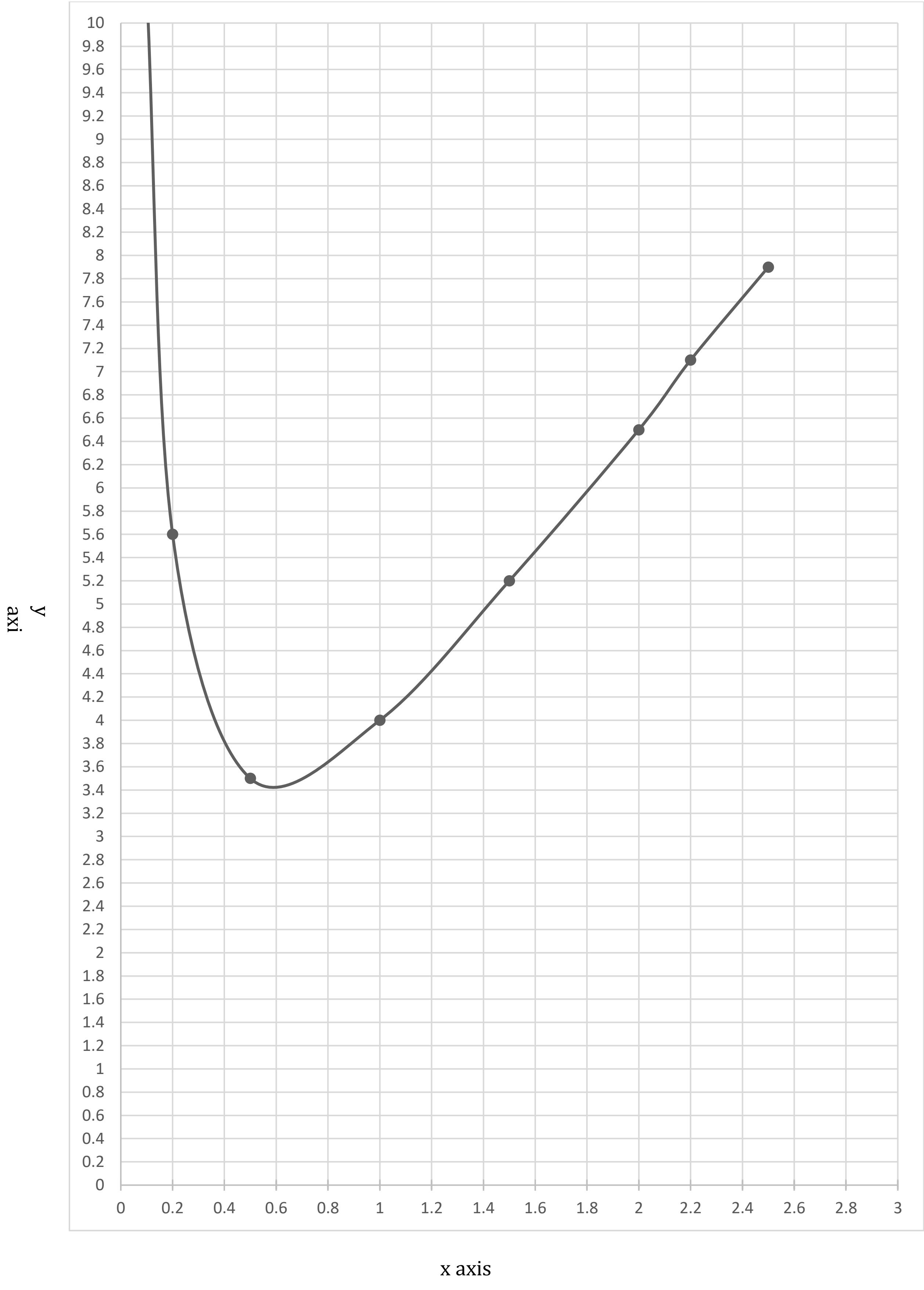
$= 6 + 0.5$

$= 6.5$

Question 8a part ii	2 marks

Data Given: *The graph provided has some points missing and some already plotted.*

Plot your new-found points and connect the points, drawing the graph.


10
9.8
9.6
9.4
9.2
9
8.8
8.6
8.4
8.2
8
7.8
7.6
7.4
7.2
7
6.8
6.6
6.4
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6
5.8
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5.4
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4.8
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3.8
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2.8
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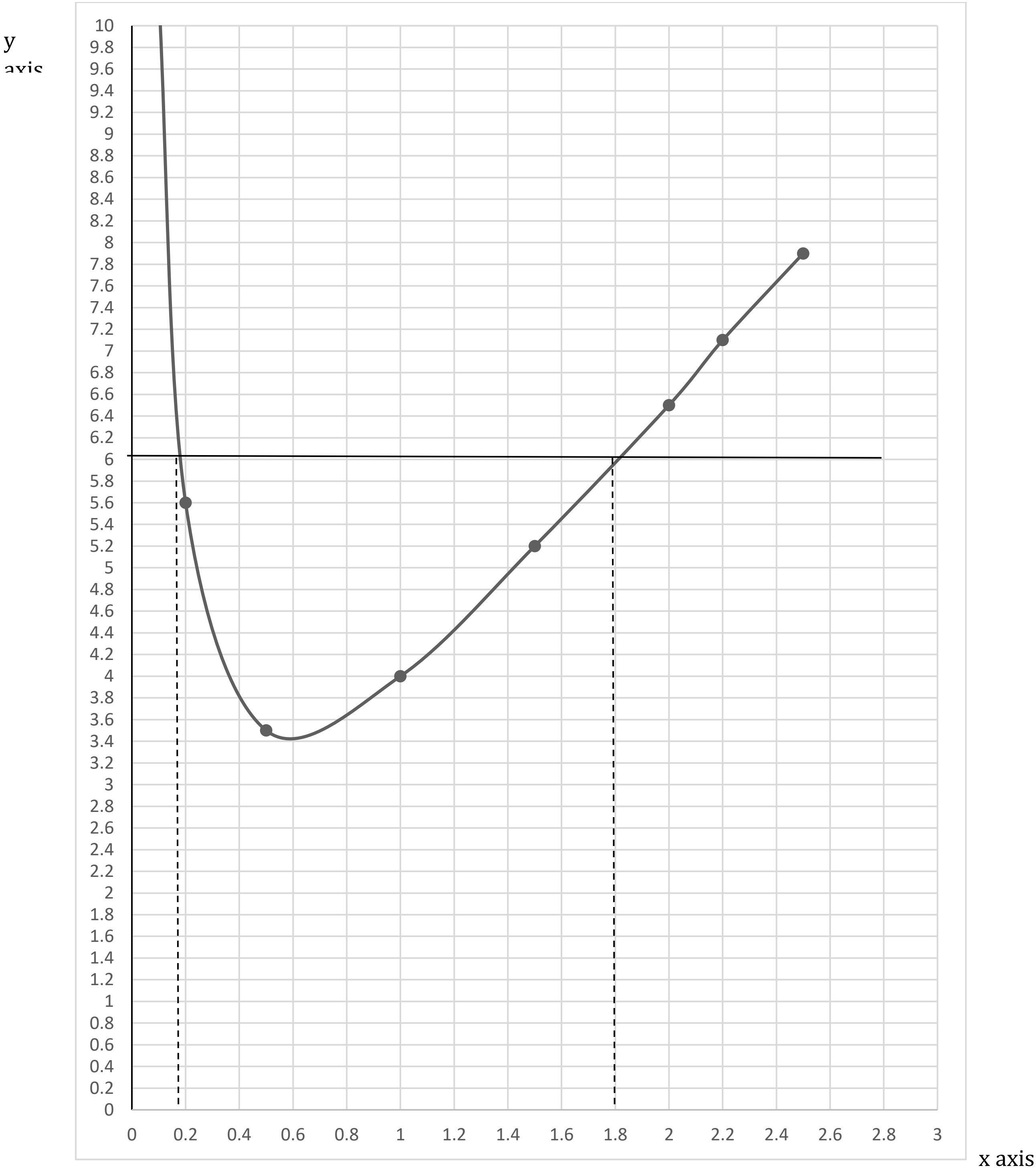
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
y
axi
x axis

Question 8a part iii	3 marks

Find solutions to $3x + \frac{1}{x} = 6$. Draw a straight line appropriately.

It is seen here that $y = 6$

We draw this line on the graph.



From this diagram, we may identify that the two possible values of x for which are 1.82 and 0.18.

Question 8b part i	1 mark

Data Given: *A graph is shown with two axes labelled speed and time(seconds). Use the graph to answer the following,*

Find acceleration, in ms^{-2}*, of car during stage B.*

To find the acceleration during stage B, we find the gradient of the graph from the beginning of stage B to the end of stage B.

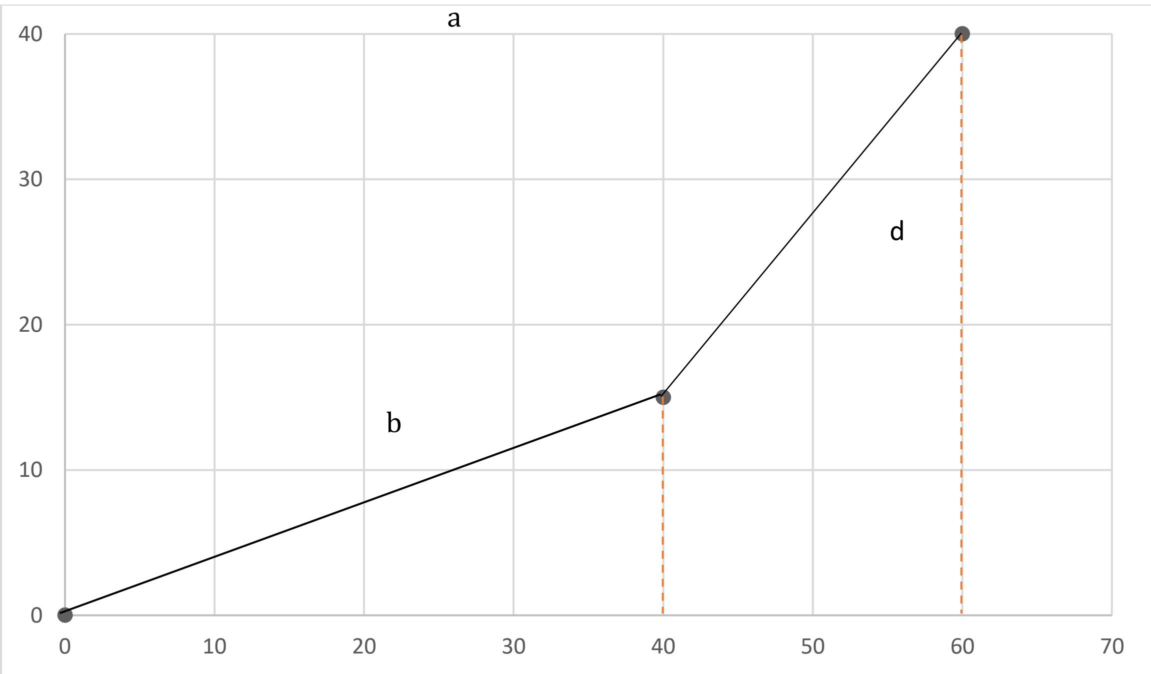
First coordinates would be (40,15).

Second set of coordinates would be (60,40).

$$\text{Gradient} = \frac{40-15}{60-40}$$
$$= \frac{25}{20}$$
$$= 1.25\ ms^{-2}$$

Question 8b part ii	3 marks

Find Average Speed of car during stage B



By finding area of the trapezium, we may find the total distance from the beginning and end of stage B.

We will then take the total time taken.

Area of trapezium $= \frac{1}{2}(a+b)d$

$$a = 40$$

$$b = 15$$

d is the height of the trapezium, h.

So,
$$d = 60 - 40$$
$$= 20$$

$$\text{Area} = \frac{1}{2}(a+b)d$$
$$= \frac{1}{2}(40+15)20$$
$$=550\text{m}$$

Therefore,

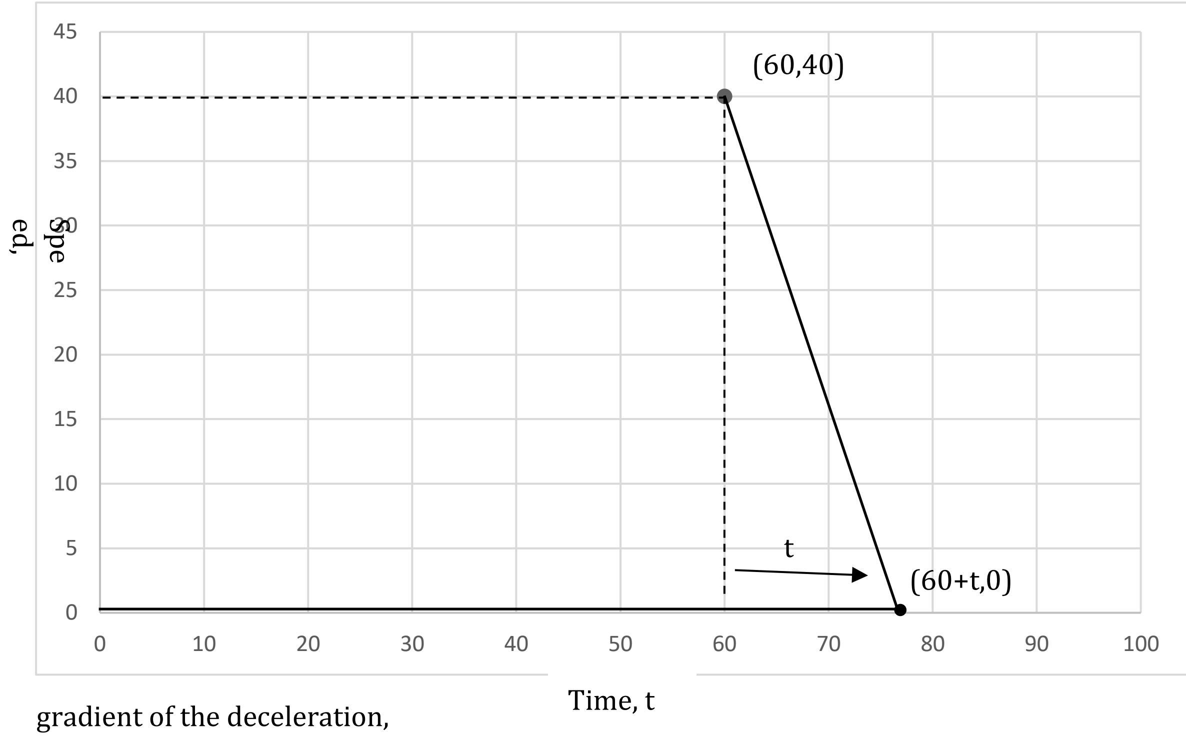
$$\text{Total time taken} = 60 - 40$$
$$=20 \text{ seconds}$$

$$\therefore \text{Average speed of car} = \frac{550}{20}$$
$$= 27.5\ ms^{-2}$$

Question 8b part iii	1 mark

Data Given: *At time, t=60s, car will decelerate uniformly at rate of 2.5*

How long will it take for car to come to a rest?



We find the gradient of the deceleration,

$$\text{Gradient} = \frac{40-0}{60-(60+t)}$$

$$= \frac{40}{-t}$$

Rate of deceleration is equal to the gradient,

$$\frac{40}{-t} = -2.5$$

$$2.5t = 40$$

$$t = \frac{40}{2.5}$$

$$t = 16\text{s}$$

When decelerating, value is negative, whereas with acceleration, value is positive.

Hence,

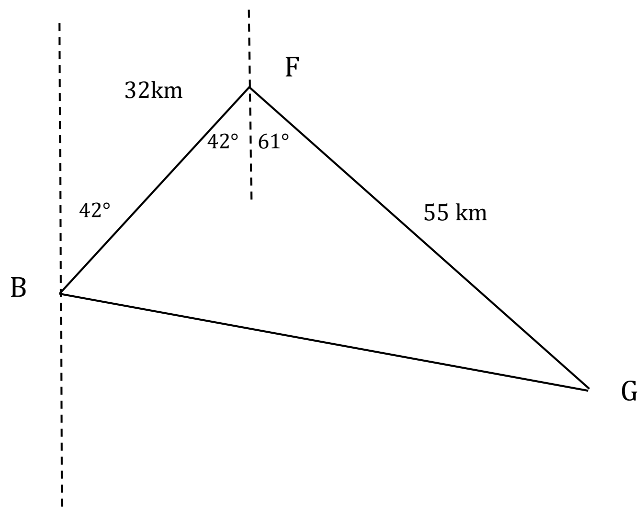
Total time to come to rest is

$$= 60 + 16$$

$= 76$ seconds from the start of travel

Question 9a part i	1 mark

Determine the bearing of B from F



Alternate angles are the same. We get 42°.

Therefore, the remaining angle is $103° - 42° = 61°$

We may determine that the bearing of B from F is 180°+42°=222°

Question 9a part ii	2 marks

Find distance BG, to 1 d.p.

Using Cosine Rule:

$$BG^2 = (32)^2 + (55)^2 - 2(32)(55)\ cos\ cos\ 103°$$

$$BG^2 = 1024 + 3025 - 3520(-0.22495)$$

$$= 4049 + 791.828$$

$$= 4840.828$$

$$BG = 69.58$$

$$= 69.6 \text{ km (to 1 d.p.)}$$

Question 9a part iii	3 marks

Calculate bearing of G from B

Using Sine Rule:

$$\frac{69.58}{sinsin\ 103°} = \frac{55}{sinsin\ \theta}$$

$$\therefore sin\ sin\ \theta = \frac{55 \times sinsin\ 103°}{69.58}$$

$$= 0.7702$$

$$\theta = sin^{-1}(0.7702)$$

$$= 50.4°$$

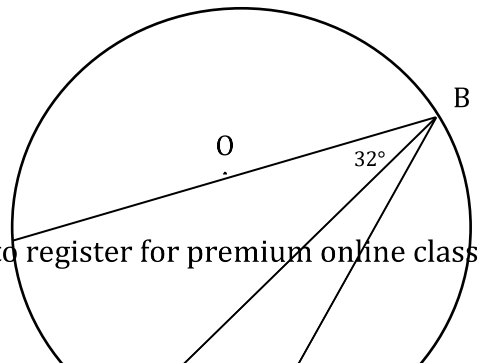
This angle plus the bearing of G from B will give us our answer,

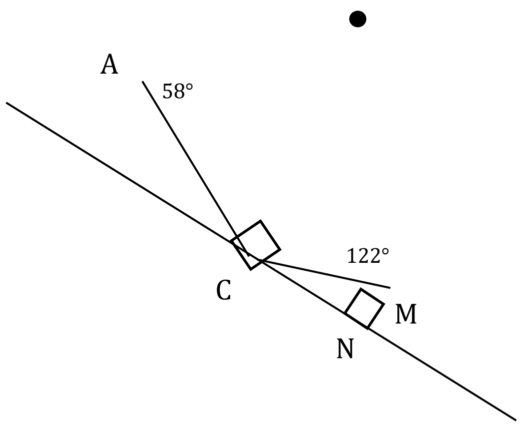
$$\text{Angle} = 42° + 50.4°$$

$$= 92.4°$$

Question 9b part i	2 marks

Find angle ABC.





Since all angles in a triangle add up to 180°,

Angle $ABC = 180° - (90° + 58°)$

$= 32°$

Question 9b part ii	4 marks

Find angle CMB.

Since opposite angles in a cyclic quadrilateral are supplementary,

Angle $CMB = 180° - 58°$

$= 122°$

Question 9b part iii	2 marks

Find Angle NCM

Since angles in a straight line add up to 180°,

Angle $CMN = 180° - 122°$

$= 58°$

Since all angles in a triangle add up to 180°,

Angle $NCM = 180° - (90° + 58°)$

$= 32°$

Question 10a part i	2 marks

Data Given:

$$T = (2\ \ -1\ 2\ 0\ )$$

Find a and b.

$$(a\ b\ ) = (2\ \ -1\ 2\ 0\ )(-2\ 3\ )$$

$$= ((2 \times -2)\ (-1 \times 3)\ (2 \times -2)\ (0 \times 3)\ )$$

$$= (-4 - 3\ \ -4 + 0\ )$$

$$= (-7\ \ -4\ )$$

So, $a = -7$

$b = -4$

Question 10a part ii	2 marks

Determine transformation matrix mapping A' to A

$$T = (2\ \ -1\ 2\ 0\ )$$

$$det\ det\ (T)\ = (2 \times 0) - (-1 \times 2)$$
$$= 2$$

$$T^{-1} = \frac{1}{2}(0\ \ -(-1)\ \ -(2)\ 2\ )$$
$$= \left(0\ \frac{1}{2}\ -1\ 1\right)$$

Question 10a part iii a   2 marks

Find 2x2 matrix representing combined transformation of T followed by P

$$P = (0\ 1\ 1\ \ -2\ )$$

The combined transformation T followed by P is,

$$PT = (0\ 1\ 1\ \ -2\ )(2\ \ -1\ 2\ 0\ )$$
$$= \big((0 \times 2) + (1 \times 2)\ (0 \times -1) + (1 \times 0)\ (1 \times 2) + (-2 \times 2)\ (1 \times -1) + (-2 \times 0)\ \big)$$
$$= (2\ 0\ \ -2\ \ -1\ )$$

Question 10a part iii b   1 mark

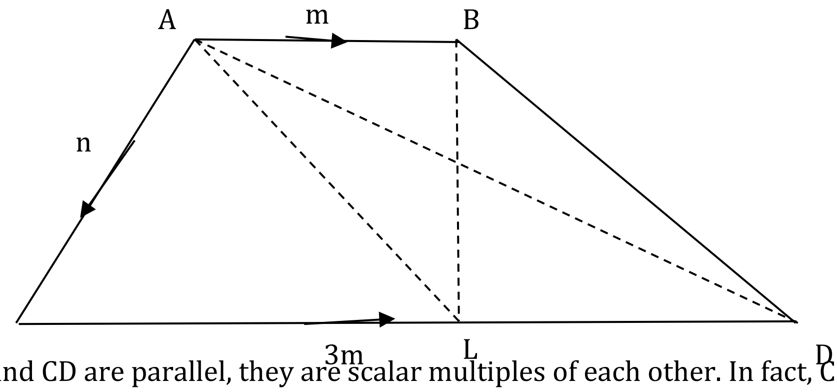
Find image of point (1,4) under this transformation

$$(2\ 0\ \ -2\ \ -1\ )(1\ 4\ ) = \ \big((2 \times 1) + (0 \times 4)\ (-2 \times 1) + (-1 \times 4)\ \big)$$
$$= (2\ \ -6\ )$$

$\therefore$ The image is $(2, -6)$.

Question 10b part i   1 mark

Complete the statement given this quadrilateral.



Since AB and CD are parallel, they are scalar multiples of each other. In fact, CD is larger than AB by a scale factor of 3.

Since $CD = 3\ AB$, then

$$|\overrightarrow{AB}| = \frac{1}{3}|\overrightarrow{DC}|$$

Therefore, the completed statement would be;

$\overrightarrow{AB}$ and $\overrightarrow{DC}$ are parallel and $|\overrightarrow{AB}|$ is $\frac{1}{3}$ times the size of $|\overrightarrow{DC}|$.

Question 10b part ii	1 mark

Express $\overrightarrow{BC}$ in terms of m and n,

According to triangle law,

$$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$$
$$= -(m) + \overrightarrow{AC}$$

Finding $\overrightarrow{AC}$,

$$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$$
$$= n + 3m$$

So,

$$\overrightarrow{BC} = -m + (n + 3m)$$
$$= n + 2m$$

Question 10b part iii	2 marks

Data Given: $L\ is\ midpoint\ of\ \overrightarrow{CD}$

Find $\overrightarrow{BL}$ in terms of m and n.

Finding $\overrightarrow{AL}$,

$$\overrightarrow{AL} = \overrightarrow{AD} + \overrightarrow{DL}$$

$$= \overrightarrow{AD} + \frac{1}{2}\overrightarrow{DC}$$

$$= n + \frac{1}{2}(3m)$$

Therefore,

$$\overrightarrow{BL} = \overrightarrow{BA} + \overrightarrow{AL}$$

$$= -(m) + \overrightarrow{AL}$$

$$= -(m) + n + \frac{1}{2}(3m)$$

$$= \frac{1}{2}m + n$$