Solutions to CSEC Maths P2 May 2019

Question 1a part (i)

Calculate the exact value of

$$\frac{\mathbf{2\frac{1}{4} - 1\frac{3}{5}}}{\mathbf{3}}$$

$$= \left(\frac{9}{4} - \frac{8}{5}\right) \div 3$$

$$= \left(\frac{5(9) - 4(8)}{20} \div \frac{3}{1}\right)$$

$$= \left(\frac{45 - 32}{20}\right) \times \frac{1}{3}$$

$$= \frac{13}{20} \times \frac{1}{3}$$

$$= \frac{13}{60}$$

Question 1a part (ii)

Calculate the exact value of

$$2.14 \sin 75^{o}$$

$= \mathit{2.07\ to\ 2\ decimal\ places}$

Question 1b part (i)

Data Given

Item	Amounts Allocated
Rent	$\$x$
Food	\$629
Other living expenses	$\$2x$
Savings	\$1,750
Total	**\$4,320**

Calculate the annual take-home pay

$$Rate\ of\ Pay\ =\ \$4{,}320\ per\ fortnight\ (every\ 2\ weeks)$$
$$One\ year\ =\ 52\ weeks$$

$$\therefore\ There\ are\ \frac{52}{2} = 26\ fortnights\ per\ year$$
$$26 \times \$4{,}320\ =\ \$112{,}320\ per\ year$$

Question 1b part (ii)

Calculate the amount of money spent on "Rent"

Required to find rent

$$x + 629 + 2x + 1{,}750 = 4{,}320$$
$$3x + 2{,}379 = 4{,}320$$
$$3x = 1{,}941$$
$$x = \frac{1{,}941}{3}$$
$$x = \$647$$

Question 1b part (iii)

Calculate how long it will take to pay off Tuition

$$Tuition\ cost\ =\ \$150{,}000$$
$$Savings\ per\ fortnight\ =\ \$1{,}750$$
$$Total\ savings\ per\ year\ =\ \$1{,}750\ \times\ 26$$
$$=\ \$45{,}500$$

$$\#\text{ of years to pay off }=\ \frac{\$150{,}000}{\$45{,}500}\ \approx\ \boldsymbol{3.30\ to\ 2\ d.p.}$$

OR

The question says that she saves the same amount of money each MONTH. There are two fortnights every month and 12 months per year. Therefore, there are 24 fortnights to consider.

$$\begin{aligned} Savings\ per\ year &= \$1{,}750 \times 24 \\ &= \$42{,}000 \end{aligned}$$

$$\textbf{\# of years to pay off} = \frac{\$150{,}000}{\$42{,}000} \approx \boldsymbol{3.57\ to\ 2\ d.p.}$$

<u>Either way she must work at least 4 years</u> **QED**

Question 2a part (i)

Simplify $3p^2 \times 4p^5$

$$= 12p^{2+5}$$

$$= 12p^7$$

Question 2a part (ii)

Simplify $\frac{3x}{4y^3} \div \frac{21x^2}{20y^2}$

$$= \frac{3x}{4y^3} \times \frac{20y^2}{21x^2}$$

$$= \frac{60xy^2}{84x^2y^3}$$

$$= \frac{5xy^2}{7x^2y^3}$$

$$= \frac{5}{7xy}$$

Question 2b part (ii)

Solve $\frac{3}{7x-1} + \frac{1}{x} = 0$

$$\frac{3x + 1\,(7x - 1)}{x(7x - 1\,)} = 0$$

If a fraction $= 0$*, then the numerator is* 0.

$$3x + 7x - 1 = 0$$

$$10x - 1 = 0$$

$$10x = 1$$

$$x = \frac{1}{10}$$

Question 2c part (i)

Simplify $(x \times 2)^2 = y$

$(2x)^2 = y$

$y = 4x^2$

Question 2c part (ii)

Calculate the values of x

$$y = x \quad \text{----------} \; equation\,1$$

$$y = 4x^2 \quad \text{----------} \; equation\,2$$

$Let\ equation\ 1 = equation\ 2.$

$$x = 4x^2$$

$$0 = 4x^2 - x$$

$$0 = x(4x - 1)$$

$Therefore,\ either\ x = 0\ \ or\ 4x - 1 = 0.$

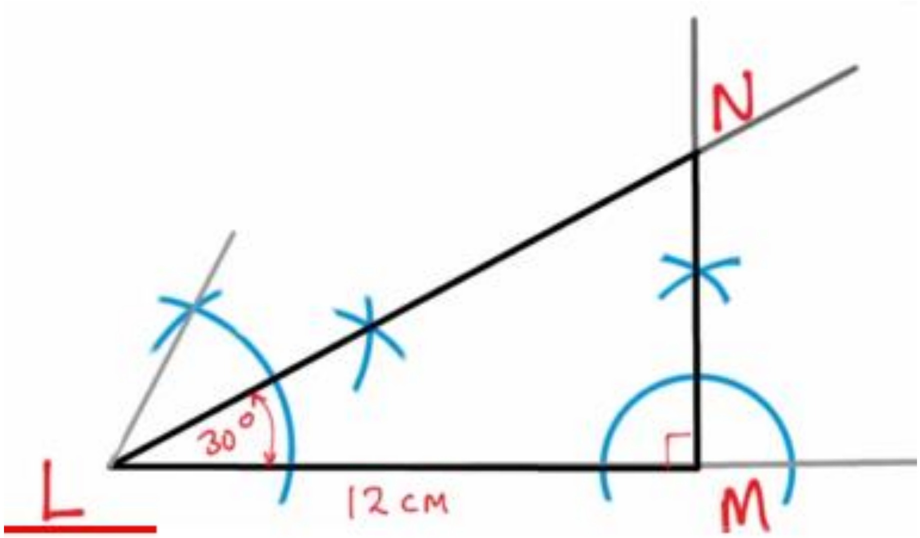
$$If \qquad 4x - 1 = 0$$

$$Then \qquad 4x = 1$$

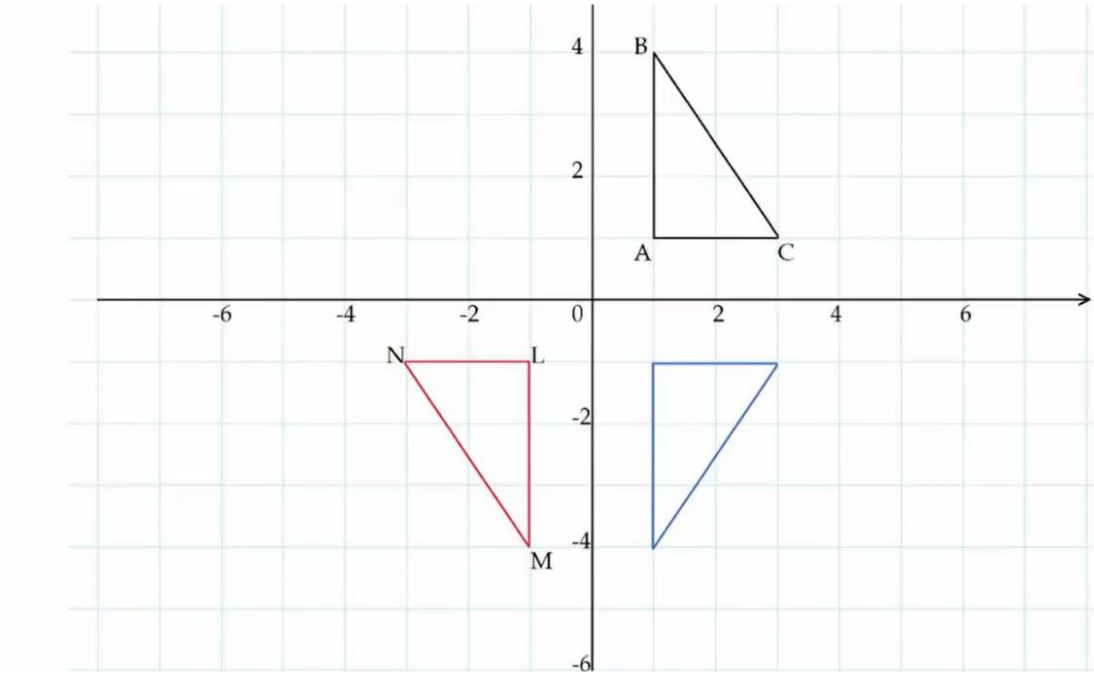
$$x = \frac{1}{4}$$

$$\therefore x = 0\ and\ \frac{1}{4}$$

Question 3a



Question 3b part (i)



Question 3b part (ii)

The single transformation that maps ΔABC onto ΔLMN is a 180° clockwise or anti-clockwise rotation about the origin OR a reflection in the origin.

Question 3b part (iii)

The 2 x 2 matrix that maps ΔABC onto ΔLMN is

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Question 4a part (i)

Express P varies inversely as the square of V

$$P \propto \frac{1}{V^2}$$

$$\therefore P = \frac{k}{V^2}$$

Question 4a part (ii)

Calculate the value of V when $P = 1$, *given that* $V = 3$ *when* $P = 4$.

$$4 = \frac{k}{3^2}$$

$$4 = \frac{k}{9}$$

$$4 \times 9 = k$$

$$\therefore k = 36$$

If $P = 1$*, then*

$$1 = \frac{36}{V^2}$$

$$V^2 = 36$$

$$V = \sqrt{36}$$

$$V = 6$$

Question 4b (i)

Calculate the value of x for $-7 < 3x + 5 \leq 7$

$Consider \quad -7 < 3x + 5$

$$-7 - 5 < 3x$$

$$-12 < 3x$$

$$-4 < x$$

Consider $\quad 3x + 5 \leq 7$

$$3x \leq 7 - 5$$

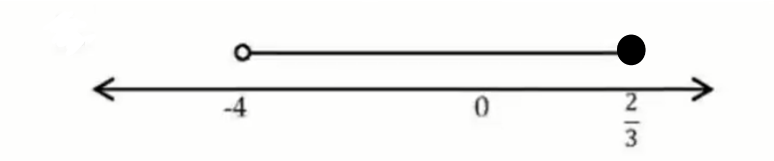
$$3x \leq 2$$

$$x \leq \frac{2}{3}$$

Hence, $\quad -4 < x \leq \frac{2}{3}.$

Question 4b (ii)

Draw the graph of $-4 < x \leq \frac{2}{3}$



Question 4c (i)

Considering,

$$\frac{x}{3} + \frac{y}{7} = 1$$

Find the coordinates of the y-intercept

State the equation in the form $y = mx + c$,

And find the value of c.

Multiplying by the LCM of the denominators gives,

$$7x + 3y = 21$$

$$3y = -7x + 21$$

$$y = -\frac{7}{3}x + \frac{21}{3}$$

$$y = -\frac{7}{3}x + 7$$

$\therefore$ *Coordinates of Q are* $(0,7)$.

Question 4c (ii)

State the gradient of the line

The gradient of the line is given by "m".

$$Gradient = -\frac{7}{3}$$

Question 5a (i)

Determine the Lower Class Boundary of the "data set

Lower class boundary of 21 - 30 is 20.5 litres.

Question 5a (ii)

Calculate the class width of the data set

Class width = Upper Class Boundary – Lower Class Boundary

$= 30.5 - 20.5$

$= 10$ vehicles

Question 5b

Calculate the range of the data set

Range = Highest Value – Lowest Value

$= 101 - 59$

$= 42$ *vehicles*

Question 5c

$$P\ (need > 50.5\ litres\ to\ fill\ tank) = \frac{Number\ of\ Desired\ Outcomes}{Total\ Number\ of\ Outcomes}$$

$$= \frac{150 - 129}{150}$$

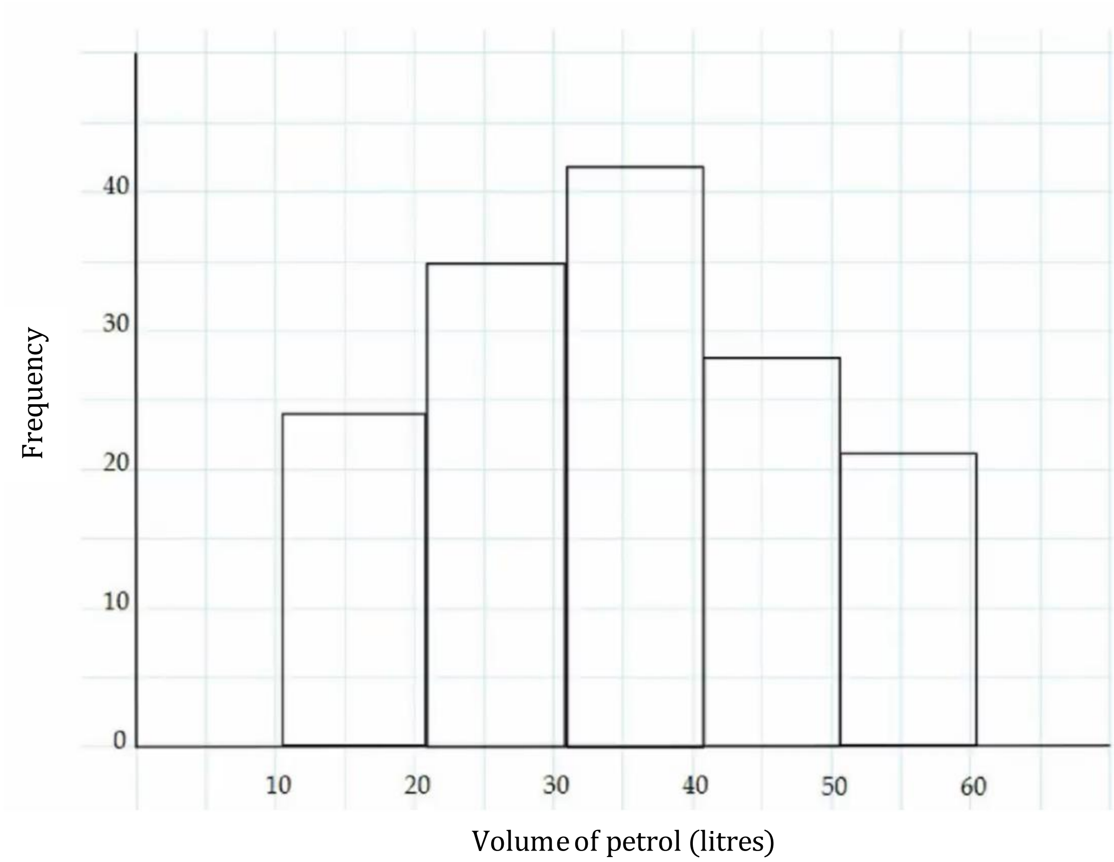
$$= \frac{21}{150}$$

$$= \frac{7}{50}$$

Question 5d

Bryon's estimate is wrong because the median would be given at the 75th vehicle (half the cumulative frequency) which would be found in the interval 31 - 40.

Question 5e



Question 6a (i)

$0.5 \times 25{,}000 = 12{,}500\ cm$

$= 0.125\ km$

Question 6a (ii)

An area of 1cm^2 on the map is represented by a $1cm \times 1cm$ square on the map. The actual area represented in real like by this would be $25{,}000 \times 25{,}000 = 625{,}000{,}000\ cm^2$.

Alternatively, since 25,000cm is one quarter of a km, then we can express this area as such:

$$0.25 \times 0.25 = 0.0625 km^2$$

An area of $2.25\ cm^2$ is 2.25 times the area of $1\ cm^2$.

$\therefore$ The area in real life will be 2.25 times the $0.0625\ km^2$.

$$2.25 \times 0.0625 = 0.140625\ km^2$$

Question 6b (i)

$Volume = \pi r^2 h$

$= \pi \times \left(\frac{3d}{2}\right)^2 \times 4$

$= \pi \times \frac{9d^2}{4} \times 4$

$= 9d^2\ \pi\, cm^3$

Question 6b (ii)

Calculate the height of Jar Y

Recall: $V = \pi r^2 h$

$$\therefore h = \frac{V}{\pi r^2}$$

$$= \frac{9d^2\pi}{\pi\left(\frac{d}{2}\right)^2}$$

$$= \frac{9d^2\pi}{1} \div \frac{d^2\pi}{4}$$

$$= \frac{9\cancel{d^2\pi}}{1} \times \frac{4}{\cancel{d^2\pi}}$$

$$= 36\text{cm}$$

Question 7 a (i)

Calculate the value of T_1

$$T_n = 3n^2 - 2$$

$$If \qquad n = 1,$$

$$Then \quad T_1 = 3(1) - 2$$

$$T_1 = 3 - 2$$

$$= 1$$

Question 7 a (ii)

Calculate the value of T_3

$$T_n = 3n^2 - 2$$

$$If \qquad n = 3,$$

$$Then \quad T_3 = 3(3)^2 - 2$$

$$T_3 = 27 - 2$$

$$= 25$$

Question 7 a (iii)

Calculate the value of n given that $T_n = 145$

$$145 = 3n^2 - 2$$

$$3n^2 = 147$$

$$n^2 = 49$$

$$n = 7$$

Question 7 b (i)

1, 1,2,3,5,8,13,21

$U(1) = 1$

$U(2) = 1$

$U(3)=2$

$U(4)=3$

$U(5)=5$

$U(6)=8$

$U(7)=13$

$U(8)=21$

Each term is the sum of the two terms that came before it (from the 3rd term onwards).

This is Fibonacci's sequence.

$$U(9) = U(8) + U(7) = 21 + 13 = 34$$
$$U(10) = U(9) + U(8) = 34 + 21 = 55$$

Question 7b (ii)

The term in the sequence which is the sum of U(18) and U(19) is U(20).

Any term is the sum of the two terms that came immediately before it.

Question 7b (iii)

RTS $U(20) - U(19) - U(17)$

Since $U(20) = U(19) + U(18)$, *then*

$U(20) - U(19) = U(18)$

Since $U(19) = U(18) + U(17)$, *then*

$U(19) - U(17) = U(18)$

Hence, $U(20) - U(19) = U(19) - U(17)$

Question 8a (i)

Given,

$$f(x) = \frac{9}{2x+1} \; and \; g(x) = x - 3$$

Calculate the value of x which cannot be in the domain of $f(x)$ is the value which makes the denominator equal to 0.

If $2x + 1 = 0, then$

$$2x = -1$$

$$x = -\frac{1}{2}$$

Question 8a (ii)

a. $Find\ fg(x)$

$$fg(x) = f(x-3)$$

$$= \frac{9}{2(x-3)+1}$$

$$= \frac{9}{2x-6+1}$$

$$= \frac{9}{2x-5}$$

b. $Find\ f^{-1}(x)$

$$Let\ y = f(x)$$

$$y = \frac{9}{2x+1}$$

$$Switch\ x\ and\ y$$

$$x = \frac{9}{2y+1}$$

$$Transpose\ for\ y$$

$$x(2y+1) = 9$$

$$2xy + x = 9$$

$$2xy = 9 - x$$

$$y = \frac{9-x}{2x}$$

$\therefore f^{-1}(x) = \frac{9-x}{2x}$

Question 8b (i)

$RTS\ that\ area\ of\ ABCD\ is\ x^2 + 2x = 4 = 0$

$Area\ of\ rectangle = l\ \times b$

Substituting $l = 4 + 3x\ and\ b = 2 + 3x$ gives

$A = (4 + 3x)(2 + 3x)$

$A = 4(2 + 3x) + 3x(2 + 3x)$

$A = 8 + 12x + 6x + 9x^2$

$A\ = 9x^2 + 18x + 8$

$Given\ that\ area = 44,$

$\therefore \quad \mathbf{9x^2 + 18x + 8 = 44}$

$9x^2 + 18x + 8 - 44 = 0$

$9x^2 + 18x - 36 = 0$

$Divide\ throughout\ by\ 9$

$x^2 + 2x - 4 = 0$ **QED**

Question 8b (ii)

Using the quadratic formula to calculate x

$x = \frac{-2 \pm \sqrt{((2)^2 - 4(1)(-4))}}{2(1)}$

$x = \frac{-2 \pm \sqrt{(4+16)}}{2}$

$x = \frac{-2 \pm \sqrt{20}}{2}$

$\boldsymbol{x = 1.236\ cm\ to\ 3\ d.p.}$

Question 8b (iii)

Perimeter of unshaded region is as follows:

$$P = GA + AB + BC + CE + EF + FG$$

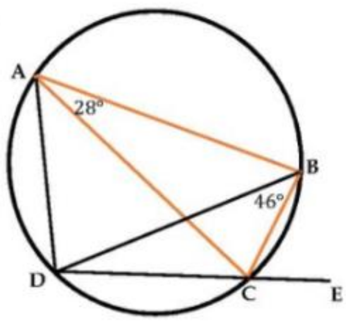
$$P = 3x + 2 + 3x + 4 + 3x + 3x + 4 + 2$$

$$P = 12x + 12$$

$$P = 12(1.236) + 12$$

$$\mathbf{P = 26.832cm \text{ to } 3\ d.p.}$$

Question 9a



i. $RTF\ \angle$DBA

DBA = ABC – DBC

$= \mathbf{90^o - 46^0}$

$= \mathbf{44^o}$

ABC is 90^o because the angle in a semicircle is a right angle.

ii. $RTF\ \angle$DAC

$\boldsymbol{DAC = DBC}$

$= \mathbf{46^o}$

Angles in the same segment standing on the same arc are equal.

iii. RTF ∠BCE

∠BCE = ∠DAB

$= 46^o + 28^o$

∠BCE = $\mathbf{74^o}$

The external angle of a cyclic quadrilateral is equal to the opposite internal angle.

Question 9b

i. $RTF\ the\ length\ of\ PS$

N. B. ∠PQS = ∠RSQ because of alternating or Z-angles.

$$\sin PQS = \frac{PS}{QS}$$

$$\therefore \sin PQS \times QS = PS$$

$$\boldsymbol{PS = \sin 30^{o} \times 8}$$

$$\boldsymbol{= 4cm}$$

ii. $RTF\ the\ length\ of\ PQ$

$Using\ Pythagoras'$ Theorem,

$$QS^2 = PQ^2 + PS^2$$

$$\therefore\ QS^2 - PS^2 = PQ^2$$

$$PQ^2 = 8^2 - 4^2$$

$$PQ = \sqrt{64 - 16}$$

$$\boldsymbol{PQ = \sqrt{48}}$$

$$\boldsymbol{= 6.93cm\ to\ 2d.p.}$$

iii. $RTF\ the\ area\ of\ PQRS$

PQRS is a trapezium

Area of trapezium $= \frac{1}{2}\ (PQ + RS^*) \times PS$

$$= \frac{1}{2}\ (6.93 + 8.54)\ \times 4$$

$$\boldsymbol{= 30.94cm\ (correct\ to\ 2\ d.p.)}$$

*Using the sine rule to find RS:

$$\frac{RS}{sinRQS} = \frac{QS}{sinQRS^{**}}$$

$$RS = \frac{8}{\sin 68^o} \times \sin 82^o$$

$$RS = 8.54cm\ to\ 2\ d.p$$

** To find ∠QRS:

Angle QRS $= 180° - (82° + 30°)$

$= 180° - 112°$

$= 68°$

Question 10a (i)

a. Find the following product

$$\begin{pmatrix}-1 & 3\\ 4 & h\end{pmatrix}\begin{pmatrix}k\\ 5\end{pmatrix} = \begin{pmatrix}(-1\times k) & + & (3\times 5)\\ (4\times k) & + & (h\times 5)\end{pmatrix}$$

$$= \begin{pmatrix}-k & + & 15\\ 4k & + & 5h\end{pmatrix}$$

b. Find the values of h and k

$$If \begin{pmatrix}-k & + & 15\\ 4k & + & 5h\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}, then$$

$$-k + 15 = 0, and$$

$$\boldsymbol{k = 15}$$

$$\therefore 4(15) + 5h = 0$$

$$60 + 5h = 0$$

$$5h = -60$$

$$h = \frac{-60}{5}$$

$$\boldsymbol{h = -12}$$

Question 10a (ii)

$$\begin{pmatrix}2 & 3\\ -5 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}5\\ 13\end{pmatrix}$$

To solve for x and y multiply the inverse of

$$\begin{pmatrix}2 & 3\\ -5 & 1\end{pmatrix} by \begin{pmatrix}5\\ 13\end{pmatrix}$$

Therefore,

$$Inverse = \frac{1}{(2\times 1)-(3\times -5)} \times \begin{pmatrix}1 & -3\\ 5 & 2\end{pmatrix}$$

$$= \frac{1}{17}\begin{pmatrix}1 & -3\\ 5 & 2\end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{17}\begin{pmatrix} 1 & -3 \\ 5 & 2 \end{pmatrix} \times \begin{pmatrix} 5 \\ 13 \end{pmatrix}$$

$$= \frac{1}{17}\begin{pmatrix} (1 \times 5) & + & (-3 \times 13) \\ (5 \times 5) & + & (2 \times 13) \end{pmatrix}$$

$$= \frac{1}{17}\begin{pmatrix} 5 & - & 39 \\ 25 & + & 26 \end{pmatrix}$$

$$= \frac{1}{17}\begin{pmatrix} -34 \\ 51 \end{pmatrix}$$

$$= \begin{pmatrix} \boldsymbol{\frac{1}{17}} & \times & \boldsymbol{-34} \\ \boldsymbol{\frac{1}{17}} & \times & \boldsymbol{51} \end{pmatrix}$$

$$= \begin{pmatrix} \boldsymbol{-2} \\ \boldsymbol{3} \end{pmatrix}$$

Question 10b

$$\overrightarrow{OA} = \begin{pmatrix} 9 \\ 0 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}, \quad \overrightarrow{AD} = \frac{1}{3}AB, \quad \overrightarrow{OE} = \frac{1}{3}OA$$

i. RTF $\overrightarrow{AB}$

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$

$$\boldsymbol{\overrightarrow{AB}} = \begin{pmatrix} \boldsymbol{-9} \\ \boldsymbol{0} \end{pmatrix} + \begin{pmatrix} \boldsymbol{3} \\ \boldsymbol{6} \end{pmatrix}$$

$$= \begin{pmatrix} \boldsymbol{-6} \\ \boldsymbol{6} \end{pmatrix}$$

ii. RTF $\overrightarrow{OD}$

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$$

$$\overrightarrow{OD} = \begin{pmatrix} 9 \\ 0 \end{pmatrix} + \frac{1}{3}\overrightarrow{AB}$$

$$\overrightarrow{OD} = \begin{pmatrix} 9 \\ 0 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -6 \\ 6 \end{pmatrix}$$

$$\boldsymbol{\overrightarrow{OD}} = \begin{pmatrix} \boldsymbol{9} \\ \boldsymbol{0} \end{pmatrix} + \begin{pmatrix} \boldsymbol{-2} \\ \boldsymbol{2} \end{pmatrix}$$

$$= \begin{pmatrix} \boldsymbol{7} \\ \boldsymbol{2} \end{pmatrix}$$

iii. RTF $\overrightarrow{BE}$ $\overrightarrow{BE} = \overrightarrow{BO} + \overrightarrow{OE}$

$$\overrightarrow{BE} = \begin{pmatrix} -3 \\ -6 \end{pmatrix} + \frac{1}{3}(\overrightarrow{OA})$$

$$\overrightarrow{BE} = \begin{pmatrix} -3 \\ -6 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 9 \\ 0 \end{pmatrix}$$

$$\boldsymbol{\overrightarrow{BE}} = \begin{pmatrix} \mathbf{-3} \\ \mathbf{-6} \end{pmatrix} + \begin{pmatrix} \mathbf{3} \\ \mathbf{0} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{0} \\ \mathbf{-6} \end{pmatrix}$$