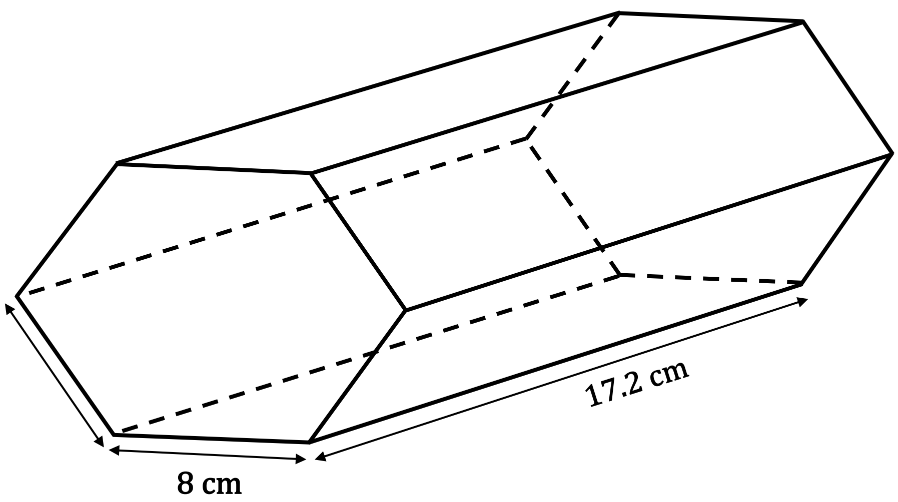
Solutions to CSEC Maths P3 June 2022

Answer ALL questions.

All working must be clearly shown.

1. $\left(\text{Use } \pi = \frac{22}{7} \text{ where required.}\right)$

The diagram below shows a solid bar of gold in the shape of a hexagonal prism of length 17.2 cm. The vertical cross-section of this prism is a **regular** hexagon with side of length 8 cm.



(a) (i) The formula for the area of any regular polygon is given by

$$\text{Area} = \frac{s^2 n}{4\tan\left(\frac{180}{n}\right)}$$

where s is the length of any side

n is the number of sides of the regular polygon, and

tan is the tangent function calculated in **degrees**.

Using the formula above, calculate the area of the vertical cross-section of the hexagonal bar of gold. [2]

$$\text{Area} = \frac{s^2 n}{4\tan\left(\frac{180}{n}\right)}$$

$$= \frac{(8)^2(6)}{4\tan\left(\frac{180}{6}\right)}$$

$$= 96\sqrt{3}\ cm^2$$

$$= 166\ cm^2 \qquad \text{(to the nearest } cm^2\text{)}$$

∴ The area of the vertical cross-section of the hexagonal bar of gold is $166\ cm^2$.

(ii) Determine the volume of the bar of gold. [1]

$$\text{Volume} = \text{Cross-sectional Area} \times \text{length}$$

$$= 96\sqrt{3} \times 17.2$$

$$= 2\ 859.96\ cm^3$$

$$= 2\ 860\ cm^3 \qquad \text{(to the nearest } cm^3\text{)}$$

∴ The volume of the bar of gold is $2\ 860\ cm^3$.

(iii) Given that the density of gold is $19.3\ \text{g/cm}^3$, calculate the mass of the bar of gold, **to the nearest kilogram.** [2]

$\left(\text{Density} = \frac{\text{mass}}{\text{volume}}\right)$

Density of gold $= 19.3\ \text{g/cm}^3$

Now,

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$19.3 = \frac{\text{mass}}{96\sqrt{3} \times 17.2}$$

$$\text{mass} = 19.3 \times 96\sqrt{3} \times 17.2$$

$$= 55\ 197.27\ \text{g} \qquad \text{(to 2 decimal places)}$$

Dividing by 1000 convert this value to kg:

$$\text{mass} = \frac{55\ 197.27}{1000}$$

$$= 56\ \text{kg} \qquad \text{(to the nearest kg)}$$

$\therefore$ The mass of the bar of gold, to the nearest kilogram, is 56 kg.

(iv) Calculate the TOTAL surface area of the bar of gold. [2]

Area of hexagonal face $= 96\sqrt{3}\ cm^2$

$$\text{Area of rectangular face} = l \times b$$

$$= 8 \times 17.2$$

$$= 137.6\ cm^2$$

Hence,

$$\text{Total surface area} = 2(\text{Area of hexagonal face}) + 6(\text{Area of rectangular face})$$

$$= 2\left(96\sqrt{3}\right) + 6(137.6)$$

$$= 1155.75\ cm^2$$

$$= 1156\ cm^2 \qquad (\text{to the nearest } cm^2)$$

(b) A jeweller melted 393 cm^3 of gold to make 6 identical spheres. Calculate the radius of EACH sphere.

$\left(\text{The volume, } V\text{, of a sphere with radius } r \text{ is } V = \frac{4}{3}\pi r^3.\right)$ [3]

Volume of a sphere, $V = \frac{4}{3}\pi r^3$

6 spheres $= 393\ cm^3$

1 sphere $= \frac{393}{6}\ cm^3$

Now,

$$\frac{393}{6} = \frac{4}{3}\pi r^3$$

$$\frac{393}{6} = \frac{4}{3} \times \frac{22}{7} \times r^3$$

$$r^3 = \frac{393}{6} \times \frac{3}{4} \times \frac{7}{22}$$

$$r^3 = \frac{2751}{176}$$

$$r = \sqrt[3]{\frac{2751}{176}}$$

$r = 2.5$ cm (to 1 decimal place)

∴ The radius of each sphere is 2.5 cm.

Total 10 marks

2. (a) The distance a bus travels on a journey from City A to City B is 800 km.

(i) Write an expression, in terms of x, for the average speed of the bus, in km/h, when the journey takes

(a) x hours [1]

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{800}{x}$$

(b) $(x + 2)$ hours [1]

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{800}{x+2}$$

(ii) The difference between the average speeds in (a)(i) is 20 km/h.

Show that $x^2 + 2x - 80 = 0$. [3]

$\text{Difference} = 20\ \text{km/h}$

$\text{Difference} = \frac{800}{x} - \frac{800}{x+2}$

Hence,

$$\frac{800}{x} - \frac{800}{x+2} = 20$$

$$\frac{(x+2)800 - 800x}{x(x+2)} = 20$$

$$\frac{800x + 1600 - 800x}{x(x+2)} = 20$$

$$\frac{1600}{x(x+2)} = 20$$

$$1600 = 20x(x+2)$$

$$1600 = 20x^2 + 40x$$

$$20x^2 + 40x - 1600 = 0$$

$(\div 20)$

$$x^2 + 2x - 80 = 0$$

Q.E.D.

(iii) Solve the quadratic equation $x^2 + 2x - 80 = 0$ and hence, determine the average speed of the bus if the journey takes $(x + 12)$ hours. [3]

$$x^2 + 2x - 80 = 0$$

$$x^2 + 10x - 8x - 80 = 0$$

$$x(x+10) - 8(x+10) = 0$$

$$(x-8)(x+10) = 0$$

Either $x - 8 = 0$ or $x + 10 = 0$

$x = 8$ $x = -10$

Since x cannot be negative, then $x = 8$ hours.

Now, the journey takes $(x + 12)$ hours.

$x + 12 = 8 + 12$

$= 20$ hours

Distance $= 800$ km

Hence,

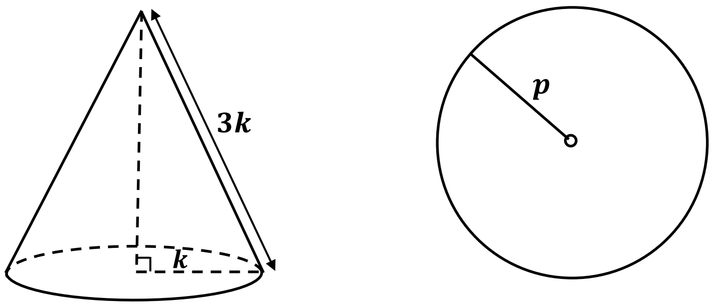
$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{800}{20}$$

$$= 40 \text{ km/h}$$

$\therefore$ The average speed of the bus is 40 km/h.

(b) The **total** surface area of a cone with radius k and slant height $3k$ is equal to the area of a circle with radius p.



Show that $p = 2k$.

[The **total** surface area, A, of a cone with radius r and slant height l is $A = \pi r^2 + \pi rl$.] [2]

Surface area of cone, $A = \pi r^2 + \pi rl$

Substituting $r = k$ and $l = 3k$ gives,

$$A = \pi r^2 + \pi rl$$

$$= \pi(k)^2 + \pi(k)(3k)$$

$$= k^2\pi + 3k^2\pi$$

Area of circle, $A = \pi r^2$

Substituting $r = p$ gives,

$A = \pi(p)^2$

$\quad = p^2\pi$

Equating both equations gives,

$k^2\pi + 3k^2\pi = p^2\pi$

Dividing by π throughout gives:

$k^2 + 3k^2 = p^2$

$4k^2 = p^2$

$\sqrt{4k^2} = \sqrt{p^2}$

$2k = p$

$\therefore p = 2k$

Q.E.D.

Total 10 marks