

CSEC Mathematics
January 2017 – Paper 2
Solutions

SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the EXACT value of:

$$(i) \quad \frac{3\frac{1}{2} \times 1\frac{2}{3}}{4\frac{1}{5}} \quad [2]$$

$$\begin{aligned} \text{Numerator} &= 3\frac{1}{2} \times 1\frac{2}{3} \\ &= \frac{7}{2} \times \frac{5}{3} \\ &= \frac{35}{6} \end{aligned}$$

$$\begin{aligned} \text{Denominator} &= 4\frac{1}{5} \\ &= \frac{21}{5} \end{aligned}$$

$$\begin{aligned} \text{Numerator} \div \text{Denominator} &= \frac{35}{6} \div \frac{21}{5} \\ &= \frac{35}{6} \times \frac{5}{21} \\ &= \frac{25}{18} \end{aligned}$$

$$\therefore \frac{3\frac{1}{2} \times 1\frac{2}{3}}{4\frac{1}{5}} = \frac{25}{18}$$

(ii) $5.47 - \sqrt{\frac{0.1014}{1.5}}$ [2]

Using the calculator,

$$5.47 - \sqrt{\frac{0.1014}{1.5}} = 5.21 \quad (\text{in exact form})$$

- (b) The table below shows the number of tickets sold for a bus tour. Some items in the table are missing.

Tickets Sold for Bus Tour			
Category	Number of Tickets Sold	Cost per Ticket in \$	Total Cost in \$
Juvenile	5	P	130.50
Youth	14	44.35	Q
Adult	R		2483.60

- (i) Calculate the value of P. [1]

$$5 \text{ Juvenile tickets cost} = \$130.50$$

$$1 \text{ Juvenile ticket cost} = \frac{\$130.50}{5}$$

$$= \$26.10$$

$$\therefore P = 26.10$$

- (ii) Calculate the value of Q . [1]

$$1 \text{ Youth ticket cost} = \$44.35$$

$$\begin{aligned} 14 \text{ Youth tickets cost} &= \$44.35 \times 14 \\ &= \$620.90 \end{aligned}$$

$$\therefore Q = 620.90$$

- (iii) An adult ticket is TWICE the cost of a youth ticket.
Calculate the value of R . [2]

An adult ticket costs twice as much as the cost of a Youth ticket.

$$\begin{aligned} \text{Hence, the cost of an adult ticket} &= \$44.35 \times 2 \\ &= \$88.70 \end{aligned}$$

The total cost of R adult tickets is \$2483.60.

$$\begin{aligned} \text{Number of adult tickets sold, } R &= \frac{\$2483.60}{\$88.70} \\ &= 28 \end{aligned}$$

$$\therefore R = 28$$

- (iv) The bus company pays taxes of 15% on each ticket sold.

Calculate the taxes paid by the bus company.

[3]

$$\begin{aligned}\text{Total cost} &= \$130.50 + \$620.90 + \$2483.60 \\ &= \$3235.00\end{aligned}$$

The bus company pays taxes of 15% on each ticket sold.

Taxes paid = 15% of \$3235

$$\begin{aligned}&= \frac{15}{100} \times \$3235 \\ &= \$485.25\end{aligned}$$

∴ The bus company paid \$485.25 in taxes.

Total: 11 marks

2. (a) Write as a single fraction:

$$\frac{2x+3}{3} + \frac{x-4}{4} \quad [2]$$

$$\begin{aligned} & \frac{2x+3}{3} + \frac{x-4}{4} \\ = & \frac{4(2x+3)+3(x-4)}{12} \\ = & \frac{8x+12+3x-12}{12} \\ = & \frac{11x}{12} \end{aligned}$$

(b) Write the following statement as an algebraic expression.

The sum of a number and its multiplicative inverse is five times the number.

[2]

Statement: The sum of a number and its multiplicative inverse is five times the number.

Let the number be x .

Its multiplicative inverse is $\frac{1}{x}$.

Expression: $x + \frac{1}{x} = 5x$

(c) Factorize completely:

(i) $x^2 - 36$ [2]

$$x^2 - 36 = (x - 6)(x + 6) \quad \text{[difference of two squares]}$$

(ii) $2x^2 + 5x - 12$ [2]

$$\begin{aligned} &2x^2 + 5x - 12 \\ &= 2x^2 + 8x - 3x - 12 \\ &= 2x(x + 4) - 3(x + 4) \\ &= (2x - 3)(x + 4) \end{aligned}$$

(d) The formula for the volume of a cylinder is given as $V = \pi r^2 h$.

Make r the subject of the formula. [2]

$$V = \pi r^2 h$$

$$\pi r^2 h = V$$

$$r^2 = \frac{V}{\pi h}$$

$$r = \sqrt{\frac{V}{\pi h}}$$

$$\therefore r = \sqrt{\frac{v}{\pi h}}$$

(e) Given that $x^2 + ax + b = (x + 2)^2 - 3$, work out the values of a and b . [2]

$$\begin{aligned} (x + 2)^2 - 3 &= (x + 2)(x + 2) - 3 \\ &= x^2 + 4x + 4 - 3 \\ &= x^2 + 4x + 1 \end{aligned}$$

So, we have,

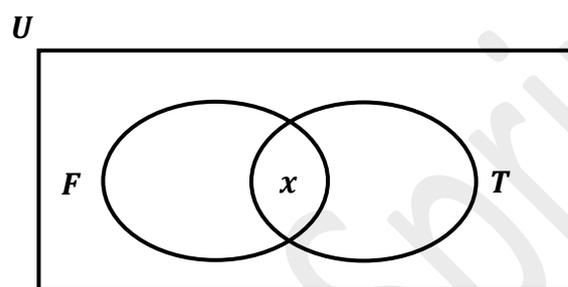
$$x^2 + ax + b = x^2 + 4x + 1$$

Comparing and equating coefficients gives:

$$a = 4 \quad \text{and} \quad b = 1$$

Total: 12 marks

3. (a) The incomplete Venn diagram below shows the number of students in a class of 28 who play football and tennis.



$U = \{\text{all students in the class}\}$

$F = \{\text{students who play football}\}$

$T = \{\text{students who play tennis}\}$

Additional information about the class is that

12 students play tennis

15 students play football

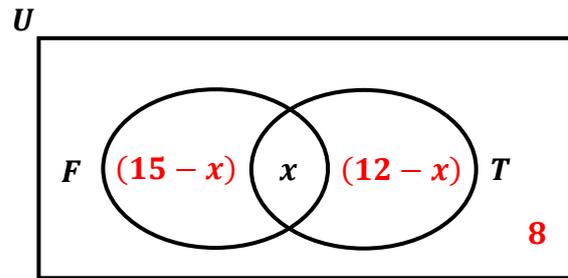
8 students play neither football nor tennis

x students play BOTH football and tennis.

- (i) Complete the Venn diagram above to represent the information, showing the number of students in EACH subset. [3]

The complete Venn diagram is shown below:

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- (ii) Calculate the value of x . [2]

The number of students in the class is 28.

Hence,

$$(15 - x) + x + (12 - x) + 8 = 28$$

$$35 - x = 28$$

$$x = 35 - 28$$

$$x = 7$$

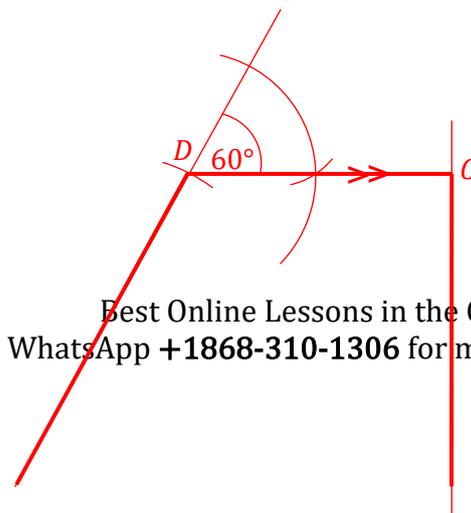
- (b) Using a ruler, a pencil and a pair of compasses, construct the trapezium

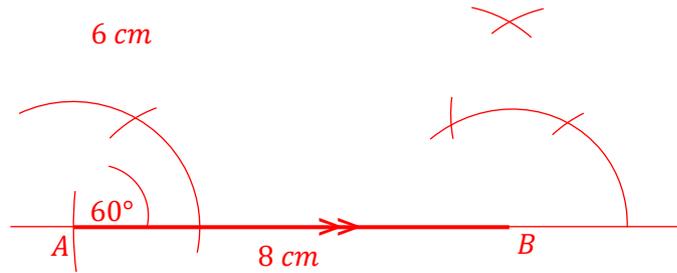
$ABCD$ with $AB = 8$ cm, $\hat{B}AD = 60^\circ$, $AD = 6$ cm, $\hat{A}BC = 90^\circ$ and AB parallel to CD .

(Credit will be given for clearly drawn construction lines.)

[7]

The construction of the trapezium $ABCD$ is given below





Total: 12 marks

4. (a) Given that $f(x) = 4x - 7$ and $g(x) = \frac{3x+1}{2}$, determine the values of:

(i) $g(0) + g(5)$ [2]

$$g(x) = \frac{3x+1}{2}$$

Now,

$$g(0) + g(5) = \frac{3(0)+1}{2} + \frac{3(5)+1}{2}$$

$$= \frac{0+1}{2} + \frac{15+1}{2}$$

$$= \frac{1}{2} + \frac{16}{2}$$

$$= \frac{1}{2} + 8$$

$$= 8\frac{1}{2}$$

(ii) $fg(5)$ [2]

$$g(x) = \frac{3x+1}{2}$$

$$g(5) = \frac{3(5)+1}{2}$$

$$= \frac{15+1}{2}$$

$$= \frac{16}{2}$$

$$= 8$$

$$f(x) = 4x - 7$$

Hence,

$$fg(5) = f[g(5)]$$

$$= f(8)$$

$$= 4(8) - 7$$

$$= 32 - 7$$

$$= 25$$

$$\therefore fg(5) = 25$$

(iii) $f^{-1}(1)$

[2]

$$f(x) = 4x - 7$$

Let $y = f(x)$.

$$y = 4x - 7$$

Interchanging variables x and y :

$$x = 4y - 7$$

Making y the subject of the formula:

$$x + 7 = 4y$$

$$\frac{x+7}{4} = y$$

$$\text{Hence, } f^{-1}(x) = \frac{x+7}{4}$$

Now,

$$f^{-1}(1) = \frac{1+7}{4}$$

$$= \frac{8}{4}$$

$$= 2$$

$$\therefore f^{-1}(1) = 2$$

(b) $P(6, -1)$ and $Q(2, 7)$ are the end point of a line segment PQ . Determine

(i) the gradient of PQ

[2]

Points are $P(6, -1)$ and $Q(2, 7)$

$$\text{Gradient of } PQ = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7 - (-1)}{2 - 6}$$

$$= \frac{8}{-4}$$

$$= -2$$

(ii) the coordinates of the midpoint of PQ [2]

Points are $P(6, -1)$ and $Q(2, 7)$

$$\text{Midpoint of } PQ = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\text{Midpoint of } PQ = \left(\frac{6+2}{2}, \frac{-1+7}{2} \right)$$

$$\text{Midpoint of } PQ = \left(\frac{8}{2}, \frac{6}{2} \right)$$

$$\text{Midpoint of } PQ = (4, 3)$$

(iii) the equation of the perpendicular bisector of PQ . [2]

$$\text{Gradient of } PQ = -2$$

$$\text{Gradient of perpendicular bisector of } PQ = \frac{-1}{-2}$$

$$\text{Gradient of perpendicular bisector of } PQ = \frac{1}{2}$$

Substituting $m = \frac{1}{2}$ and point $(4, 3)$ into $y - y_1 = m(x - x_1)$ gives:

$$y - 3 = \frac{1}{2}(x - 4)$$

$$y - 3 = \frac{1}{2}x - 2$$

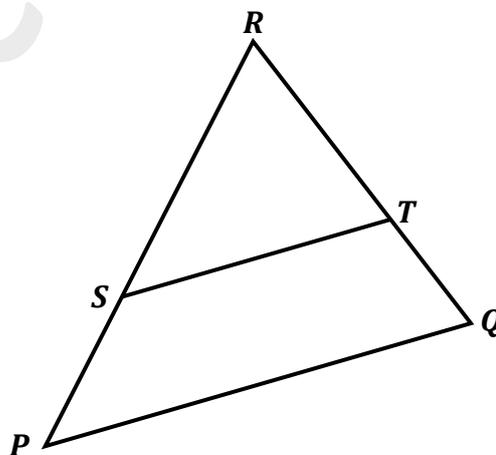
$$y = \frac{1}{2}x - 2 + 3$$

$$y = \frac{1}{2}x + 1$$

∴ The equation of the perpendicular bisector of PQ is: $y = \frac{1}{2}x + 1$

Total: 12 marks

5. (a)



Triangles PQR and STR are similar triangles.

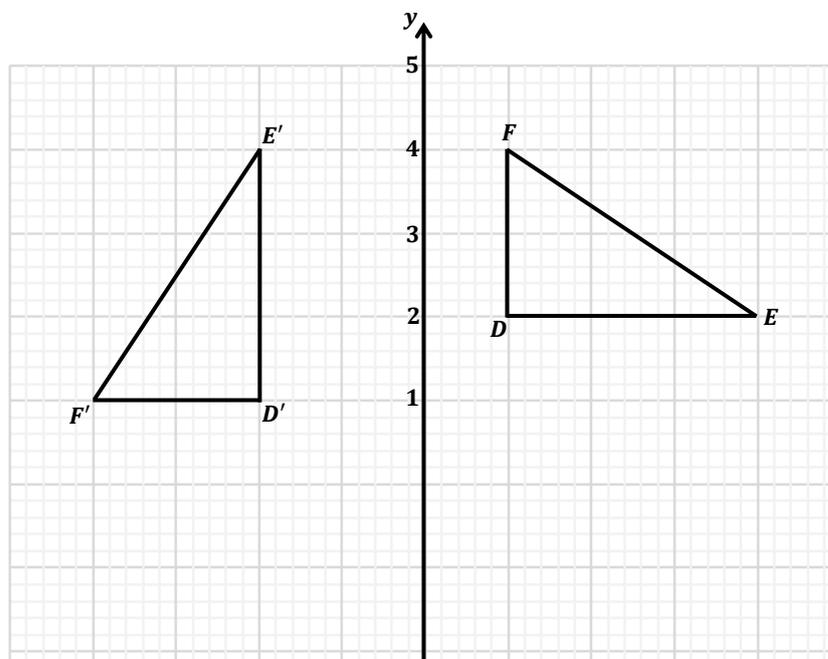
- (i) Complete the following statement:

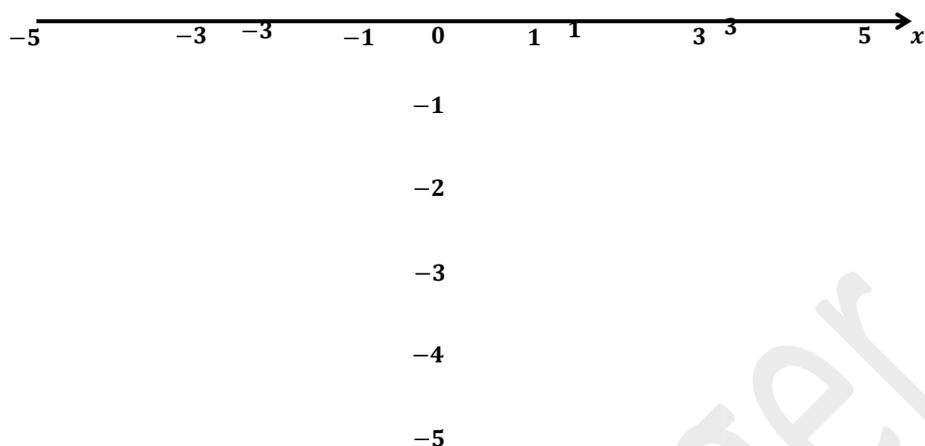
In the diagram above, the corresponding angles of $\triangle PQR$ and $\triangle STR$ are and the of their corresponding sides are the same. [2]

In the diagram above, not drawn to scale, $RS = 15\text{ cm}$, $SP = 9\text{ cm}$ and $ST = 12\text{ cm}$.

- (ii) Determine the length of PQ . [3]

- (b) The graph below shows triangle DEF and its image $D'E'F'$ after a transformation.





- (i) State the coordinates of the point E . [1]
- (ii) Describe fully the transformation that maps triangle DEF to its image, $D'E'F'$. [3]
- (iii) On the grid above, draw triangle $D''E''F''$, the reflection of triangle $D'E'F'$ in the x -axis. [2]

Total: 11 marks

6. (a) The scale on a map is 1: 25 000.

- (i) Anderlin and Jersey are 31.8 cm apart on the map.

Determine, in km, the actual distance between Anderlin and

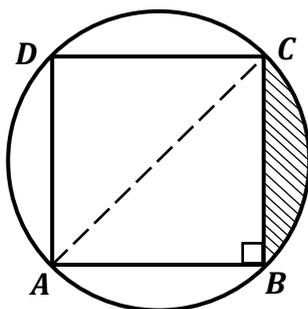
Jersey.

[2]

- (ii) The actual distance between Clifton and James Town is 2.75 km.

How many units apart are they on the map?

- (b) The diagram below shows a square $ABCD$ drawn inside a circle. The vertices of the square lie on the circumference of the circle. The length of a side of the square is 11 cm.



- (i) Show that the diameter of the circle is $11\sqrt{2}$ cm. [2]

Write your answers correct to 2 decimal places.

Calculate

- (ii) the area of the circle [2]
 (iii) the area of the square [1]
 the area of the shaded section [2]

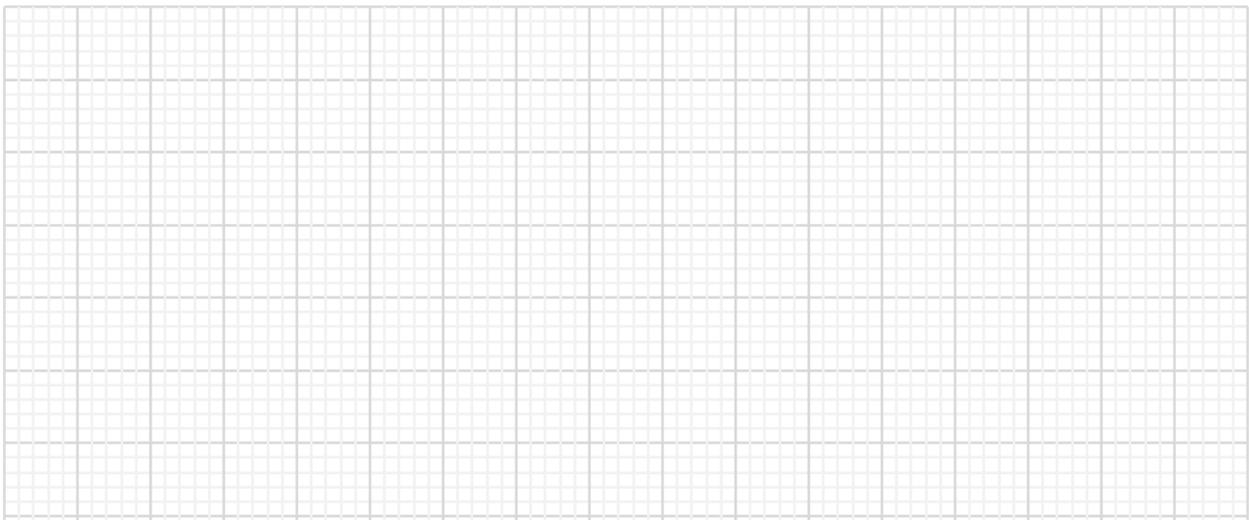
Total: 11 marks

7. The table below shows the number of bananas, to the nearest tonne, produced annually on a farm over a period of 6 years.

Year	2010	2011	2012	2013	2014	2015
Production (tonnes)	150	275	100	40	125	210

- (a) **On the graph paper provided on page 21**, draw a bar chart to represent the data given in the table above using a scale of 1 cm to represent 1 year on the x -axis and 1 cm to represent on the y -axis. [4]
- (b) Determine the range of the number of bananas produced between 2010 and 2015. [2]
- (c) (i) During which year was there the greatest production of bananas? [1]
- (ii) How is this information shown on the bar chart? [1]
- (d) (i) Between which two consecutive years was there the greatest **change** in the production of bananas? [1]
- (ii) How is this information shown on the bar chart? [1]
- (e) Give ONE reason why the bar chart is unsuitable for predicting the number of bananas produced in 2016. [1]

Total: 11 marks



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8. A sequence of figures is made up of unit squares with unit sides. The first three figures in the sequence are shown below.

Figure 1



Figure 2



Figure 3

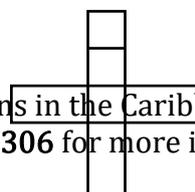
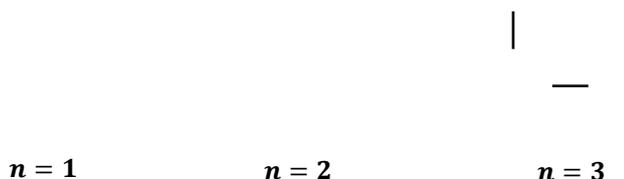


Figure 4

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(a) Draw Figure 4 of the sequence in the space provided above. [2]

(b) Study the pattern of numbers in each row of the table below. Each row relates to one of the figures in the sequence. Some rows have not been included in the table.

Complete the rows numbered (i), (ii), (iii) and (iv).

Figure	Number of Unit Squares	Perimeter of Figure
1	1	4
2	5	12
3	9	20
(i) 4		
(ii)	45	
(iii) 30		
(iv) n		

Total: 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

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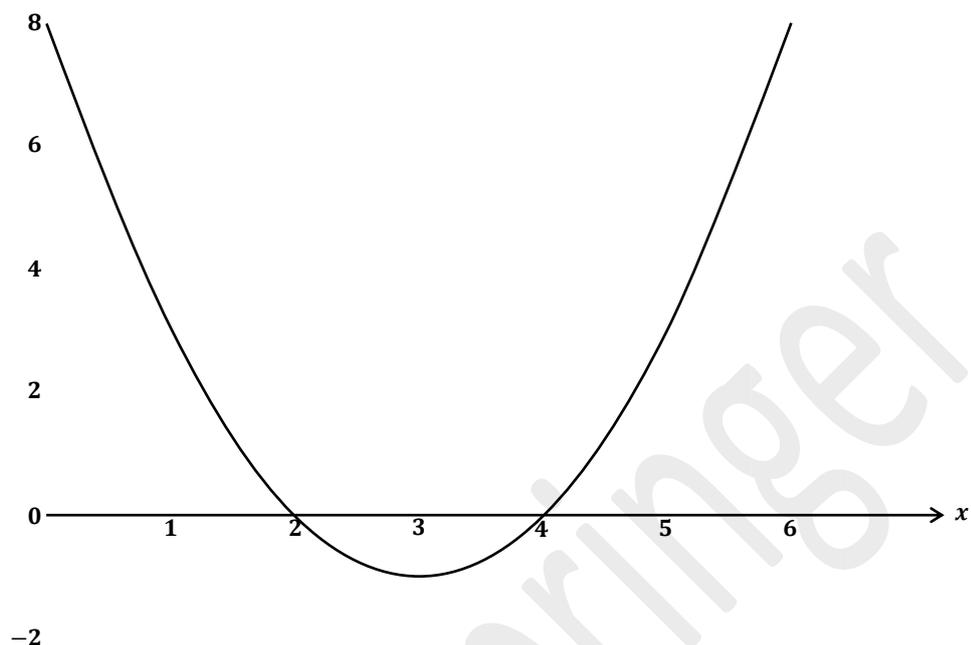
9. (a) The table below shows pairs of values of x and y , where y is inversely proportional to x .

x	3	4	a	20
y	2	1.5	1.2	b

- (i) Express y in terms of x and a constant k . [1]
- (ii) Calculate the value of the constant k . [1]
- (iii) Determine the values of a and b . [2]

- (c) The diagram below shows the graph of the function $f(x) = x^2 - 6x + 8$ for values of x from 0 to 6.



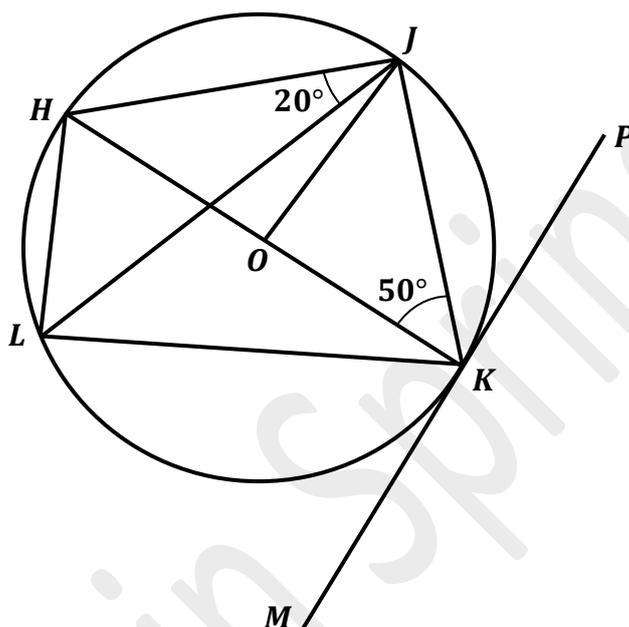


- (i) Use the graph to solve the equation $x^2 - 6x + 8 = 0$. [2]
- (ii) Write down the coordinates of the minimum point in the form (x, y) . [1]
- (iii) Write $x^2 - 6x + 8$ in the form $a(x + h)^2 + k$ where a, h and k are constants. [3]
- (iv) On the same axes, draw the graph of the straight line $g(x) = x - 2$. [3]
- (v) Hence, solve the equation $x^2 - 6x + 8 = x - 2$. [2]

Total: 15 marks

MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) The diagram below, **not drawn to scale**, shows a circle with centre O . The vertices H, J, K and L of a quadrilateral lie on the circumference of the circle and PKM is a tangent to the circle at K . The measure of angle $H\hat{J}L = 20^\circ$ and $J\hat{K}H = 50^\circ$.

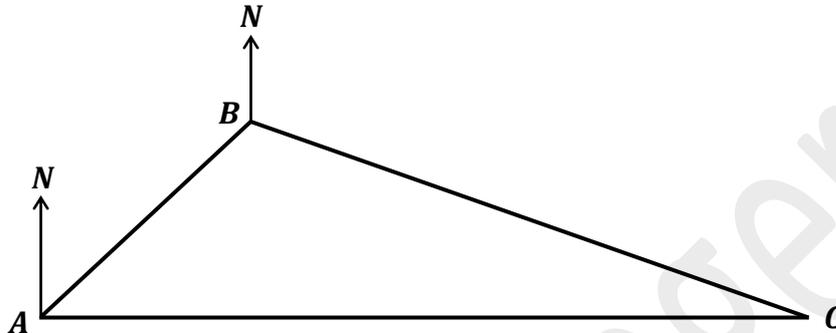


Calculate, **giving reasons for each step of your answer**, the measure of

- | | | |
|-------|-------------|-----|
| (i) | $H\hat{K}L$ | [2] |
| (ii) | $J\hat{O}K$ | [2] |
| (iii) | $J\hat{H}K$ | [2] |

- (b) A ship travels from Akron (A) on a bearing of 030° to Bellville (B), 90 km

away. It then travels to Comptin (C) which is 310 km due east of Akron (A), as shown in the diagram below.



- (i) Indicate on the diagram the bearing of 030° and the distances 90 km and 310 km. [2]
- (ii) Calculate, to the nearest km, the distance between Bellville (B) and Comptin (C). [2]
- (iii) Calculate, to the nearest degree, the measure of $\hat{A}BC$. [2]
- (iv) Determine the bearing of Comptin (C) from Bellville (B). [3]

Total: 15 marks

VECTORS AND MATRICES

11. (a) The matrix $T = \begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix}$ maps the point $P(2, 3)$ onto the point $Q(2, -3)$.

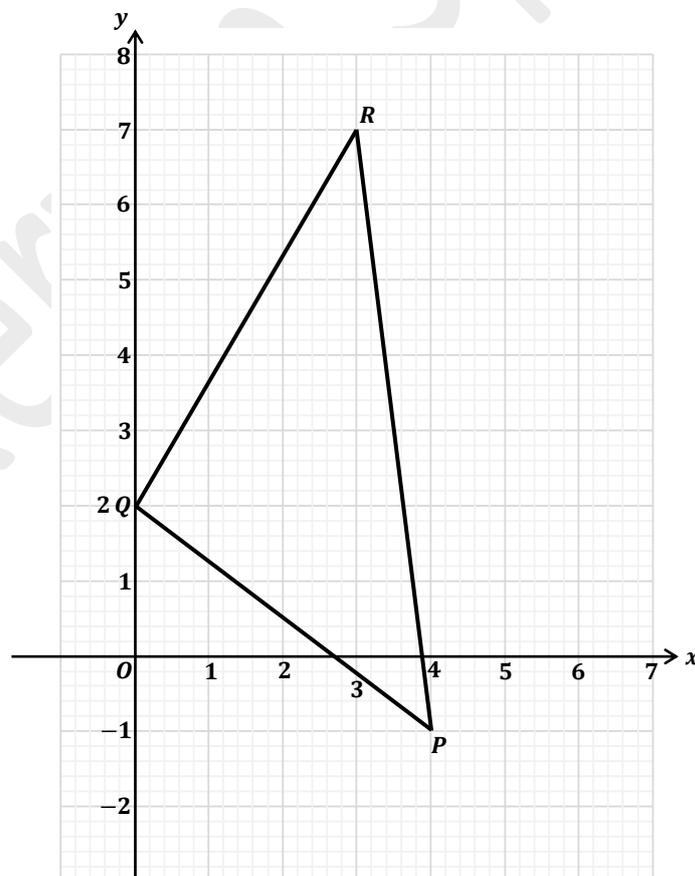
(i) Determine the values of c and d . [2]

(ii) Determine the image of $(-5, 4)$ under the transformation T . [1]

(iii) Describe fully the transformation T . [2]

(iv) Find the matrix that maps the point Q back onto the point P . [2]

(b) The graph below shows three points P , Q and R , relative to the origin, O .



- (i) Write as a column vector in the form $\begin{pmatrix} x \\ y \end{pmatrix}$
- the vector \overrightarrow{OP} [1]
 - the vector \overrightarrow{QR} [2]
- (ii) Determine the magnitude of the vector \overrightarrow{QR} . [2]
- (iii) On the graph provided on page 33, draw the vector $\overrightarrow{OS} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$. Show that $PQRS$ is a parallelogram. [3]

Total: 15 marks

END OF TEST

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IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

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