STUDENT HUB

CSEC Mathematics

June 2012 – Paper 2

Solutions

SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Calculate the EXACT value of

$$\frac{3\frac{1}{5}-\frac{2}{3}}{2\frac{4}{5}}$$

giving your answer as a fraction in its lowest terms. [3]

$$\text{Numerator} = 3\frac{1}{5}-\frac{2}{3}$$

$$= \frac{16}{5}-\frac{2}{3}$$

$$= \frac{48-10}{15}$$

$$= \frac{38}{15}$$

$$\text{Denominator} = 2\frac{4}{5}$$

$$= \frac{14}{5}$$

$$\text{Numerator} \div \text{Denominator} = \frac{38}{15} \div \frac{14}{5}$$

$$= \frac{38}{15} \times \frac{5}{14}$$

$$= \frac{19}{21}$$

$\therefore \frac{3\frac{1}{5}-\frac{2}{3}}{2\frac{4}{5}} = \frac{19}{21}$

(b) The table below shows the cost price, selling price and profit or loss as a percentage of the cost price.

Copy and complete the table below, inserting the missing values at (i) and (ii). [4]

	Cost Price	Selling Price	Percentage Profit or Loss
(i)	\$55.00	\$44.00	_____
(ii)	_____	\$100.00	25% profit

(i) Selling Price = \$44.00

Cost Price = \$55.00

Since the selling price is less than the cost price, a loss has occurred.

Loss = Cost Price − Selling Price

= \$55.00 − \$44.00

= \$11.00

$\text{Percentage Loss} = \frac{Loss}{Cost\ Price} \times 100$

$= \frac{11}{55} \times 100$

$= 20\%$

∴ The percentage loss is 20%.

(ii) A 25% profit implies that the selling price is 125% of the cost price.

$$125\% = \$100$$

$$1\% = \frac{100}{125}$$

$$100\% = \frac{100}{125} \times 100$$

$$= \$80$$

∴ The cost price is \$80.

(c) The table below shows some rates of exchange:

US \$1.00 = EC \$2.70
TT \$1.00 = EC \$0.40

Calculate the value of

(i) EC \$1 in TT\$ [1]

$$EC\ \$0.40 = TT\ \$1.00$$

$$EC\ \$1 = \frac{TT\ \$1.00}{0.40}$$

$$= TT\ \$2.50$$

∴ EC \$1 = TT \$2.00

(ii) US $80 in EC$ [1]

US $1.00 = EC $2.70

US $80 = EC $2.70 × 80

= EC $216

∴ US $80 = EC $216

(iii) TT $648 in US$ [3]

TT $1.00 = EC $0.40

TT $648 = EC $0.40 × 648

= EC $259.20

Now,

US $1.00 = EC $2.70

EC $2.70 = US $1.00

$$\text{EC } \$1.00 = \frac{\text{US } \$1.00}{2.70}$$

$$\text{EC } \$259.20 = \frac{\text{US } \$1.00}{2.70} \times 259.20$$

= US $96.00

∴ TT $648 = US $96.00

Total: 11 marks

2. (a) Factorise completely:

(i) $2x^3y + 6x^2y^2$ [2]

$2x^3y + 6x^2y^2 = 2x^2y(x + 3y)$

(ii) $9x^2 - 4$ [1]

$9x^2 - 4 = (3x - 2)(3x + 4)$ [difference of two squares]

(iii) $4x^2 + 8xy - xy - 2y^2$ [2]

$4x^2 + 8xy - xy - 2y^2$

$= 4x(x + 2y) - y(x + 2y)$

$= (x + 2y)(4x - y)$

(b) Solve for x:

$\frac{2x-3}{3} + \frac{5-x}{2} = 3$ [3]

$\frac{2x-3}{3} + \frac{5-x}{2} = 3$

$\frac{2(2x-3)+3(5-x)}{6} = 3$

$\frac{4x-6+15-3x}{6} = 3$

$\frac{x+9}{6} = 3$

$x + 9 = 6 \times 3$

$x + 9 = 18$

$x = 18 - 9$

$x = 9$

(c) Solve the simultaneous equations:

$3x - 2y = 10$

$2x + 5y = 13$ [4]

$3x - 2y = 10$ → Equation 1

$2x + 5y = 13$ → Equation 2

Multiplying Equation 1 by 2 gives:

$6x - 4y = 20$ → Equation 3

Multiplying Equation 2 by 3 gives:

$6x + 15y = 39$ → Equation 4

So, we have,

$6x - 4y = 20$ → Equation 3

$6x + 15y = 39$ → Equation 4

Equation 3 − Equation 4 gives,

$-19y = -19$

$y = \frac{-19}{19}$

$y = 1$

Now,

Substituting $y = 1$ into Equation 1 gives,

$3x - 2(1) = 10$

$3x - 2 = 10$

$3x = 10 + 2$

$3x = 12$

$x = \frac{12}{3}$

$x = 4$

$\therefore x = 4$ and $y = 1$.

Total: 12 marks

3. (a) In a survey of 36 students, it was found that

30 play tennis,

x play volleyball ONLY,

$9x$ play BOTH tennis and volleyball,

4 play neither tennis nor volleyball.

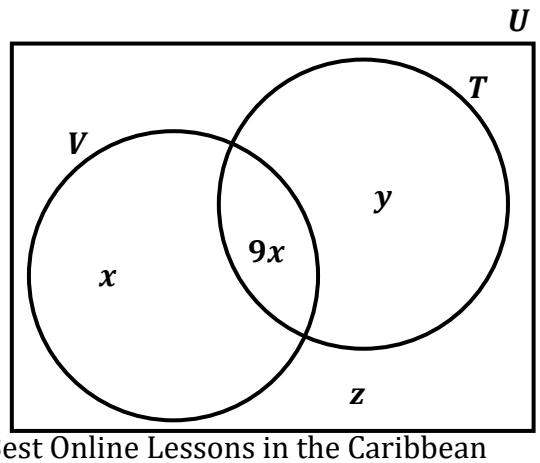
(i) Given that:

$U = \{\text{students in the survey}\}$

$V = \{\text{students who play Volleyball}\}$

$T = \{\text{students who play Tennis}\}$

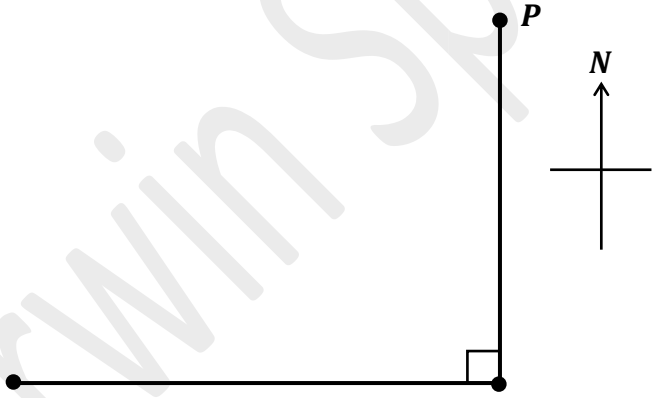
Copy and complete the Venn diagram below to show the number of students in the subsets marked y and z. [2]



(ii) (a) Write an **expression** in x to represent the TOTAL number of students in the survey. [1]

(b) Write an **equation** in x to represent the total number of students in the survey and hence solve for x. [2]

(b) The diagram below, not drawn to scale, shows the journey of a ship which started at port P, sailed 15 km due south to port Q, and then a further 20 km due west to port R.



(i) Copy the diagram and label it to show the points Q and R, and the distances 20 km and 15 km. [2]

(ii) Calculate PR, the **shortest** distance of the ship from the port where the journey started. [2]

Using Pythagoras' Theorem,

$PR^2 = PQ^2 + RQ^2$

$PR^2 = (15)^2 + (20)^2$

$PR^2 = 225 + 400$

$PR^2 = 625$

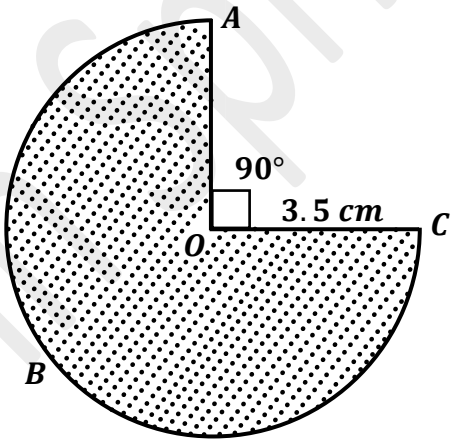
$PR = \sqrt{625}$

$PR = 25\ km$

(iii) Calculate the measure of angle QPR, giving your answer to the nearest degree. [3]

Total: 12 marks

4. The diagram below, **not drawn to scale**, shows the cross section of a prism in the shape of a sector of a circle, centre O, and radius 3.5 cm. The angle at the centre is 270°.



Use $\pi = \frac{22}{7}$

(a) Calculate

(i) the length of the arc ABC [2]

Length of the arc $ABC = \frac{270°}{360°} \times 2\pi r$

$= \frac{3}{4} \times 2 \times \frac{22}{7} \times 3.5$

$= 16.5\ cm$

(ii) the perimeter of the sector $OABC$ [2]

Perimeter of the sector $OABC$

$=$ Length of AO $+$ Arc length ABC $+$ Length of radius CO

$= 16.5 + 3.5 + 3.5$

$= 23.5\ cm$

(iii) the area of the sector $OABC$ [2]

Area of the sector $OABC = \frac{270°}{360°} \times \pi r^2$

$= \frac{3}{4} \times \frac{22}{7} \times (3.5)^2$

$= 28.875\ cm^2$

$\therefore$ The area of the sector $OABC$ is $28.875\ cm^2$.

(b) The prism is 20 cm long and is a solid made of tin. Calculate

(i) the volume of the prism [2]

Volume of the prism $=$ Cross-sectional Area $\times$ Height

$= 28.875 \times 20$

$= 577.5\ cm^3$

$\therefore$ The volume of the prism is $577.5\ cm^3$.

(ii) the mass of the prism, **to the nearest kg**, given that 1 cm^3 of tin has a mass of 7.3 kg. [2]

$1\ cm^3$ of tin weighs $= 7.3\ g$

$577.5\ cm^3$ of tin weighs $= 7.3 \times 577.5$

$= 4\ 215.75\ g$

Now,

$1000\ g = 1\ kg$

$1\ g = \frac{1}{1000}\ kg$

$4\ 215.75\ g = \frac{1}{1000} \times 4\ 215.75$

$= 4.21575\ kg$

$= 4.2\ kg$ (to the nearest kg)

∴ The mass of the prism is 4.2 kg.

Total: 10 marks

5. (a) (i) Using a ruler, a pencil and a pair of compasses, construct triangle PQR with $PQ = 8\ cm$, $\angle PQR = 60°$ and $\angle QPR = 45°$. [4]

(ii) Measure and state the length of RQ. [1]

(b) The line l passes through the points $S(6, 6)$ and $T(0, -2)$.

Determine

(i) the gradient of the line, l [2]

(ii) the equation of the line, l [2]

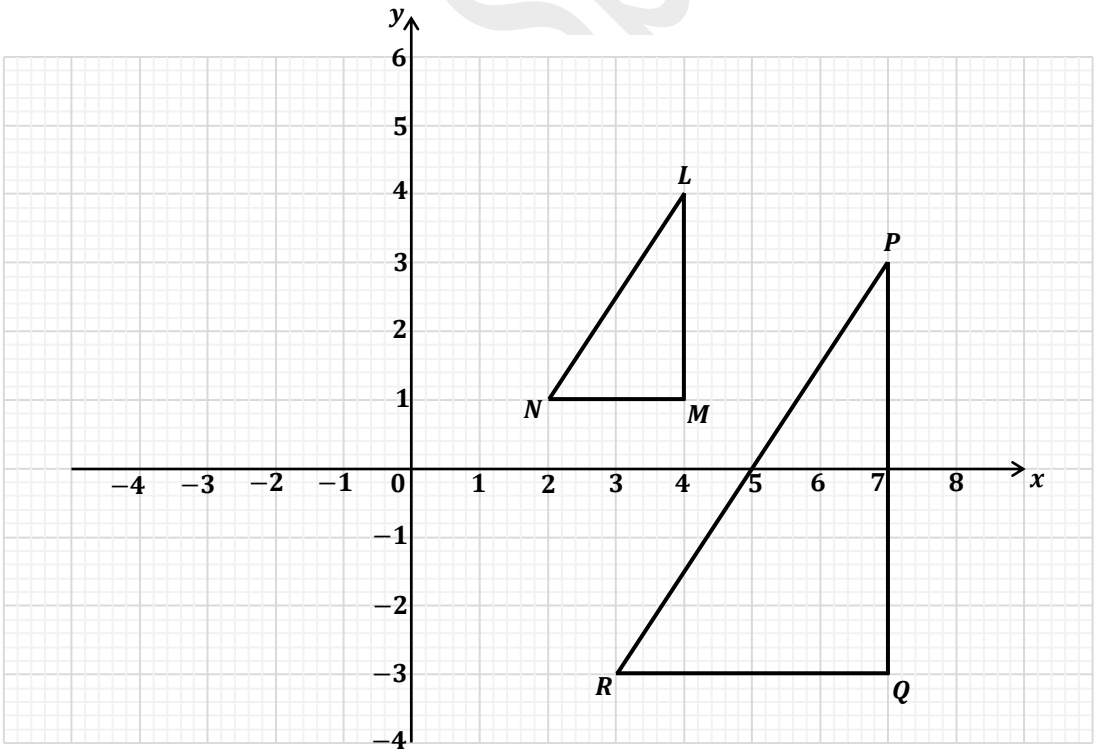
(iii) the midpoint of the line segment, TS [1]

(iv) the length of the line segment, TS. [2]

Total: 12 marks

6. **An answer sheet is provided for this question.**

The graph below shows triangle LMN and its image PQR after an enlargement.



On the answer sheet provided,

(a) Locate the centre of enlargement, showing your method clearly. [2]

(b) State the scale factor and the coordinates of the centre of the enlargement. [2]

(c) Determine the value of $\frac{Area\ of\ PQR}{Area\ of\ LMN}$. [2]

(d) Draw and label triangle ABC with coordinates $(-4, 4)$, $(-1, 4)$ and $(-1, 2)$ respectively. [2]

(e) Describe fully the single transformation which maps triangle LMN onto triangle ABC. [3]

Total: 11 marks

7. The table below shows the ages, to the nearest year, of the persons who visited the clinic during a particular week.

Age (yrs)	Number of persons	Cumulative Frequency
$40 - 49$	4	4
$50 - 59$	11	15
$60 - 69$	20	_____
$70 - 79$	12	_____
$80 - 89$	3	50

(a) Copy and complete the table to show the cumulative frequency. [2]

(b) **Using a scale of 2 cm to represent 10 years on the x-axis and 1 cm to represent 5 persons on the y-axis**, draw the cumulative frequency curve for the data. [5]

(c) Use your graph drawn at (b) above to estimate

(i) the median age for the data [2]

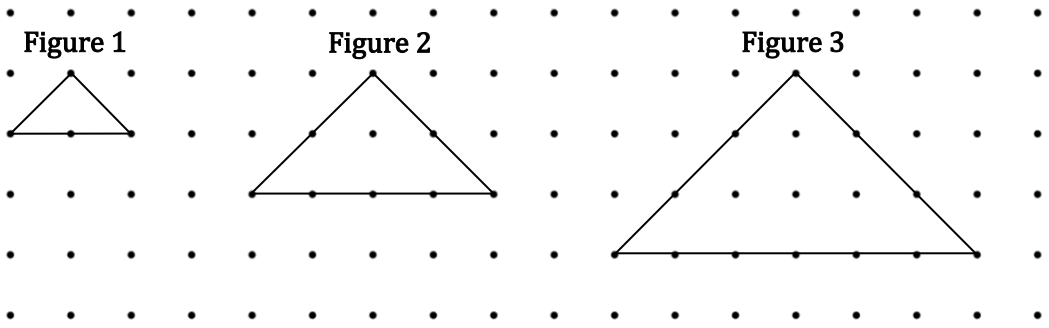
(ii) the probability that a person who visited the clinic was 75 years or younger. [2]

Draw lines on your graph to show how these estimates were obtained.

Total: 11 marks

8. **An answer sheet is provided for this question.**

The diagram below shows the first three figures in a sequence of figures. Each figure is an isosceles triangle made of a rubber band stretched around pins on a geo-board. The pins are arranged in rows and columns, one unit apart.



(a) **On the answer sheet provided**, draw the fourth figure (Figure 4) in the sequence. [2]

(b) Study the pattern in the table below, and on your answer sheet, complete the rows numbered (i), (ii), (iii) and (iv). The breaks in the columns indicate that the rows do not follow one after the other.

	Figure	Area of Triangle	No. of Pins on Base	
	1	1	$2 \times 1 + 1 = 3$	
	2	4	$2 \times 2 + 1 = 5$	
	3	9	$2 \times 3 + 1 = 7$	
(i)	4	_____	_____	[2]
(ii)	_____	100	_____	[2]
(iii)	20	_____	_____	[2]
(iv)	n	_____	_____	[2]

Total: 10 marks

SECTION II

There are THREE questions in this section.

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) (i) Solve the pair of simultaneous equations:

$$y = 8 - x$$

$$2x^2 + xy = -16$$ [5]

(ii) State, giving the reason for your answer, whether the line $y = 8 - x$ is tangent to the curve $2x^2 + xy = -16$. [2]

(b) **An answer sheet is provided for this question.**

A florist makes bouquets of flowers, each consisting of x roses and y orchids.

For each bouquet, she applies the following constraints:

- the number of orchids must be at least half the number of roses
- there must be at least 2 roses
- there must be no more than 12 flowers

(i) Write THREE inequalities for the constraints given. [3]

(ii) **On the answer sheet provided**, shade the region of the graph which represents the solution set for the inequalities in (b)(i). [1]

(iii) State the coordinates of the points which represent the vertices of the region showing the solution set. [1]

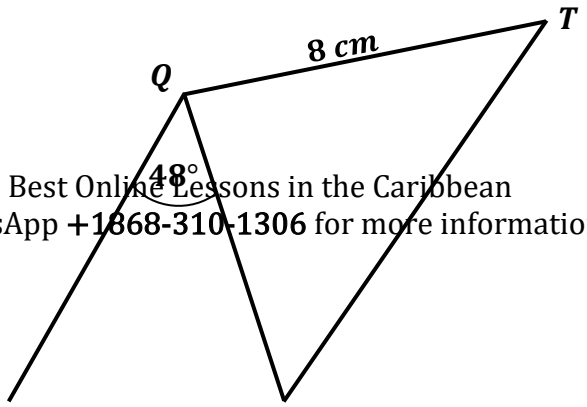
(iv) The florist sells a bouquet of flowers to make a profit of $3 on each rose and $4 on each orchid. Determine the MAXIMUM possible profit on the sale of a bouquet. [3]

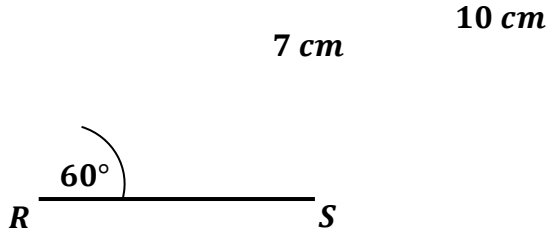
Total: 15 marks

MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) The diagram below, **not drawn to scale**, shows a quadrilateral $QRST$ in which

$QS = 7\ cm, ST = 10\ cm, QT = 8\ cm, \angle SRQ = 60°$ and $\angle RQS = 48°$.



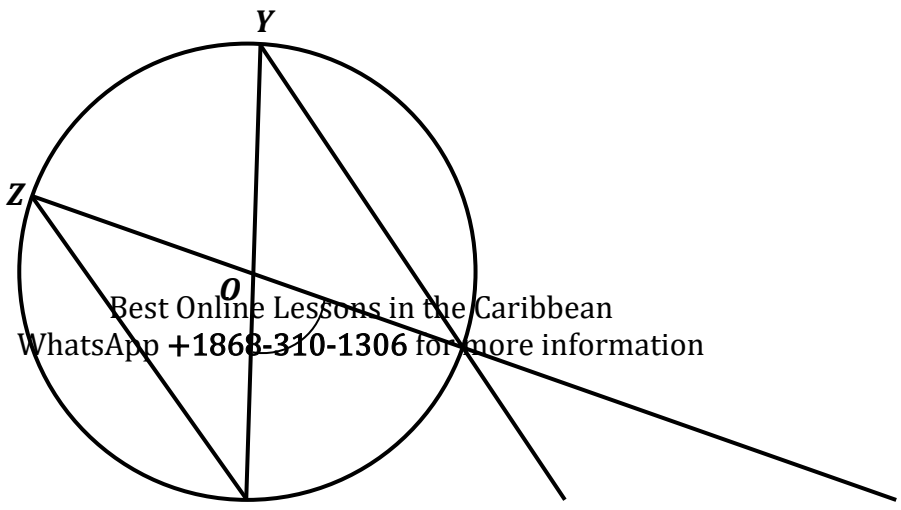


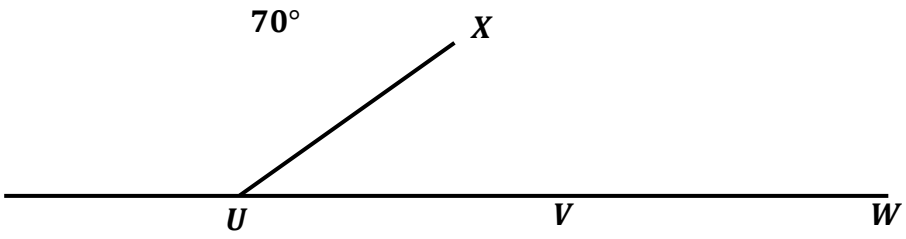
Calculate

(i) the length of RS [3]

(ii) the measure of $\angle QTS$ [3]

(b) The diagram below, **not drawn to scale**, shows a circle, centre O. The line UVW is a tangent to the circle, $ZOXW$ is a straight line and angle $UOX = 70°$.





(i) Calculate, showing working where necessary, the measure of angle

(a) OUZ [2]

(b) UVY [3]

(c) UWO [2]

(ii) Name the triangle in the diagram which is congruent to triangle

(a) ZOU [1]

(b) YXU [1]

Total: 15 marks

VECTORS AND MATRICES

11. (a) The points A, B and C have position vectors $\overrightarrow{OA} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 12 \\ -2 \end{pmatrix}$ respectively.

(i) Express in the form $\begin{pmatrix} x \\ y \end{pmatrix}$ the vector

(a) $\overrightarrow{BA}$ [2]

(b) $\overrightarrow{BC}$ [2]

(ii) State ONE geometrical relationship between BA and BC. [1]

(iii) Draw a sketch to show the relative positions of A, B and C. [2]

(b)(i) Calculate the values of a and b such that $\begin{pmatrix} a & -4 \\ 1 & b \end{pmatrix}\begin{pmatrix} 2 & -4 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$. [3]

(ii) Hence, or otherwise, write down the inverse of $\begin{pmatrix} 2 & -4 \\ 1 & -3 \end{pmatrix}$. [2]

(iii) Use the inverse of $\begin{pmatrix} 2 & -4 \\ 1 & -3 \end{pmatrix}$ to solve for x and y in the matrix equation

$$\begin{pmatrix} 2 & -4 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 7 \end{pmatrix}.$$ [3]

Total: 15 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.