STUDENT HUB

CSEC Mathematics

June 2017 – Paper 2

Solutions

SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, calculate the EXACT value of

(i) $\left(4\frac{1}{3} - 1\frac{2}{5}\right) \div \frac{4}{15}$ [2]

$$\left(4\frac{1}{3} - 1\frac{2}{5}\right) \div \frac{4}{15} = \left(\frac{13}{3} - \frac{7}{5}\right) \div \frac{4}{15}$$

$$= \left(\frac{(5\times13)-(7\times3)}{15}\right) \div \frac{4}{15}$$

$$= \left(\frac{65-21}{15}\right) \div \frac{4}{15}$$

$$= \frac{44}{15} \div \frac{4}{15}$$

$$= \frac{44}{15} \times \frac{15}{4}$$

$$= 11$$

(ii) $\frac{(3.1-1.15)^2}{0.005}$ [2]

$$\frac{(3.1-1.15)^2}{0.005} = \frac{(1.95)^2}{0.005}$$

$$= \frac{3.8025}{0.005}$$

$$= 760.5$$

(b) A store is promoting a new mobile phone under two plans: Plan A and Plan B.

The plans are advertised as shown in the table below.

	Plan A	**Plan B**
Deposit	\$400	\$600
Monthly instalment	\$65	\$80
Number of months to repay	12	6
Tax on ALL payments	0%	5%

(i) Calculate the TOTAL cost of a phone under Plan A. [2]

Total cost of phone

$= \text{Deposit} + (\text{Monthly instalments} \times \text{number of months}) + \text{Tax}$

$= \$400 + (\$65 \times 12) + \$0$

$= \$400 + \$780$

$= \$1180$

$\therefore$ The total cost of a phone under Plan A is \$1180.

(ii) Determine which of the two plans, A or B, is the better deal.

Justify your answer. [2]

Total cost of phone

$= \text{Deposit} + (\text{Monthly instalments} \times \text{number of months}) + \text{Tax}$

$= \$600 + (\$80 \times 6) + 0.05(600 + (80 \times 6))$

$= \$600 + \$480 + 0.05(600 + 480)$

$= \$1134$

$\therefore$ Plan B is the better deal as it has an overall cost

$(\$1180 - \$1134 = \$46)$ cheaper than that of Plan A.

(c) John's monthly electricity bill is based on the number of kWh of electricity that he consumes for that month. He is charged \$5.10 per kWh of electricity consumed. For the month of March 2016, two meter readings are displayed in the table below.

	Meter Readings (kWh)
Beginning 01 March	0 3 0 1 1
Ending 31 March	0 3 3 0 7

(i) Calculate the TOTAL amount that John pays for electricity consumption for the month of March 2016. [2]

Number of kWh used

= Final reading on 31st March − Initial reading on 1st March

= 0 3 3 0 7 − 0 3 0 1 1

= 296 kWh

If 1 kWh costs = \$5.10,

Then, 296 kWh cost = $\$5.10 \times 296$

= \$1509.60

∴ John pays $1509.60 for electricity consumption for the month of March 2016.

(ii) For the next month, April 2016, John pays $2351.10 for electricity consumption.

Determine his meter reading at the end of April 2016. [2]

$$\text{Number of kWh used on April} = \frac{\$2351.10}{\$5.10}$$

$$= 461 \text{ kWh}$$

At the end of April, the meter reading should read $= 0\ 3\ 3\ 0\ 7 + 4\ 6\ 1$

$= 0\ 3\ 7\ 6\ 8$

Total: 12 marks

2. (a) Factorize the following expressions completely.

(i) $6y^2 - 18xy$ [2]

$6y^2 - 18xy = 6y(y - 3x)$

(ii) $4m^2 - 1$ [1]

$4m^2 - 1 = (2m + 1)(2m - 1)$ [difference of two squares]

(iii) $2t^2 - 3t - 2$ [2]

$2t^2 - 3t - 2$

$= 2t^2 - 4t + t - 2$

$= 2t(t - 2) + 1(t - 2)$

$= (2t + 1)(t - 2)$

(b) Write as a single fraction and simplify

$\frac{5p+2}{3} - \frac{3p-1}{4}$ [2]

$\frac{5p+2}{3} - \frac{3p-1}{4}$

$= \frac{4(5p+2)-3(3p-1)}{12}$

$= \frac{20p+8-9p+3}{12}$

$= \frac{20p-9p+8+3}{12}$

$= \frac{11p+11}{12}$

$= \frac{11(p+1)}{12}$

(c) A formula is given as $d = \sqrt{\frac{4h}{5}}$.

(i) Determine the value of d when $h = 29$. Give your answer correct to 3 significant figures. [2]

$d = \sqrt{\frac{4h}{5}}$

When $h = 29$,

$d = \sqrt{\frac{4(29)}{5}}$

$= \sqrt{\frac{116}{5}}$

$= \sqrt{23.2}$

$= 4.82$ (to 3 significant figures)

(ii) Make h the subject of the formula. [2]

$d = \sqrt{\frac{4h}{5}}$

Squaring both sides gives:

$d^2 = \frac{4h}{5}$

$5d^2 = 4h$

$h = \frac{5d^2}{4}$

Total: 11 marks

3. (a) The universal set U is defined as follows:

$U = \{x: x \in \mathbb{N}, 2, x < 12\}$.

The sets M and R are subsets of U such that

$M = \{\text{odd numbers}\}$

$R = \{\text{square numbers}\}$

(i) List the members of the subset M. [1]

The universal set is $U = \{3, 4, 5, 6, 7, 8, 9, 10, 11\}$.

The members of set $M = \{\text{odd numbers}\}$

$= \{3, 5, 7, 9, 11\}$

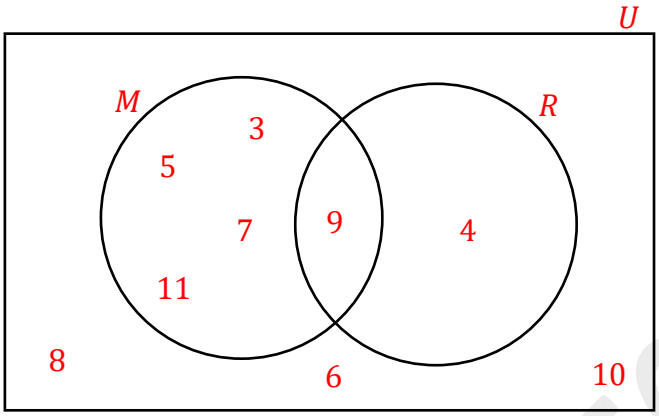
(ii) List the members of the subset R. [1]

The members of set $R = \{\text{square numbers}\}$

$= \{4, 9\}$

(iii) Draw a Venn diagram that represents the relationship among the defined subsets of U. [4]

The Venn diagram is as follows:



(b) Using **a ruler, a pencil and a pair of compasses,**

(i) construct accurately, the square $ABCD$, with sides 6 cm [3]

(ii) construct, as an extension of your drawing in (b)(i), the trapezium $DABQ$ so that $\angle ABQ = 120°$. [2]

[Note: Credit will be given for clearly drawn construction lines.]

(iii) Hence, measure and state the length of BQ. [1]

Total: 12 marks

4. (a) The function f is defined as $f(x) = \frac{1}{3}x - 2$.

(i) Find the value of $f(3) + f(-3)$. [2]

$f(3) = \frac{1}{3}(3) - 2$

$= 1 - 2$

$= -1$

$f(-3) = \frac{1}{3}(-3) - 2$

$= -1 - 2$

$= -3$

$\therefore f(3) + f(-3) = -1 + (-3)$

$= -1 - 3$

$= -4$

(ii) Calculate the value of x for which $f(x) = 5$. [2]

$f(x) = 5$

So, we have,

$\frac{1}{3}x - 2 = 5$

$\frac{1}{3}x = 5 + 2$

$\frac{1}{3}x = 7$

$x = 7 \times 3$

$x = 21$

(iii) Determine the inverse function $f^{-1}(x)$. [2]

$f(x) = \frac{1}{3}x - 5$

Let $y = f(x)$.

$y = \frac{1}{3}x - 5$

Interchanging the variables x and y.

$x = \frac{1}{3}y - 5$

Making y the subject of the formula.

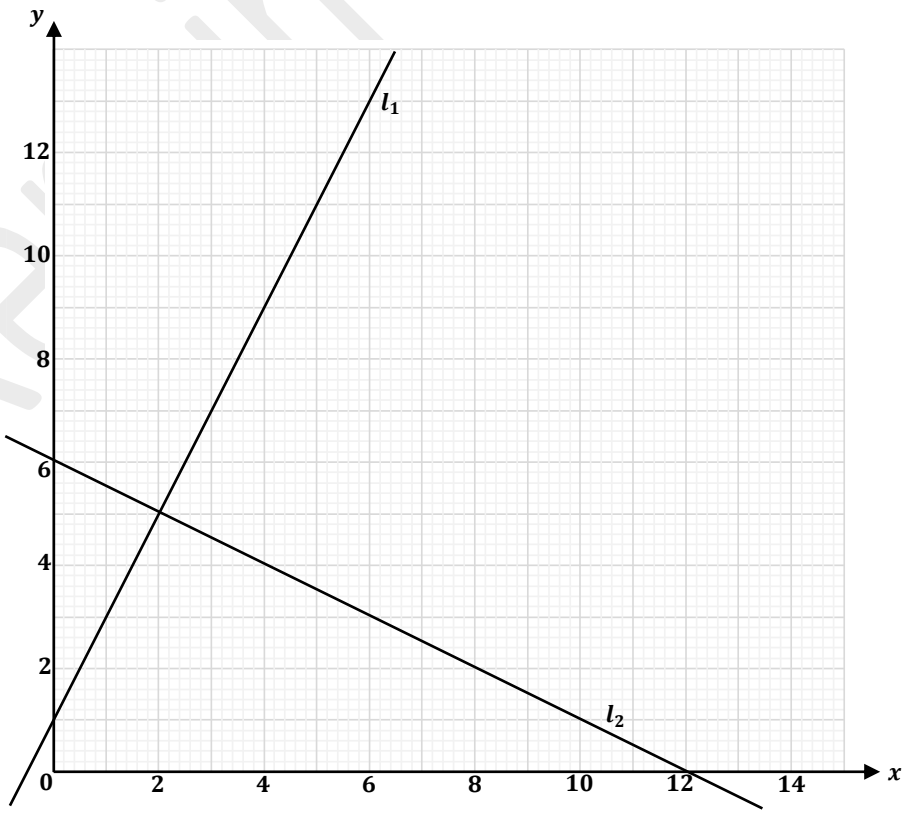
$x + 5 = \frac{1}{3}y$

$y = 3(x + 2)$

$y = 3x + 6$

$\therefore f^{-1}(x) = 3x + 6$

(b) The graph below shows two straight lines, l_1 and l_2. Line l_1 intercepts the y-axis at $(0, 1)$. Line $l_2$intercepts the x and y axes at $(12, 0)$ and $(0, 6)$ respectively.



(i) Calculate the gradient of the lines l_1 and l_2. [2]

For the line l_1: Points are $(0, 1)$ and $(2, 5)$.

Gradient of line $l_1 = \frac{y_2 - y_1}{x_2 - x_1}$

Gradient of line $l_1 = \frac{5-1}{2-0}$

Gradient of line $l_1 = \frac{4}{2}$

Gradient of line $l_1 = 2$

For the line l_2: Points are $(12, 0)$ and $(0, 6)$.

Gradient of line $l_2 = \frac{y_2 - y_1}{x_2 - x_1}$

Gradient of line $l_2 = \frac{6-0}{0-12}$

Gradient of line $l_2 = \frac{6}{-12}$

Gradient of line $l_2 = -\frac{1}{2}$

(ii) Determine the equation of the line l_1. [2]

The general equation of a straight line is of the form $y = mx + c$.

Gradient of line $l_1 = 2$

y-intercept of line $l_1 = 1$

$\therefore$ The equation of the line l_1 is $y = 2x + 1$.

(iii) What is the relationship between l_1 and l_2? Give a reason for your answer. [2]

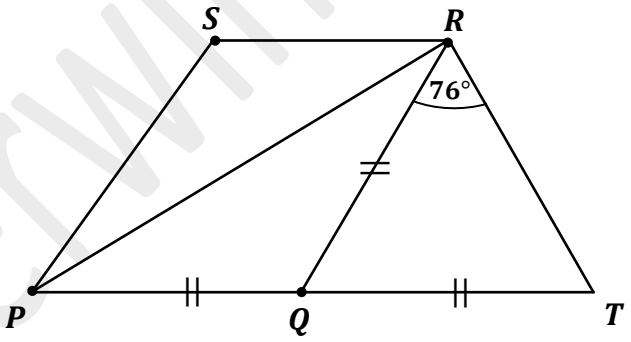
Gradient of line $l_1 = 2$ and Gradient of line $l_2 = -\frac{1}{2}$

The gradient of line l_2 is the negative reciprocal of the gradient of line l_1.

$\therefore$ Line l_1 is perpendicular to line l_2.

Total: 12 marks

5. (a) $PTRS$, **not drawn to scale**, is a quadrilateral. Q is a point on PT such that $QT = QR = QP$ and Angle $QRT = 76°$.



Determine, giving a reason for each step of your answer, the measure of

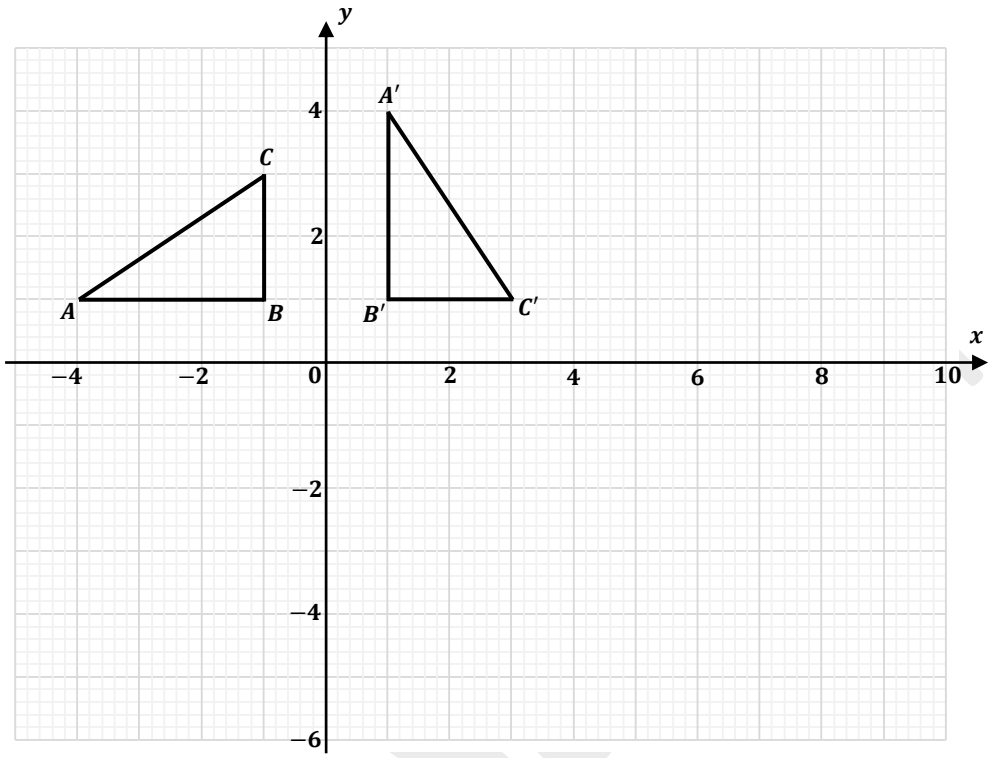
(i) angle RQT [2]

(ii) angle PRT [2]

(iii) angle SPT, given that angle $SRT = 145°$ and angle $PSR = 100°$ [2]

(b) The diagram below shows triangle ABC and its image, $A'B'C'$, under a single

transformation.



(i) Describe **completely** the transformation that maps ΔABC to $\Delta A'B'C'$. [3]

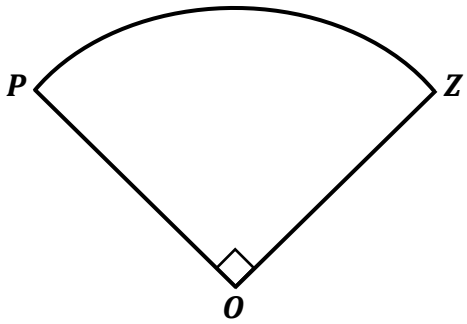
(ii) The translation vector $T = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$ maps $\Delta A'B'C'$ to $\Delta A"B"C"$.

On the diagram above, draw the $\Delta A"B"C"$. [2]

Total: 11 marks

6. (a) **In this problem, take π to be $\frac{22}{7}$.**

The diagram below, **not drawn to scale**, shows a field in the shape of a sector of a circle with centre O and diamter 28 m. Angle POZ is 90°.



Calculate

(i) the area of the field [2]

Radius $= \frac{Diameter}{2}$

$= \frac{28}{2}$

$= 14\ m$

Area of the field $= \frac{\theta}{360°}\pi r^2$

$= \frac{90°}{360°} \times \frac{22}{7} \times (14)^2$

$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$

$= 154\ m^2$

∴ The area of the field is $154\ m^2$.

(ii) the perimeter of the field [3]

Perimeter of the field = Length of arc $PZ + 2r$

$= \left(\frac{\theta}{360°} \times \pi d\right) + 2r$

$= \left(\frac{90°}{360°} \times \frac{22}{7} \times 28\right) + 2(14)$

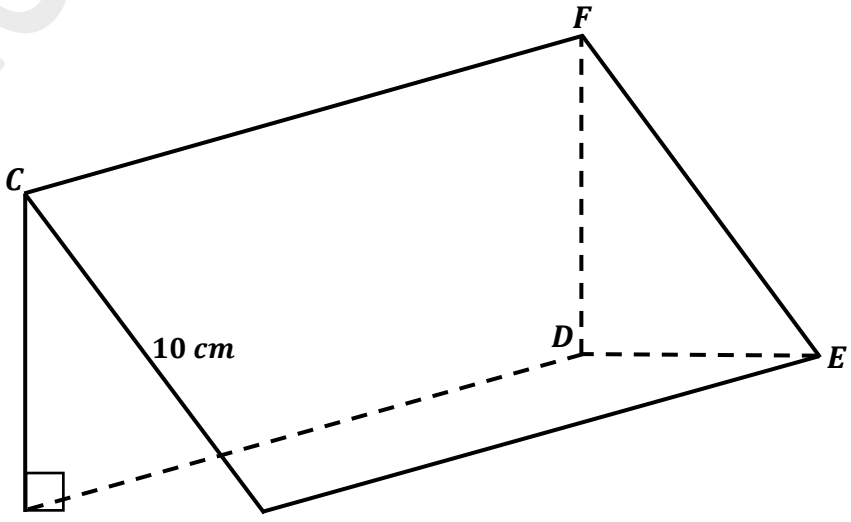
$= 22 + 28$

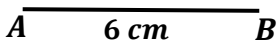
$= 50\ m$

∴ The perimeter of the field is $50\ m$.

(b) The diagram below, **not drawn to scale**, shows a triangular prism $ABCDEF$.

The cross section is the right-angled triangle, ABC, where $AB = 6$ cm and $BC = 10$ cm.





Calculate

(i) the area of the triangle ABC [2]

Using Pythagoras' Theorem,

$BC^2 = AC^2 + AB^2$

$(10)^2 = AC^2 + (6)^2$

$100 = AC^2 + 36$

$AC^2 = 100 - 36$

$AC^2 = 64$

$AC = \sqrt{64}$

$AC = 8\ cm$

Now,

Area of $\Delta ABC = \frac{bh}{2}$

$= \frac{6 \times 8}{2}$

$= \frac{48}{2}$

$= 24\ cm^2$

∴ The area of the triangle ABC is $24\ cm^2$.

(ii) the length of the prism, if the volume is 540 cm^3 [2]

Volume of the prism = Area of cross-section × Length of prism

$$540 = 24 \times \text{Length of prism}$$

$$\text{Length of prism} = \frac{540}{24}$$

$$\text{Length of prism} = 22.5\ cm$$

∴ The length of the prism is 22.5 cm.

(iii) the surface area of the prism [2]

$$\text{Area of } \Delta ABC = \frac{bh}{2}$$

$$= \frac{6 \times 8}{2}$$

$$= \frac{48}{2}$$

$$= 24\ cm^2$$

$$\text{Area of } \Delta DEF = \frac{bh}{2}$$

$$= \frac{6 \times 8}{2}$$

$$= \frac{48}{2}$$

$$= 24\ cm^2$$

$$\text{Area of rectangle } ABED = 6 \times 22.5$$

$$= 135\ cm^2$$

Area of rectangle $ADFE = 8 \times 22.5$

$= 180\ cm^2$

Area of rectangle $BEFC = 10 \times 22.5$

$= 225\ cm^2$

Hence,

Surface area of the prism = Sum of the area of all five sides

$= 24 + 24 + 135 + 180 + 225$

$= 588\ cm^2$

Total: 11 marks

7. The table below shows the speeds, to the nearest $\boldsymbol{kmh^{-1}}$, of 90 vehicles that pass a checkpoint.

Speed (in $\boldsymbol{kmh^{-1}}$)	Frequency	Cumulative Frequency
$0 - 19$	5	5
$20 - 39$	11	16
$40 - 59$	26	
$60 - 79$	37	

$80 - 99$	9	
$100 - 119$	2	

(a) For the class interval 20-39, as written in the table above, complete the following sentences.

(i) The upper class limit is …… [1]

(ii) The class width is …… [1]

(iii) Sixteen vehicles passed a checkpoint at no more than ……. kmh^{-1}. [1]

(b) Complete the table shown above by inserting the missing values for the cumulative frequency column. [2]

(c) On the grid provided **on page 21**, using a scale of 2 cm to represent 20 kmh^{-1} on the x-axis, and 2 cm to represent 10 vehicles on the y-axis, draw the cumulative frequency curve to represent the information in the table. [4]

(d) (i) On your graph, draw reference lines to estimate the speed at which no more than 50% of the vehicles drove as they passed the check point. [1]

(ii) What is the estimated speed? [1]

Total: 11 marks

8. The first four figures in a sequence are shown below. Figure 1 is a single black dot, while each of the others consist of black dots arranged in an equilateral manner.

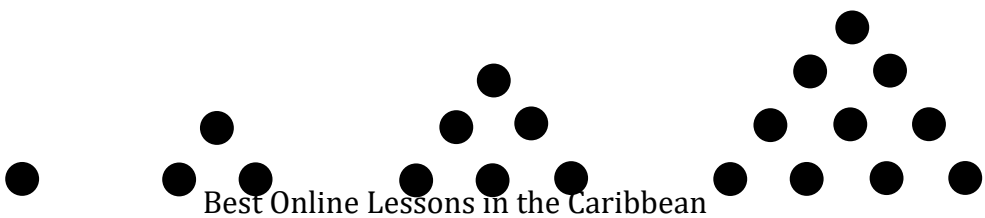


Figure 1 **Figure 2** **Figure 3** **Figure 4**

(a) Draw Figure 5 of the sequence in the space below. [2]

(b) How many dots would be in Figure 6? [1]

The table below refers to the figures and the number of dots in each figure. Study the patterns shown.

Figure, n	**Number of Dots, d, in terms of n**	**Number of Dots Used, d**
1	$\frac{1}{2} \times 1 \times (1 + 1)$	1
2	$\frac{1}{2} \times 2 \times (2 + 1)$	3
3	$\frac{1}{2} \times 3 \times (3 + 1)$	6
⋮		
11		
⋮		
n		

(c) Complete the row which corresponds to Figure 11 in the table above. [2]

(d) Determine which figure in the sequence has 210 dots. [2]

(e) Write a simplified algebraic expression for the number of dots, d, in the Figure n. [1]

(f) Show that there is no diagram that has exactly 1000 dots. [2]

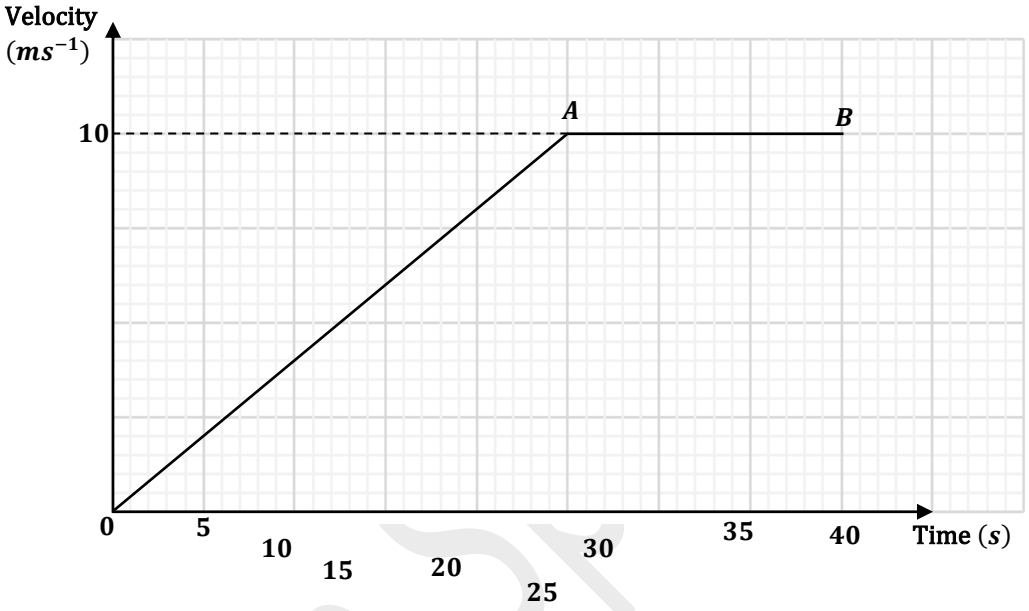
Total: 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) The velocity-time graph below shows the motion of a cyclist over a period of 40 seconds.



(i) Calculate the gradient of

(a) OA [1]

(b) AB [1]

(ii) Complete the following statements.

The cyclist started from rest, where his velocity was …….. ms^{-1}, and steadily increased his velocity by …….. ms^{-1} each second during the first 25 seconds.

During the next 15 seconds, his velocity remained constant, that is, his acceleration was …….. ms^{-1}. [3]

(iii) Determine the average speed of the cyclist over the 40-second

period. [3]

(b) Consider the following pair of simultaneous equations:

$x^2 + 2xy = 5$

$x + y = 3$

(i) WITHOUT solving, show that $(1, 2)$ is a solution for the pair of simultaneous equations. [2]

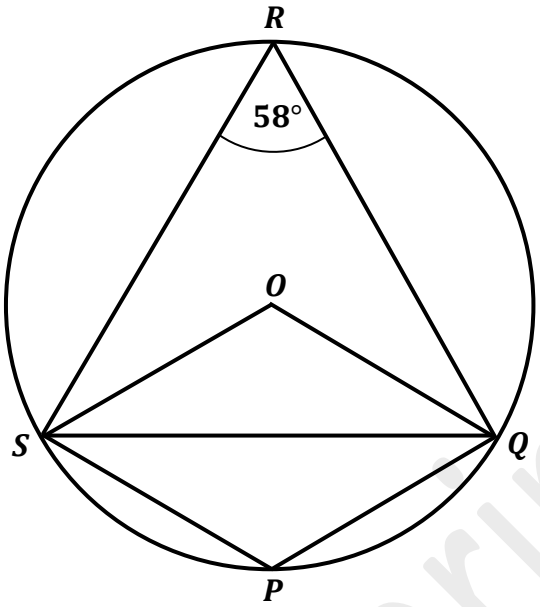
(ii) Solve the pair of simultaneous equations above to determine the **other** solution. [5]

Total: 15 marks

MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) P, Q, R and S are four points on the circumference of the circle shown below.

Angle $QRS = 58°$



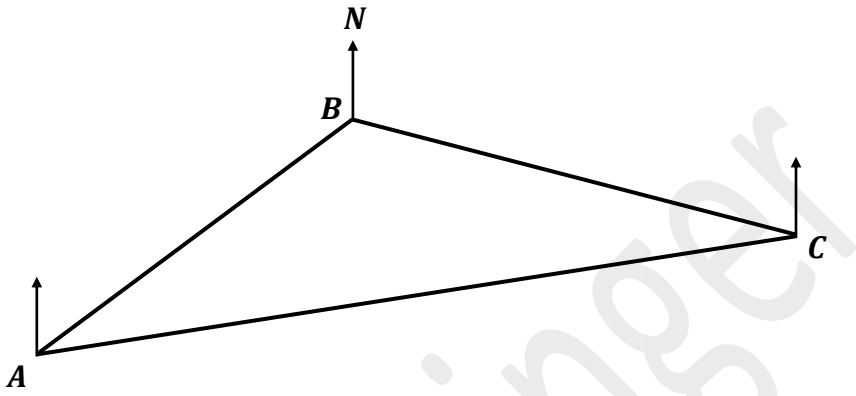
Using the geometrical properties of a circle to give reasons for each step of your answer, determine the measure of

(i) $\angle SPQ$ [2]

(ii) $\angle OQS$ [3]

(b) A ship leaves Port A and sails 52 km on a bearing of 044° to Port B. The ship then changes course to sail to Port C, 72 km away, on a bearing of 105°.

(i) On the diagram below, **not drawn to scale**, label the known distances travelled and the known angles. [2]



(ii) Determine the measure of $\angle ABC$. [2]

(iii) Calculate, to the nearest km, the distance AC. [3]

(iv) Show that the bearing of A from C, to the nearest degree, is 260°. [3]

Total: 15 marks

11. (a) Matrices A and B are such that

$$A = \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix} \text{ and } B = \begin{pmatrix} 4 & 0 \\ 3 & -1 \end{pmatrix}.$$

(i) Show by multiplying A and B, that $AB \neq BA$. [2]

(ii) Find A^{-1}, the inverse of A. [2]

(iii) Write down the 2×2 matrix representing the matrix product AA^{-1}. [1]

(b) (i) Write the following pair of simultaneous equations as a matrix equation.

$3x + 2y = 1$

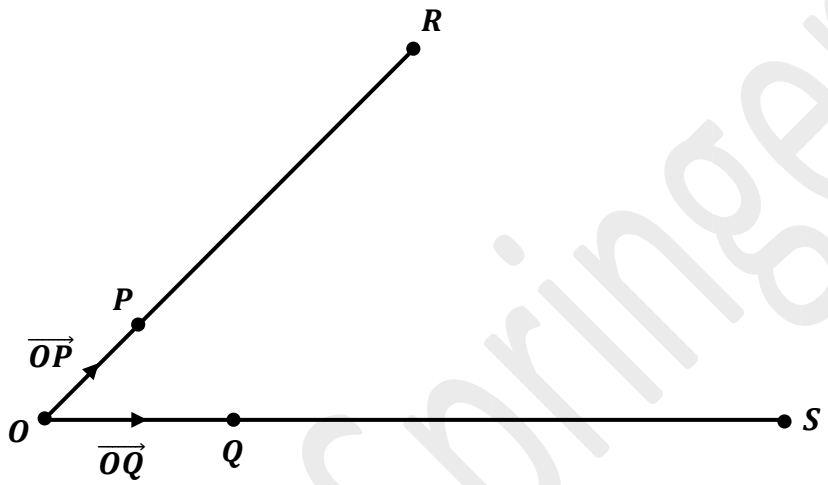
$5x + 4y = 5$ [1]

(ii) Write the solution of your matrix equation in (b)(i) as a product of two matrices. [2]

(c) The position vectors of the points P and Q relative to an origin, O, are

$\overrightarrow{OP} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\overrightarrow{OQ} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ respectively.

The diagram below shows that $PR = 3\ OP$ and $QS = 3\ OQ$.



(i) Express in the form $\begin{pmatrix} x \\ y \end{pmatrix}$, vector

- $\overrightarrow{OS}$ [1]
- $\overrightarrow{PQ}$ [2]
- $\overrightarrow{RS}$ [2]

(ii) State TWO geometrical relationships between PQ and RS. [2]

Total: 15 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.