CSEC Mathematics

June 2025 - Paper 2

Solutions

SECTION I

Answer ALL questions.

ALL working MUST be clearly shown.

1. (a) Using a calculator, or otherwise, evaluate EACH of the following, giving your answers in EXACT form.

(i) $\left(\frac{3}{2}\right)^2 - 1 + 6$ [1]

$= \left(\frac{9}{4}\right) - 1 + 6$

$= \frac{9}{4} + 5$

$= \frac{9}{4} + \frac{20}{4}$

$= \frac{29}{4} \; or \; 7\frac{1}{4} \; or \; 7.25$

(ii) $\left(2.1 \times \frac{10}{7}\right) + 12 \div 1\frac{3}{5}$ [2]

$= (3) + 12 \div 1\frac{3}{5}$

$= 3 + \left(12 \div 1\frac{3}{5}\right)$

$= 3 + \left(\frac{12}{1} \div \frac{8}{5}\right)$

$= 3 + \left(\frac{12}{1} \times \frac{5}{8}\right)$

$= 3 + \left(\frac{15}{2}\right)$

$= \frac{3}{1} + \frac{15}{2}$

$= \frac{6 + 15}{2}$

$= \frac{21}{2} \quad or \quad 10\frac{1}{2} \quad or \quad 10.5$

(b) Maranda earns \$8 316 per month. She spends $\frac{4}{7}$ of her earnings on utility bills, food and personal items, and **saves the remainder.**

(i) What **percentage** of her monthly salary does she save? [1]

Maranda spends $\frac{4}{7}$ of her earnings.

$\therefore$ She saves $\frac{7}{7} - \frac{4}{7}$

$= \frac{3}{7}$

Percentage saved $= \frac{3}{7} \times \frac{100}{1}$

$= \frac{300}{7}$

$= 42\frac{6}{7}\%$

$= 42.9\%$ (to 3 significant figures)

(ii) The portion of her earnings spent on utility bills, food and personal items is divided in the ratio 5:3:4 respectively. Calculate the amount of money she spends on personal items. [2]

Total amount spent $= \frac{4}{7} \times 8316$

$= \$4752$

Ratio of earnings spent:

Utility bills : Food : Personal Items

5 : 3 : 4

Total parts $= 5 + 3 + 4$

$= 12$ parts

Since 12 parts $= \$4752$,

$$1 \text{ part} = \frac{4752}{12}$$

$= \$396$

Since Personal Items $= 4$ parts,

Amount of money spent on personal items $= 4 \times \$396$

$= \$1584$

(iii) Maranda saves the same amount of money each month. Show that her annual savings from her earning is \$42 768. [1]

Amount of money saved in one month $= \frac{3}{7} \times \$8316$

$= \$3564$

Since there are 12 months in one year,

Amount of money saved in one year $= \$3564 \times 12$

$= \$42,768$

Q.E.D.

(iv) She invests the $42 768 in her credit union which pays compound interest of 5% per annum. What is the TOTAL interest earned after 2 years?

[The final amount, A, when principal, P, is invested compound interest at rate, r for n number of years is given by $A = P\left(1 + \frac{r}{100}\right)^n$.]

[2]

Method 1:

Substituting Principal $(P) = \$42\ 768$, Rate $(r) = 5$ and Number of years $(n) = 2$ into:

$$A = P\left(1 + \frac{r}{100}\right)^n$$

$$= 42\ 768\left(1 + \frac{5}{100}\right)^2$$

$$= 42\ 768\left(\frac{441}{400}\right)$$

$$= \$47\ 151.72$$

Interest Earned $=$ Amount $-$ Principal

$= \$47\ 151.72 - \$42\ 768$

$= \$4\ 383.72$

Method 2:

Amount after 1st year $= \frac{105}{100} \times \$42\ 768$

$= \$44\ 906.40$

Amount after 2nd year $= \frac{105}{100} \times \$44\ 906.40$

$= \$47\ 151.72$

Total interest earned = Amount after 2nd year − Principal

$= \$47\ 151.72 - \$42\ 768$

$= \$4\ 383.72$

Total: 9 marks

2. (a) Factorize, completely, EACH of the following expressions.

(i) $xy^2 - x^2y$ [1]

$xy^2 - x^2y$

$= xy(\, y - x)$

(ii) $3x^2 + x - 10$ [2]

$3x^2 + x - 10$

$= 3x^2 + 6x - 5x - 10$

$= 3x(x + 2) - 5(x + 2)$

$= (3x - 5)(x + 2)$

(b) Solve for n.

$$4^n \div 4^3 = \frac{1}{4}$$

$$4^n \div 4^3 = \frac{1}{4}$$

$$\frac{4^n}{4^3} = \frac{1}{4}$$

$$4^{n-3} = \frac{1}{4}$$

$$4^{n-3} = 4^{-1}$$

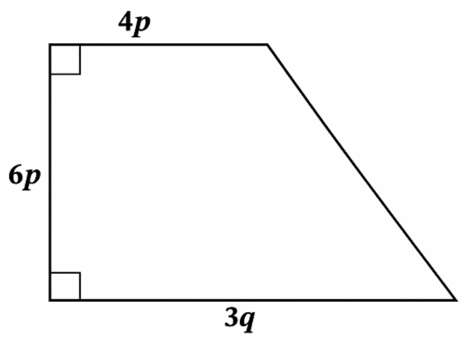
$$n - 3 = -1$$

$$n = -1 + 3$$

$$n = 2$$

(c) The lengths of three sides of a trapezium are shown on the diagram below.

The area of the trapezium is 750 square units.



(i) Write down an expression in terms of p and q for the area of the trapezium. [1]

Area of trapezium $= \frac{1}{2}(sum\ of\ the\ parallel\ sides) \times\ height$

$= \frac{1}{2}(4p + 3q)\ \times 6p$

$= 3p\ (4p + 3q)$

$= 12p^2 + 9pq$

$\therefore$ An expression in terms of p and q for the area of the trapezium is $12p^2 + 9pq$ square units.

(ii) Given that $q = 2p$, determine the value of p. [3]

From part (i), we found that Area of trapezium $= 12p^2 + 9pq$.

From the question, Area of the trapezium $= 750$ square units.

$\therefore 12p^2 + 9pq = 750$

Since $q = 2p$,

$12p^2 + 9p(2p) = 750$

$12p^2 + 18p^2 = 750$

$30p^2 = 750$

$p^2 = \frac{750}{30}$

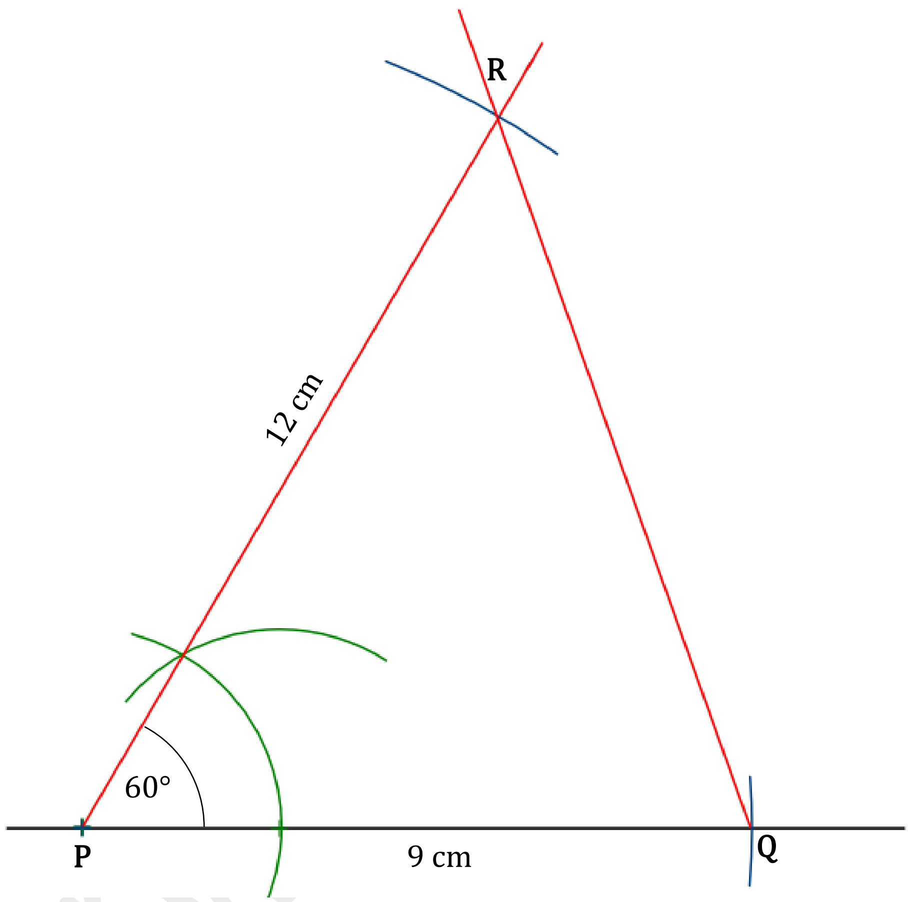
$p^2 = 25$

$p = \pm\sqrt{25}$

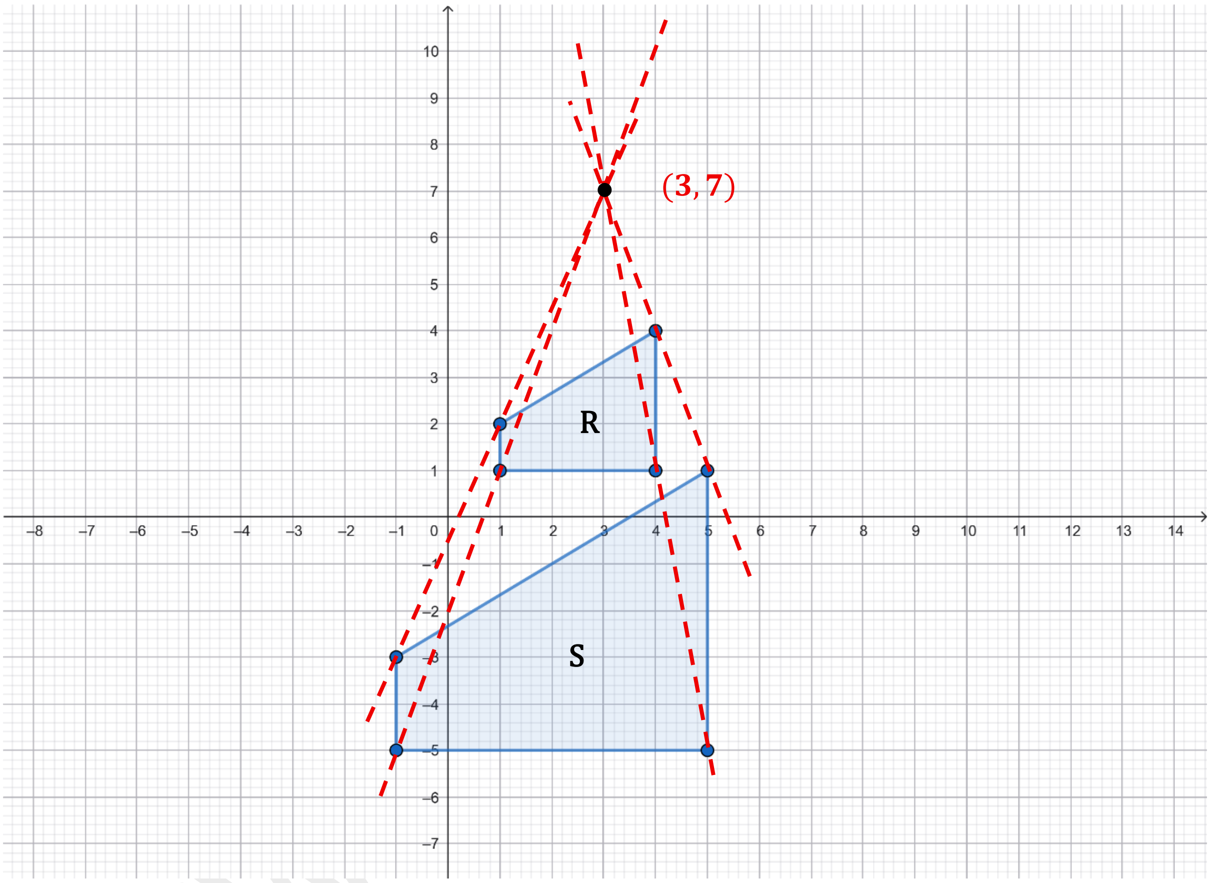
$p = 5$ *units* only (since length cannot be negative)

Total: 9 marks

3. (a) Using a ruler, a pencil and a pair of compasses, construct the triangle PQR, such that $PQ = 9\ cm$, $\angle QPR = 60°$ and $PR = 12\ cm$. [3]



(b) The diagram below shows two quadrilaterals, R and S, where S is the image of R under a transformation.



(i) Describe, fully, the **single** transformation that maps Quadrilateral R onto Quadrilateral S. [3]

$$\text{Scale Factor} = \frac{\text{Image length}}{\text{Object Length}}$$

$$= \frac{6}{3}$$

$$= 2$$

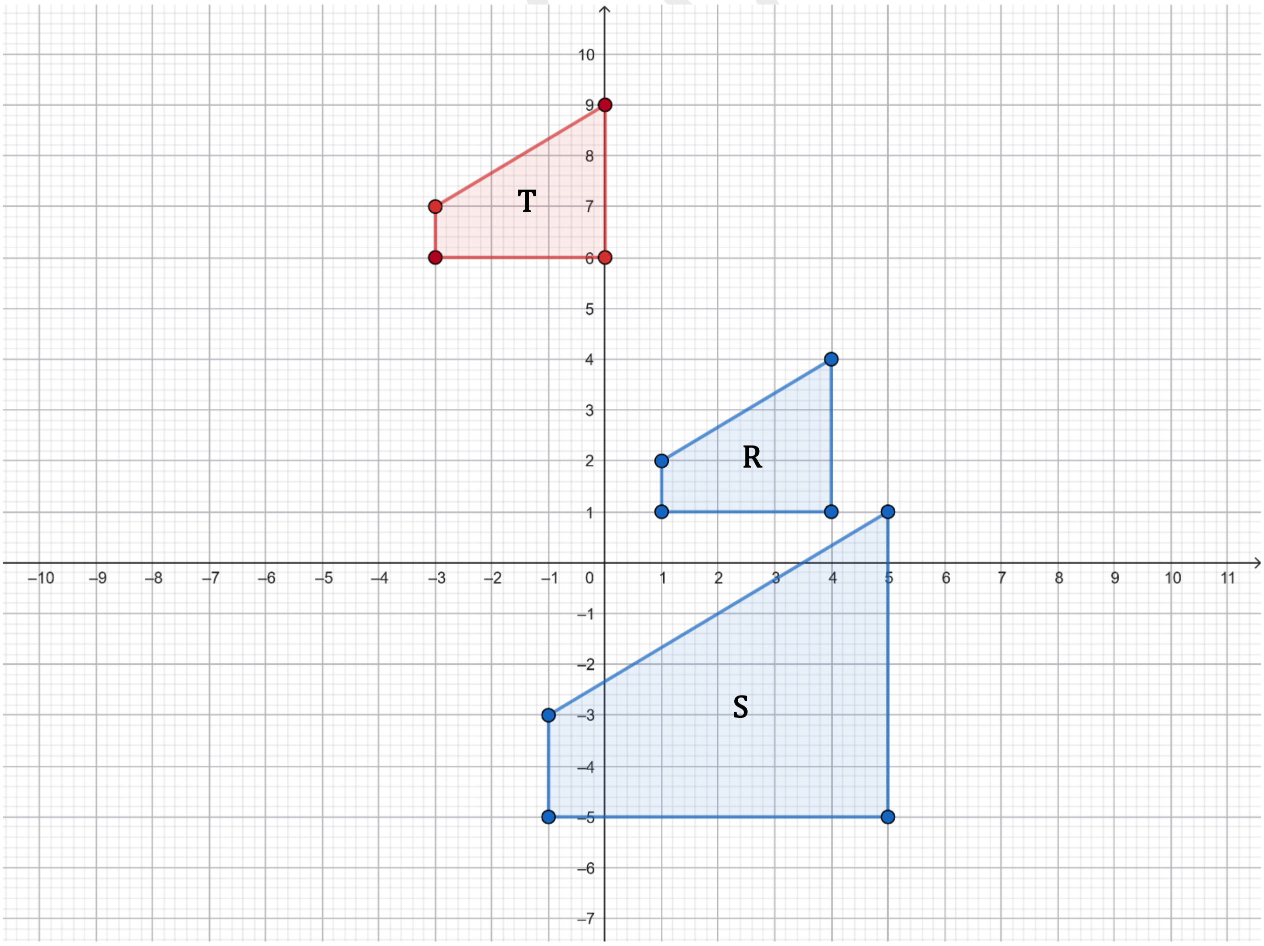
∴ The single transformation that maps R onto S is an enlargement of scale factor 2 where the centre of enlargement is $(3, 7)$,

(ii) On the grid **on page 12**, draw the image of Quadrilateral R when it is translated by the vector $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$. Name the image T. [1]

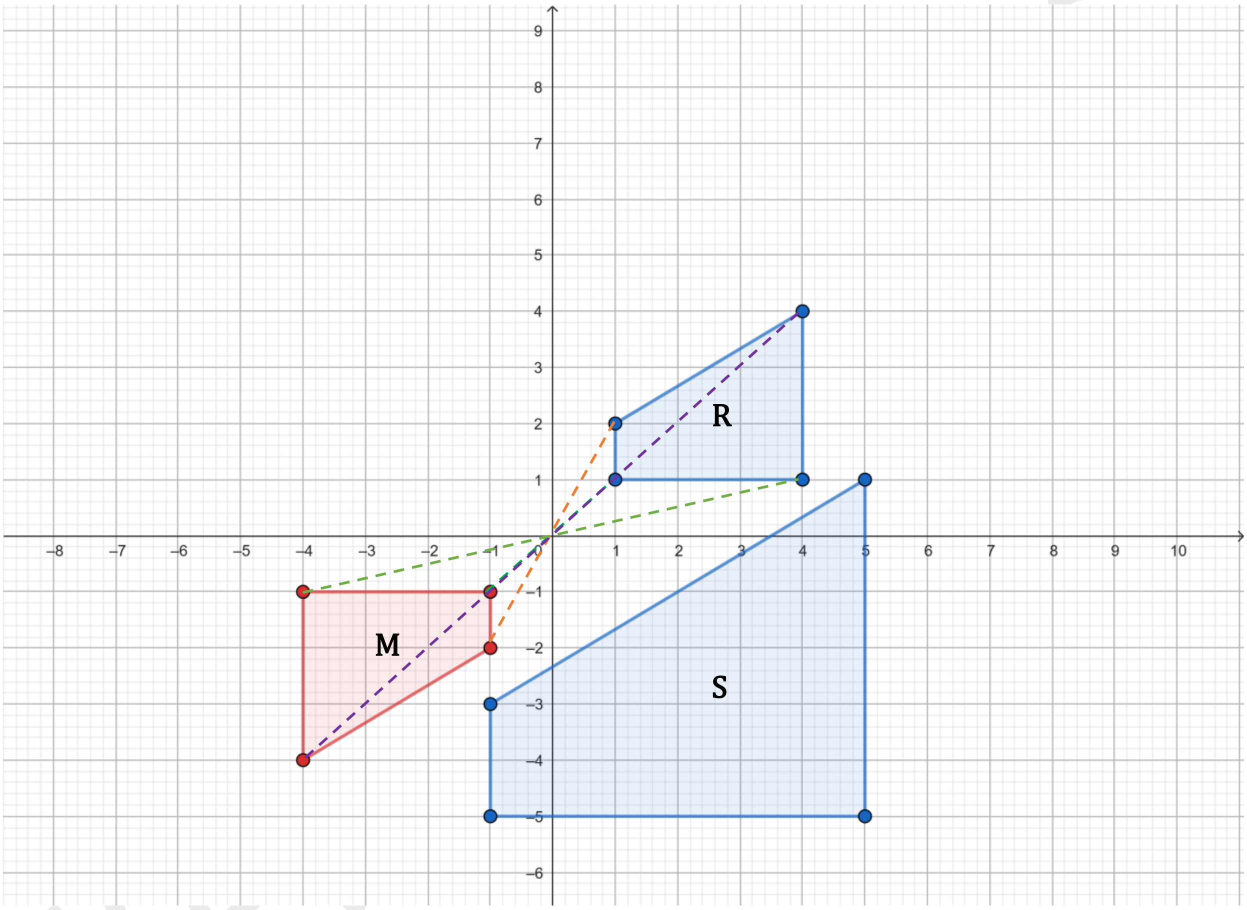
Translation vector $= \begin{pmatrix} -4 \\ 5 \end{pmatrix}$

∴ We need to move the vertices of R 4 units to the left and then 5 units upwards.

The coordinates of T will therefore be: $(-3, 6), (-3, 7), (0, 9)$ and $(0, 6)$.



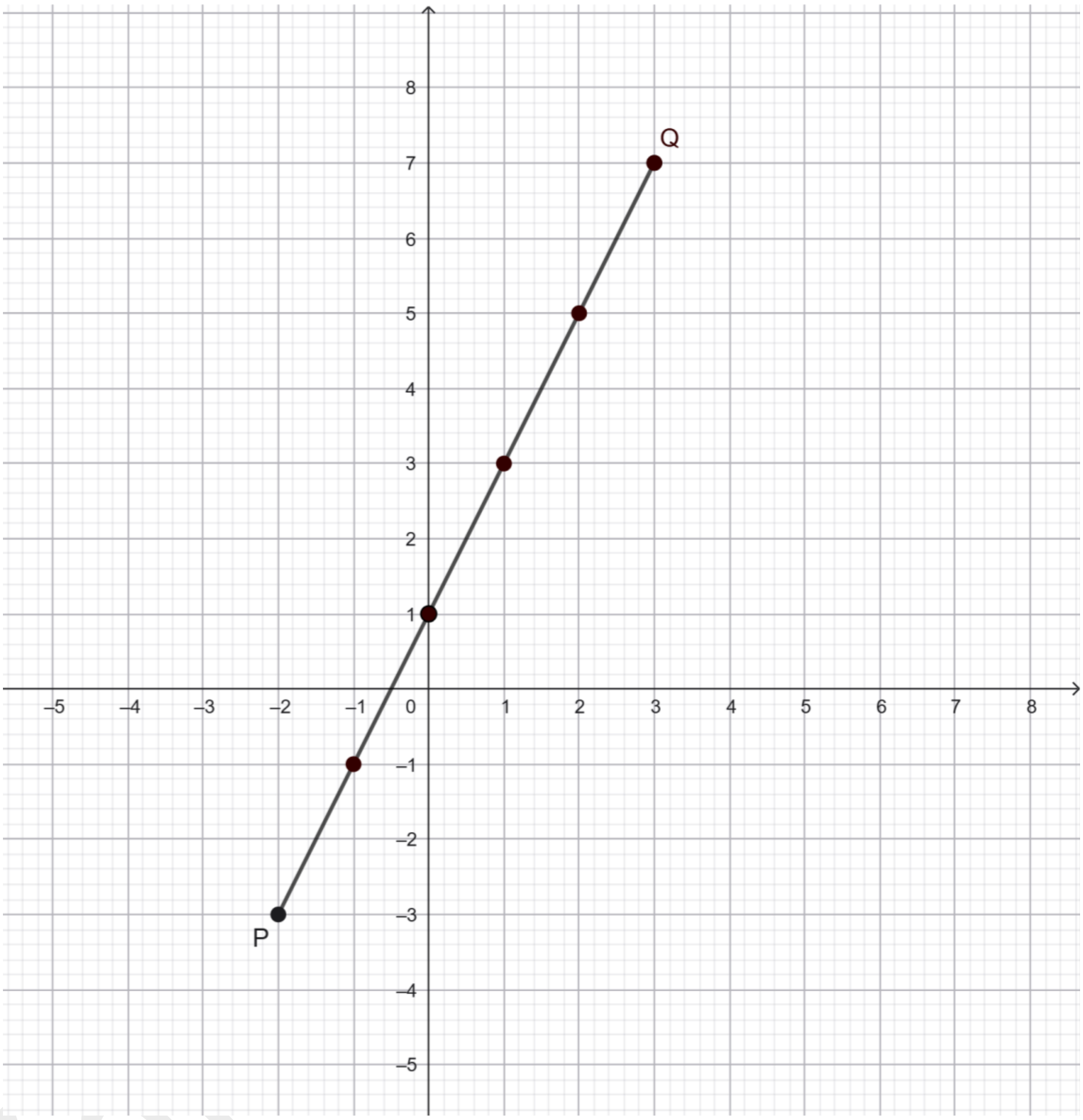
(iii) On the grid **on page 12**, draw the image of Quadrilateral R when it is rotated 180° about the origin. Name the image M. [2]



The coordinates of M will therefore be $(-4,-4), (-4,-1), (-1,-1)$ and $(-1,-2)$.

Total: 9 marks

4. (a) The graph below shows the straight line PQ.



(i) Find the gradient of the line PQ. [1]

Using the coordinates $P = (-2, -3)$ and $Q = (3, 7)$,

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7 - (-3)}{3 - (-2)}$$

$$= \frac{7 + 3}{3 + 2}$$

$= \frac{10}{5}$

$= 2$

(ii) Write down the equation of the line through PQ in the form

$y = mx + c$ [1]

Reading off the graph, the y-intercept, $c = 1$.

Substituting the gradient, $m = 2$ and y-intercept, $c = 1$ into

$y = mx + c$.

$y = (2)x + (1)$

$y = 2x + 1$

(b)Two functions, f and g are defined as

$f(x) = x^2 - 1$ and $g(x) = 3x + 2$

Find

(i) $g(-6)$ [1]

$g(-6) = 3(-6) + 2$

$= -18 + 2$

$= -16$

(ii) The inverse function, $g^{-1}(x)$. [2]

$g(x) = 3x + 2$

Let $y = g(x)$

$y = 3x + 2$

Make x the subject of the formula:

$$y = 3x + 2$$

$$y - 2 = 3x$$

$$\frac{y-2}{3} = x$$

$$x = \frac{y-2}{3}$$

Interchange x and y:

$$y = \frac{x-2}{3}$$

$$\therefore g^{-1}(x) = \frac{x-2}{3}$$

(iii) a) Show that $fg(x) = 3(3x+1)(x+1)$ [3]

$f(x) = x^2 - 1$ $g(x) = 3x + 2$

$$\begin{aligned}
fg(x) &= f[g(x)] \\
&= f[3x+2] \\
&= (3x+2)^2 - 1 \\
&= (3x+2)(3x+2) - 1 \\
&= 9x^2 + 6x + 6x + 4 - 1 \\
&= 9x^2 + 12x + 3 \\
&= 3(3x^2 + 4x + 1) \\
&= 3(3x^2 + 3x + x + 1) \\
&= 3(3x(x+1) + 1(x+1)) \\
&= 3(3x+1)(x+1)
\end{aligned}$$

Q.E.D.

b) Hence, solve the equation $fg(x) = 0$. [1]

$fg(x) = 3(3x+1)(x+1)$

Since $fg(x) = 0$,

$3(3x+1)(x+1) = 0$

Either

$3x + 1 = 0$ or $x + 1 = 0$

$3x = -1$ or $x = -1$

$x = -\frac{1}{3}$

Total: 9 marks

5. Thirty students took a Mathematics test. The marks they scored are shown in the table below.

Mark	Tally	Frequency	Cumulative Frequency
1	|||	3	3
2	||||	4	7
3	~~||||~~ |||	8	15
5	|||	3	18
7	|	1	19
8	~~||||~~ |	6	25
10	~~||||~~	5	30

(a) Complete the Cumulative Frequency column in the table above. [1]

The Cumulative Frequency column has been completed above.

(b) Using the information in the table above, determine the

(i) Range [1]

Range $= Highest\ Value - Lowest\ Value$

$= 10 - 1$

$= 9$ marks

(ii) Modal mark [1]

The modal mark is the mark that appears most frequently.

∴ The modal mark is 3 marks.

(iii) Median mark [2]

$$\text{Median, } Q_2 = \frac{1}{2}(n+1)^{th} \text{ value}$$

$$= \frac{1}{2}(30+1)$$

$$= \frac{1}{2}(31)$$

$$= 15.5^{th} \text{ value}$$

Reading off the table, the 15th value is 3 and the 16th value is 5.

$$\therefore Median = \frac{3+5}{2}$$

$$= \frac{8}{2}$$

$$= 4 \text{ marks}$$

(iv) Mean mark [2]

Mark (x)	**Frequency (f)**	$f \times x$
1	3	3
2	4	8
3	8	24
5	3	15
7	1	7
8	6	48
10	5	50
	$\sum f = 30$	$\sum fx = 155$

$$Mean, \bar{x} = \frac{\sum fx}{\sum f}$$

$$= \frac{155}{30}$$

$$= 5\frac{1}{6} \text{ marks}$$

(c) The following two-way table shows the gender distribution of the student's performance on the Mathematics test.

	Male	**Female**	**Total**
Pass	4	8	12
Fail	5	13	18
Total	9	21	30

(i) A student is chosen at random. Find the probability that the student is a female who failed the test. [1]

Number of female students who failed the test = 13

Total number of students = 30

$$\text{Probability} = \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{13}{30}$$

(ii) A male student is chosen at random. What is the probability that he passed the test? [1]

Number of male students who passed the test $= 4$

Number of male students $= 9$

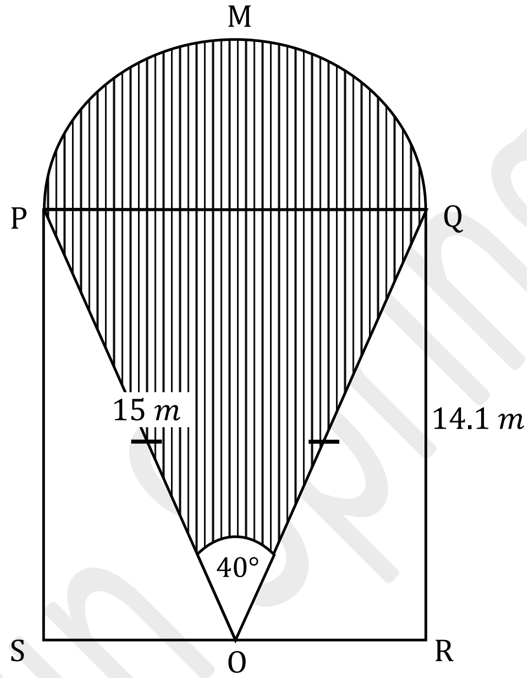
$$\text{Probability} = \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{4}{9}$$

Total: 9 marks

6. $PMQROS$ is the cross-section of a play area in a park. $PQRS$ is a rectangle and PMQ is a semi-circle. O is the mid-point of RS. $OP = OQ = 15\ m$ and Angle $POQ = 40°$.

$\left(\text{Use } \pi = \frac{22}{7}\right)$



(a) (i) Determine the value of Angle OPQ. [1]

Triangle OPQ is an isosceles triangle which means that the base angles are equal.

Since $Q\hat{O}P = 40°$,

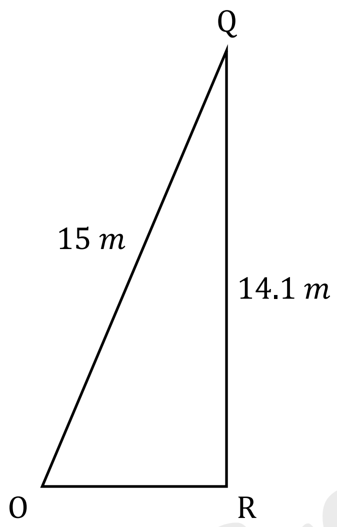
$$O\hat{P}Q = \frac{180° - 40°}{2}$$

$$= \frac{140°}{2}$$

$$= 70°$$

(ii) Calculate the length of OR. [2]

A sketch of triangle OQR can be seen below:



According to Pythagoras' Theorem,

$OR^2 + QR^2 = OQ^2$

$OR^2 = OQ^2 - QR^2$

$OR^2 = (15)^2 - (14.1)^2$

$OR^2 = 225 - 198.81$

$OR^2 = 26.19$

$OR = \sqrt{26.19}$

$OR = 5.12\ m$ (correct to 3 significant figures)

(b) Calculate the area of the shaded portion of the diagram. [3]

Area of the triangle

Base of the shaded triangle, $PQ = 2 \times OR$

$= 2 \times 5.12$

$= 10.24\ m$

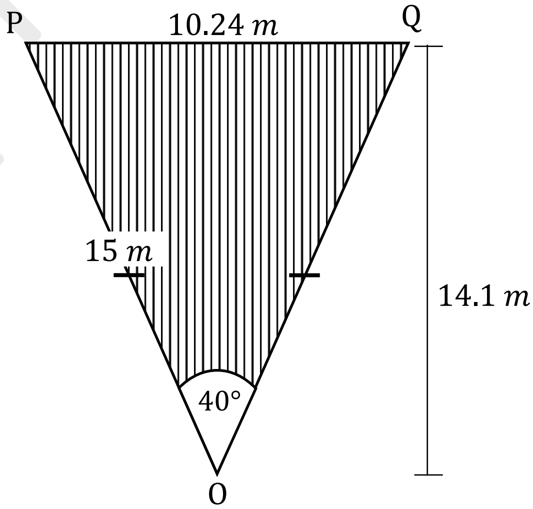
The height of the shaded triangle was given as $14.1\ m$.

Substituting base, $b = 10.24m$ and $height, h = 14.1m$ into:

Area of a triangle $= \frac{1}{2}\ bh$

$= \frac{1}{2}(10.24)(14.1)$

$= 72.192\ m^2$



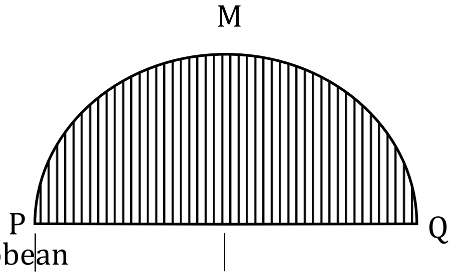
Area of the semi-circle

Substituting $\pi = \frac{22}{7}$ and radius, $r = 5.12\ m$ into:

Area of the semi-circle $= \frac{1}{2}\pi r^2$

$= \frac{1}{2}\left(\frac{22}{7}\right)(5.12)^2$

$= 41.194\ m^2$ (correct to 3 decimal places)



$$5.12\ m$$

Altogether,

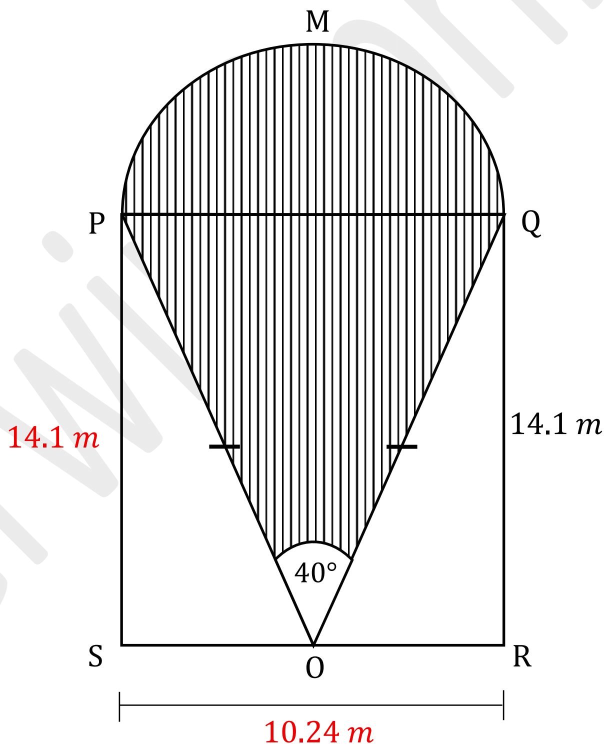
Area of shaded region $= \textit{Area of triangle} + \textit{Area of semi-circle}$

$= 72.192\ m^2 + 41.194\ m^2$

$= 113.386\ m^2$

(c) Find the perimeter of the cross-section $PMQROS$. [3]

A sketch of $PMQROS$ is shown below with the sides that we know so far:



To find the length of the arc PMQ, we can substitute $\pi = \frac{22}{7}$ and radius, $r = 5.12\ m$ into:

Length of arc $PMQ = \frac{1}{2} \times 2\pi r$

$= \frac{1}{2} \times 2 \times \frac{22}{7} \times 5.12$

$= 16.09$ (to 2 decimal places)

Perimeter of the cross-section $PMQROS = 16.09 + 14.1 + 14.1 + 10.24$

$= 54.53\ m$

Total: 9 marks

7. A sequence of figures is made up of regular pentagons, using sticks of unit length. The first three figures in the sequence are shown below. The vertices in each figure are marked with dots.

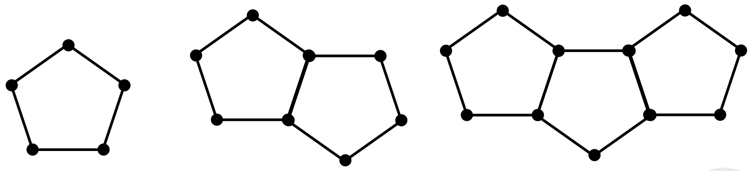


Figure 1 Figure 2 Figure 3

(a) An incomplete diagram of Figure 4 of the sequence is shown below. Complete Figure 4 by adding more sticks and dots. [2]

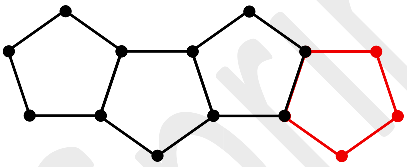


Figure 4

(b) Study the pattern of numbers in each row of the table below. Each row relates to one of the figures in the sequence of figures shown **on page 22**. Some rows have not been included in the table.

Complete the rows corresponding to (i), (ii) and (iii) in the table below.

	Figure	**Number of Sticks (S)**	**Number of Dots (D)**	
	1	5	5	
	2	9	8	
	3	13	11	
	⋮	⋮	⋮	
(i)	20	81	62	[2]
(ii)	42	169	128	[2]
(iii)	n	$4n + 1$	$3n + 2$	[2]

(iii) Looking at Figure 1, 2 and 3, we can deduce that

Number of Sticks, $S = 4n + 1$

Number of Dots, $D = 3n + 2$

Explanation for S:

Figure	Number of Sticks (S)	Number of Dots (D)
1	5	5
2	9	8
3	13	11

The number of sticks increase by 4.

$\therefore$ In terms of n, we can deduce that $S = 4n + k$ (where k is a constant)

Substituting $n = 1$ and $S = 5$ into:

$S = 4n + k$

$5 = 4(1) + k$

$5 = 4 + k$

$5 - 4 = k$

$1 = k$

$k = 1$

Substituting $n = 2$ and $S = 9$ into:

$S = 4n + k$

$9 = 4(2) + k$

$9 = 8 + k$

$9 - 8 = k$

$1 = k$

$k = 1$

Substituting $n = 3$ and $S = 13$ into:

$S = 4n + k$

$13 = 4(3) + k$

$13 = 12 + k$

$13 - 12 = k$

$1 = k$

$k = 1$

$\therefore S = 4n + 1$

Explanation for D:

Figure	Number of Sticks (S)	Number of Dots (D)
1	5	5
2	9	8
3	13	11

The number of dots increase by 3.

$\therefore$ In terms of n, we can deduce that $D = 3n + k$ (where k is a constant)

Substituting $n = 1$ and $D = 5$ into:

$D = 3n + k$

$5 = 3(1) + k$

$5 = 3 + k$

$5 - 3 = k$

$2 = k$

$k = 2$

Substituting $n = 2$ and $D = 8$ into:

$D = 3n + k$

$8 = 3(2) + k$

$8 = 6 + k$

$8 - 6 = k$

$2 = k$

$k = 2$

Substituting $n = 3$ and $D = 11$ into:

$D = 3n + k$

$11 = 3(3) + k$

$11 = 9 + k$

$11 - 9 = k$

$2 = k$

$k = 2$

$\therefore\ D = 3n + 2$

(ii) When $n = 20$,

Number of Sticks, $S = 4n + 1$

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$= 4(20) + 1$

$= 80 + 1$

$= 81$

Number of Dots, $D = 3n + 2$

$= 3(20) + 2$

$= 60 + 2$

$= 62$

(ii) When $S = 169$,

$169 = 4n + 1$

$169 - 1 = 4n$

$168 = 4n$

$\frac{168}{4} = n$

$42 = n$

$n = 42$

When $n = 42$,

$D = 3(42) + 2$

$= 126 + 2$

$= 128$

(c) Write an equation in S, D and n to show the relationship between the number of sticks, S, and the number of dots D, for Figure n. [2]

Number of Sticks, $S = 4n + 1$

Number of Dots, $D = 3n + 2$

Method 1 ($S - D$):

$S - D = 4n + 1 - (3n + 2)$

$S - D = 4n + 1 - 3n - 2$

$S - D = 4n - 3n + 1 - 2$

$S - D = n - 1$

$S = n - 1 + D$

Method 2 ($S \times D$):

$SD = (4n + 1)(3n + 2)$

$SD = 12n^2 + 8n + 3n + 2$

$SD = 12n^2 + 11n + 2$

$SD = 12n^2 + 11n + 2$

Method 3 ($S \div D$):

$$\frac{S}{D} = \frac{4n + 1}{3n + 2}$$

$$S = \frac{4n + 1}{3n + 2} \times D$$

Total: 10 marks

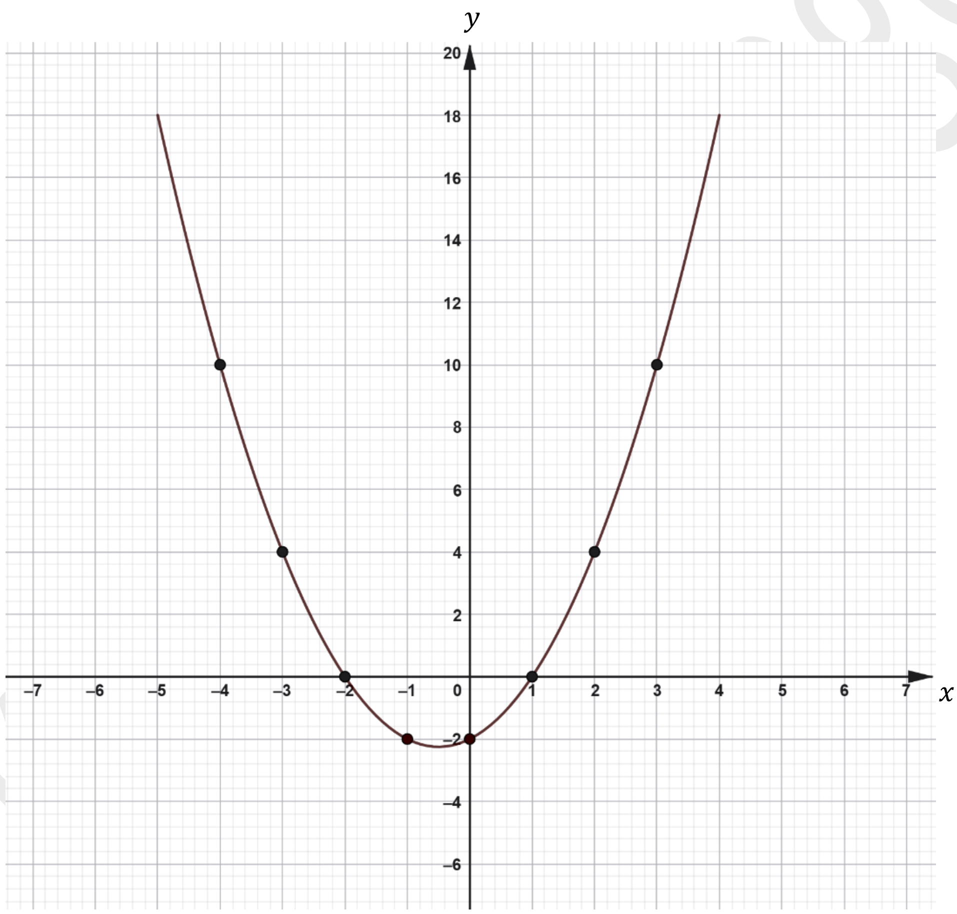
SECTION II

Answer ALL questions.

ALL working MUST be clearly shown.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS.

8. (a) The following diagram shows the graph of the function $y = x^2 + x - 2$.



(i) From the graph, state the

a) Roots of the function [1]

The graph cuts the x-axis at $x = 1$ and $x = -2$.

∴ The roots are: $x = 1$ and $x = -2$.

b) Coordinates of the y-intercept of the function [1]

The graph cuts the y-axis at $y = -2$.

∴ The coordinates of the y-intercept is $(0, -2)$

c) Minimum value of the function [1]

Reading off the graph, the minimum value is -2.25 or $-2\frac{1}{4}$.

d) Equation of the axis of symmetry of the function [1]

The equation of the axis of symmetry is $x = -0.5$

(b) (i) On the same pair of axes **on page 24**, draw and label the line $y = 2x + 4$.

[2]

From the equation $y = 2x + 4$, we can tell that the y-intercept is 4.

$\therefore$ The coordinates of the y-intercept is $(0, 4)$

When $x = -2$,

$y = 2(-2) + 4$

$= -4 + 4$

$= 0$

When $x = 2$,

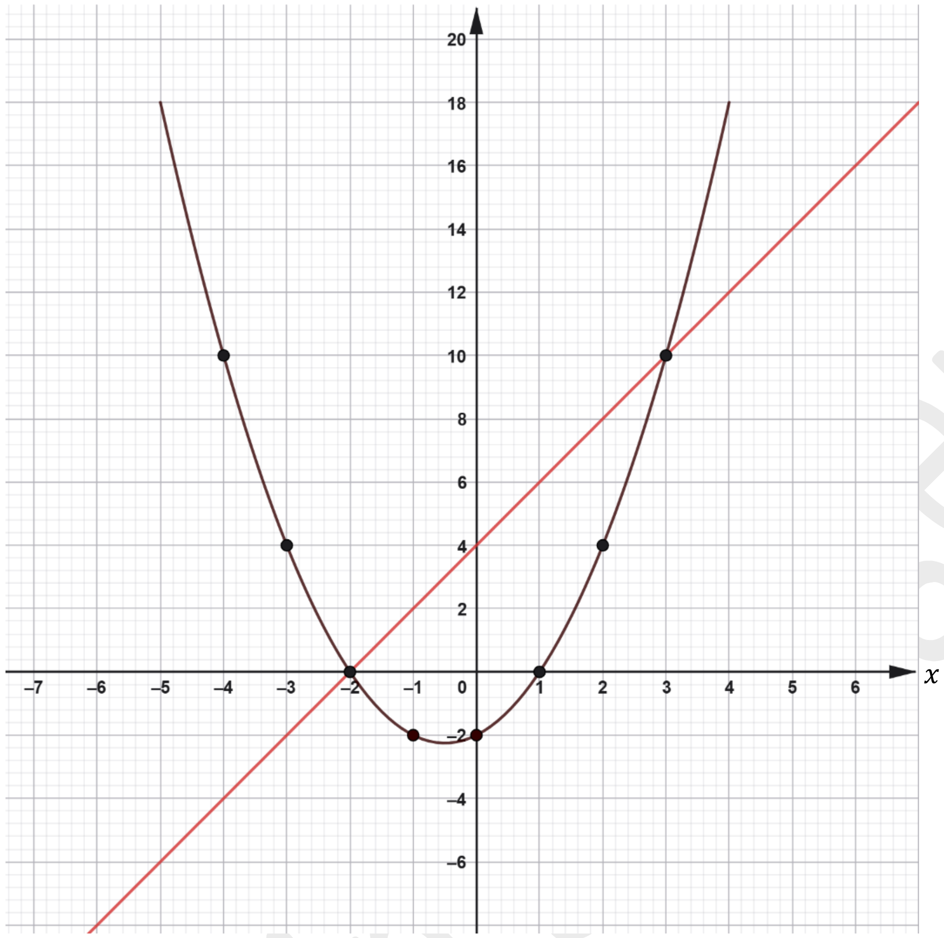
$y = 2(2) + 4$

$= 4 + 4$

$= 8$

$\therefore$ We can draw a line to connect the points $(0, 4)$, $(-2, 0)$, and $(2, 8)$ to get the line $y = 2x + 4$.

y



(ii) Using your graphs, determine the solutions to the following pair of simultaneous equations.

$$y = x^2 + x - 2$$

$$y = 2x + 4$$

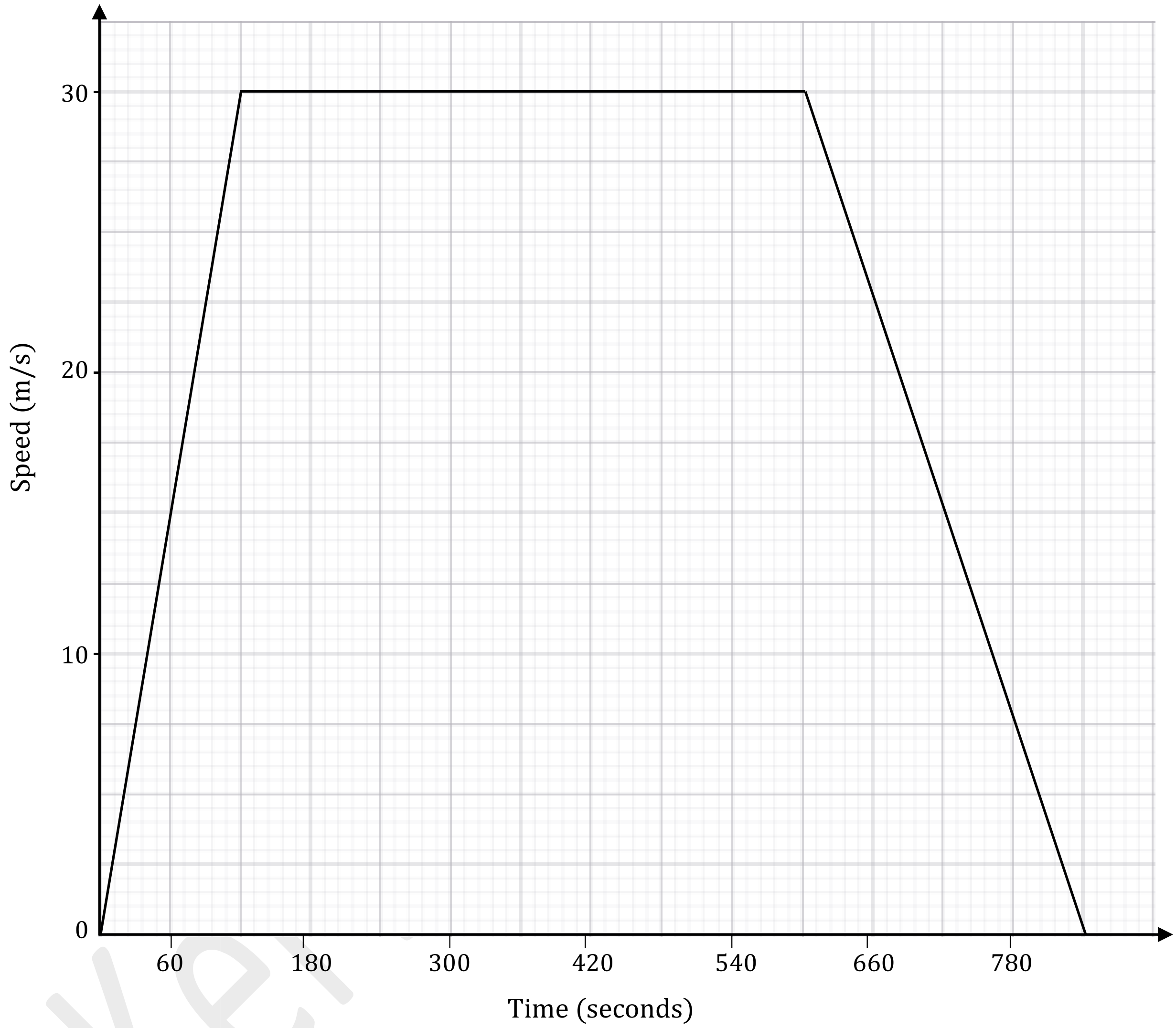
[2]

The solutions to the pair of simultaneous equations will be the points of intersection. Reading off the graph, the line $y = 2x + 4$ and the curve $y = x^2 + x - 2$ intersects at the points $(-2, 0)$ and $(3, 10)$.

$\therefore$ The solutions to the pair of simultaneous equations are:

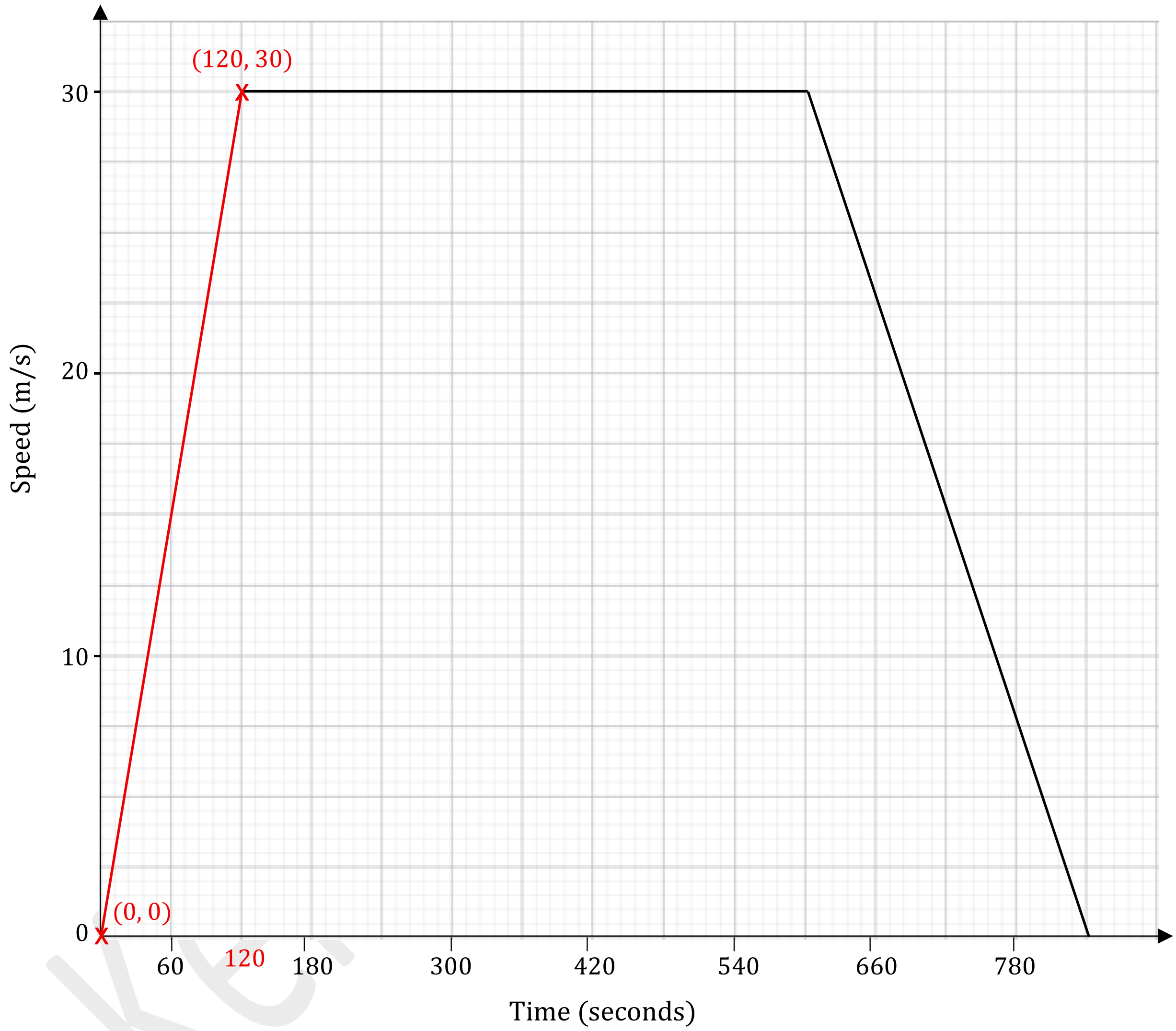
$x = -2$ and $y = 0$ or $x = 3$ and $y = 10$

(c) The following diagram shows the velocity-time graph of a car's journey from Town R to Town Q. The car accelerates for two minutes, travels at a constant maximum speed, then slows to a stop.



(i) Determine the initial acceleration of the car. [2]

To find the initial acceleration, we need to find the gradient.



Substituting the points $(\underset{x_1}{0}, \underset{y_1}{0})$ and $(\underset{x_2}{120}, \underset{y_2}{30})$ into:

$$Gradient, m = \frac{y_2 - y_1}{x_2 - x_1}$$

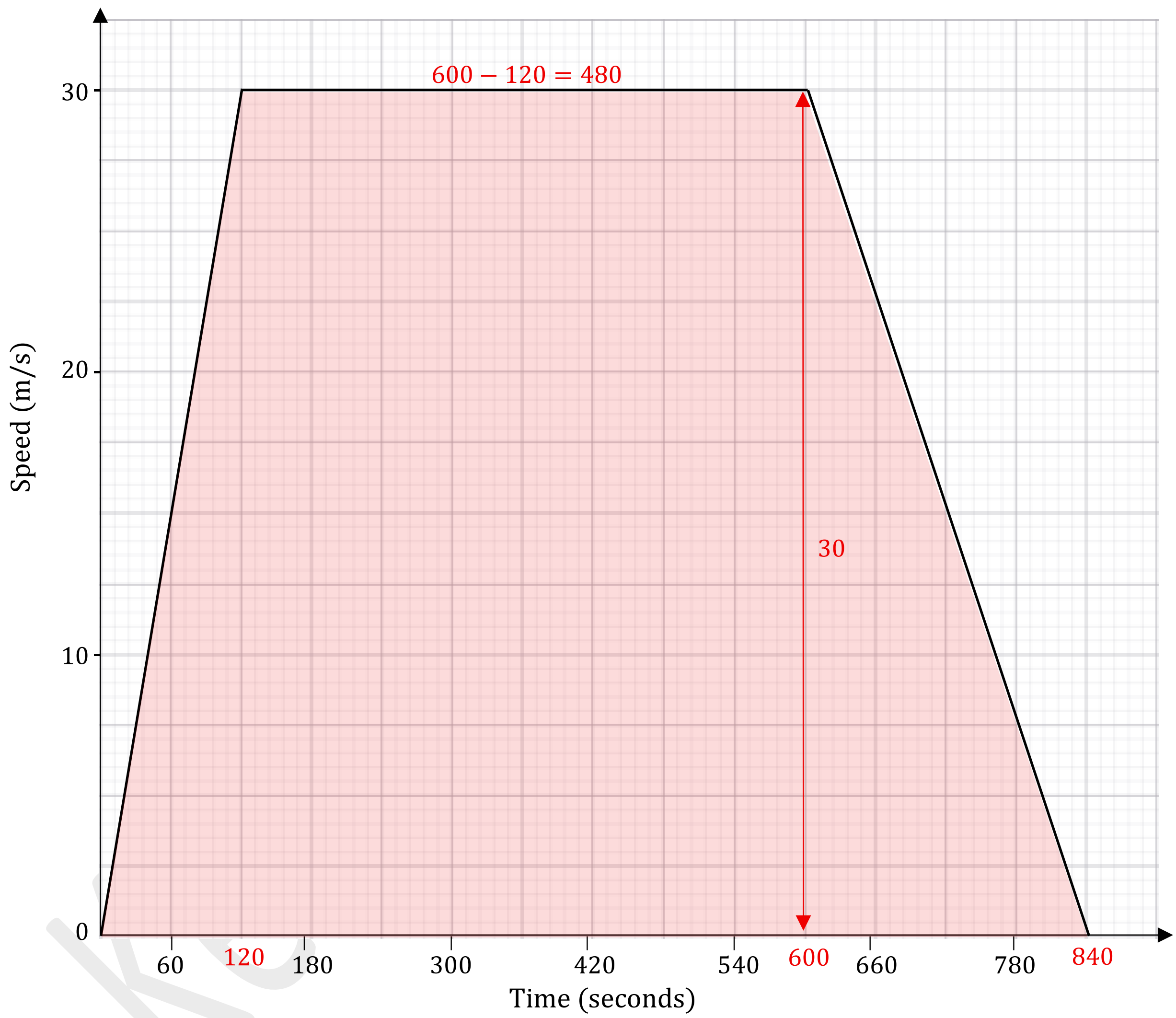
$$= \frac{30 - 0}{120 - 0}$$

$$= \frac{30}{120}$$

$$= \frac{1}{4} \quad or \quad 0.25\ ms^{-2}$$

(ii) Calculate the distance between the two towns. [2]

The distance travelled between the two towns can be found by finding the area under the graph. This shape is a trapezium.



$$\text{Distance travelled} = \frac{1}{2}(a+b)h$$

$$= \frac{1}{2}(480 + 840)(30)$$

$$= \frac{1}{2}(1320)(30)$$

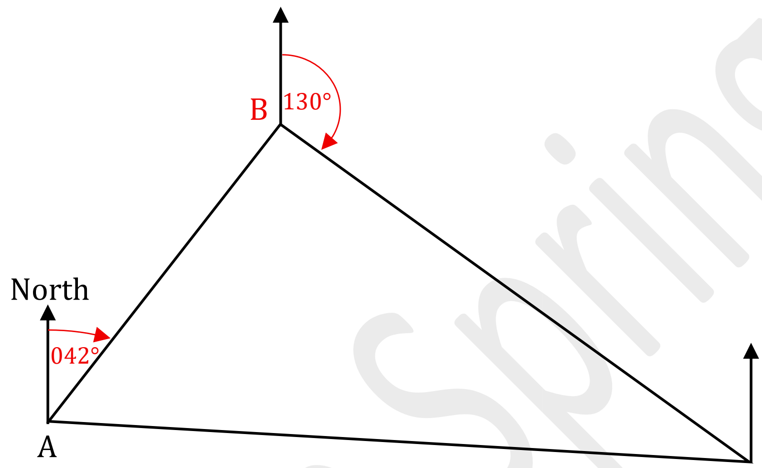
$$= \frac{1}{2}(39\,600)$$

$$= 19\,800\ m$$

Total: 12 marks

GEOMETRY AND TRIGONOMETRY.

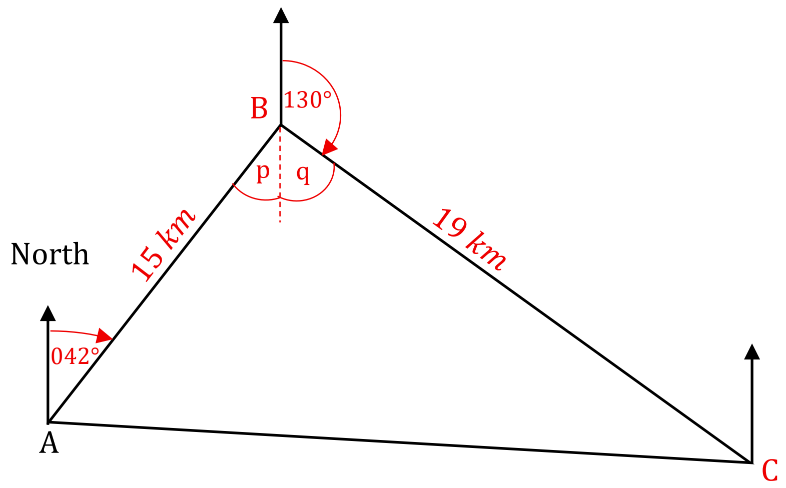
9. (a) A ship sails from Point A to Point B, which is 15 km from A on a bearing of 042°. The ship then sails to Point C, which is 19 km from B on a bearing of 130°. The following diagram shows a sketch of the ship's journey.



(i) On the diagram, insert the bearings 042° and 130°. [1]

The bearings 042° and 130° have been inserted on the diagram above.

(ii) Calculate the distance between Town A and Town C. [2]



Since alternate angles are equal,

$p = 42°$.

Since angles on a straight line add up to 180°,

$q = 180° - 130°$

$q = 50°$

$A\hat{B}C = p + q$

$= 42° + 50°$

$= 92°$

Using the cosine rule,

$(AC)^2 = (AB)^2 + (BC)^2 - 2(AB)(BC)\cos A\hat{B}C$

$(AC)^2 = (15)^2 + (19)^2 - 2(15)(19)\cos 92°$

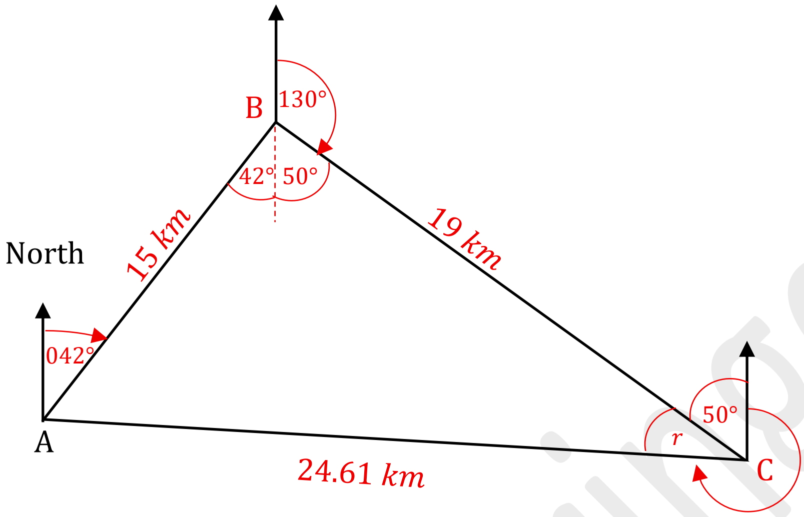
$(AC)^2 = 605.89$

$AC = \sqrt{605.89}$

$AC = 24.61\ km$ (correct to 2 decimal places)

$\therefore$ The distance between Town A and Town C is 24.61 km.

(iii) Determine the bearing of Town A from Town C. [3]



Substituting $A\hat{C}B = r$, $AB = 15$, $\sin A\hat{B}C = 92°$, $AC = 24.61$ into the sine rule:

$$\frac{\sin A\hat{C}B}{AB} = \frac{\sin A\hat{B}C}{AC}$$

$$\frac{\sin r}{15} = \frac{\sin 92°}{24.61}$$

$$\sin r = \frac{\sin 92°}{24.61} \times 15$$

$$r = \sin^{-1}\left(\frac{\sin 92°}{24.61} \times 15\right)$$

$$r = \sin^{-1}\left(\frac{\sin 92° \times 15}{24.61}\right)$$

$r = 37.53°$ (correct to 2 decimal places)

Bearing of Town A from Town $C = 360° - (50° + 37.53)$

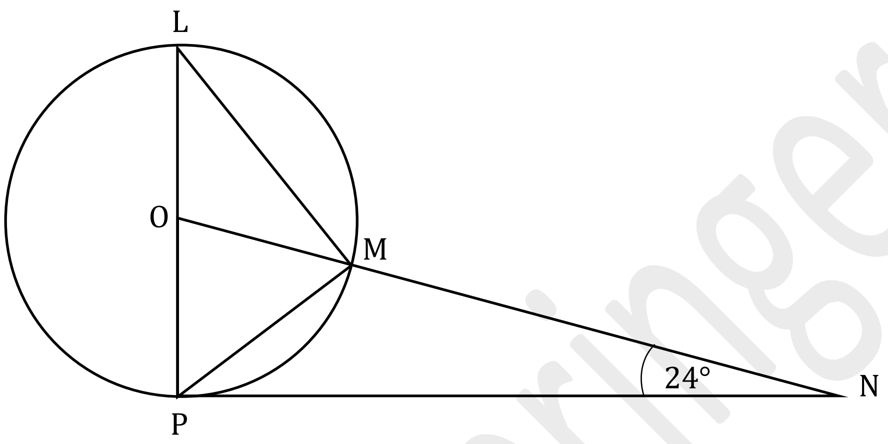
$= 360° - 87.53°$

$= 272.47°$

$\therefore$ The bearing of Town A from Town C is $272.47°$.

(b) The points L, M and P lie on the circumference of a circle whose centre is O.

The line NP is a tangent to the circle at P and OMN is a straight line. The line PL is a diameter and Angle $ONP = 24°$.

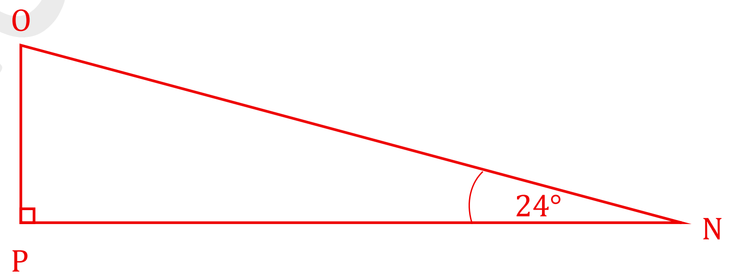


Calculate the value of EACH of the following angles. Show detailed working where necessary and give a reason to support your answers.

(i) Angle PON [2]

The angle made by a tangent (PN) and a radius (OP) is 90°.

$\therefore O\hat{P}N = 90°$



Since the angles in a triangle add up to 180°,

$P\hat{O}N = 180° - (90° + 24°)$

$= 66°$

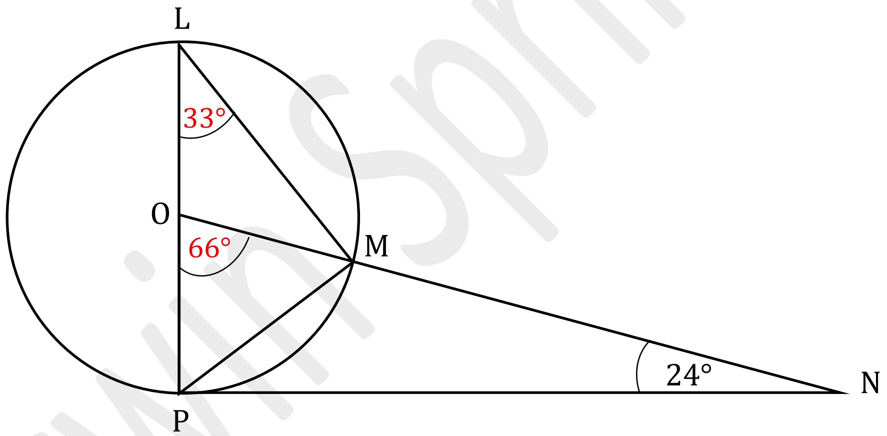
(ii) Angle PLM [2]

The angle at the centre of the circle $(P\hat{O}M)$ is twice the angle at the circumference $(P\hat{L}M)$ subtended from the same chord (PM).

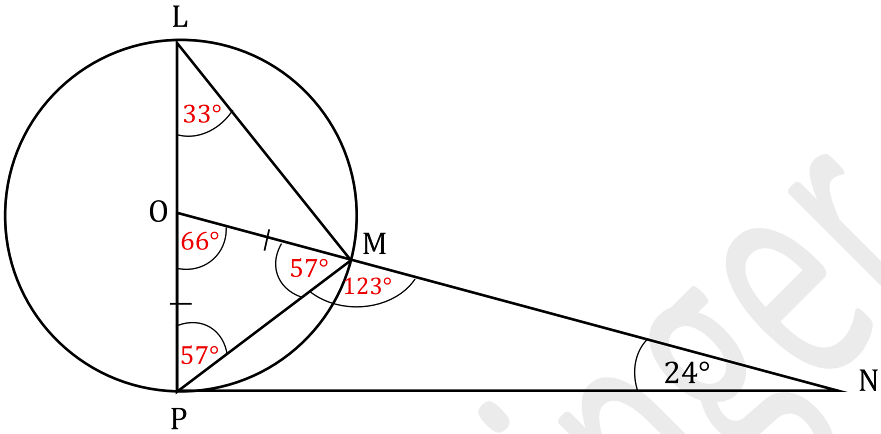
$$P\hat{L}M = \frac{1}{2}\,(P\hat{O}M)$$

$$= \frac{1}{2}\,(66°)$$

$$= 33°$$



(iii) Angle PMN [2]



$OP = OM$ since they are both radii of the same circle.

Triangle OPM is an isosceles triangle so the base angles are equal.

$$P\hat{M}O = \frac{180° - 66°}{2}$$

$$= \frac{114°}{2}$$

$$= 57°$$

Since angles on a straight line add up to 180°,

$$P\hat{M}N = 180° - 57°$$

$$= 123°$$

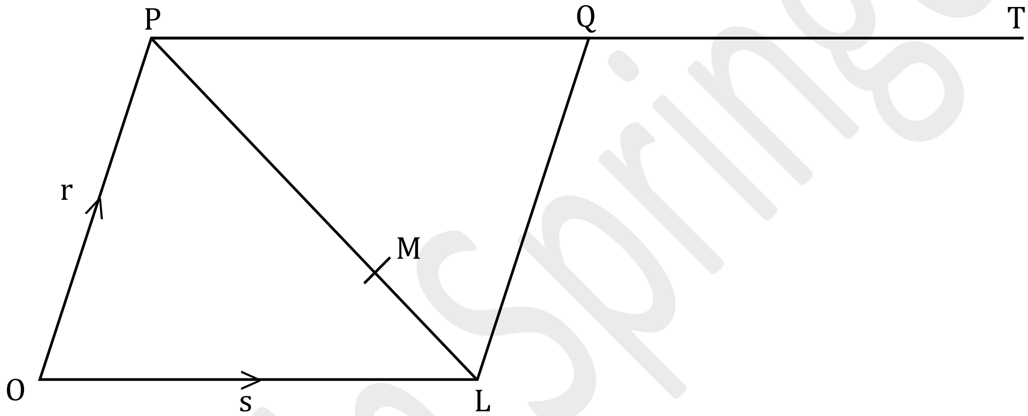
Total: 12 marks

10. (a) In the diagram below $OPQL$ is a parallelogram.

$\overrightarrow{OP} = \boldsymbol{r}$ and $\overrightarrow{OL} = \boldsymbol{s}$

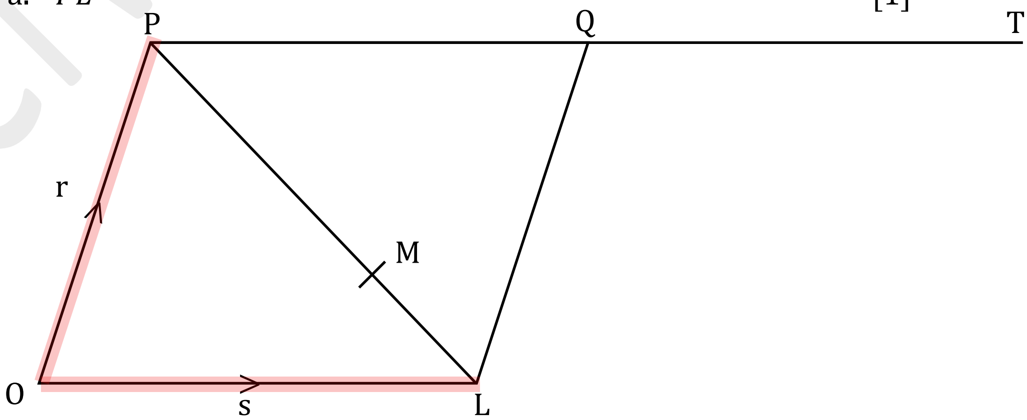
T is the point such that $\overrightarrow{PQ} = \overrightarrow{QT}$.

The point M divides PL in the ratio 2:1.



(i) Find, in terms of **r** and **s**,

a. $\overrightarrow{PL}$ [1]

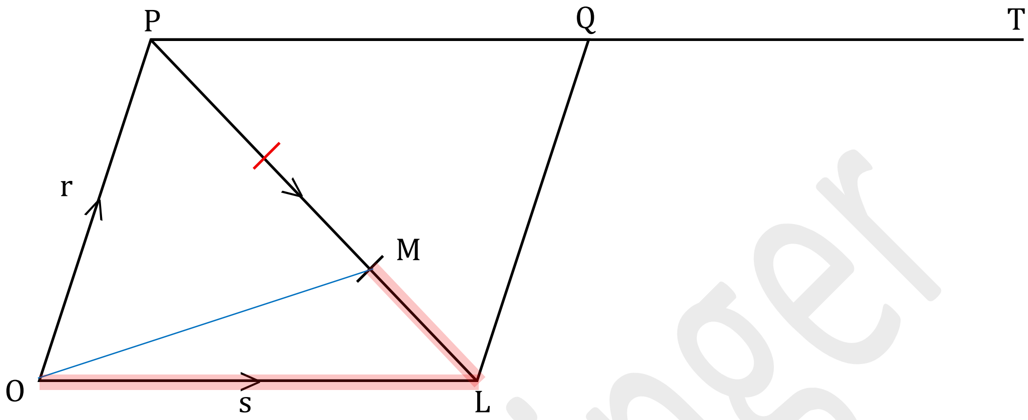


Using the triangle law,

$\overrightarrow{PL} = \overrightarrow{OL} - \overrightarrow{OP}$

$= s - r$

b. $\overrightarrow{OM}$ [2]



$\overrightarrow{PL} = s - r$

The point M divides PL in the ratio 2:1.

$$\overrightarrow{LM} = -\frac{1}{3}\overrightarrow{PL}$$

$$= -\frac{1}{3}(s - r)$$

Using the triangle law,

$$\overrightarrow{OM} = \overrightarrow{OL} + \overrightarrow{LM}$$

$$= \boldsymbol{s} + \left(-\frac{1}{3}(\boldsymbol{s} - \boldsymbol{r})\right)$$

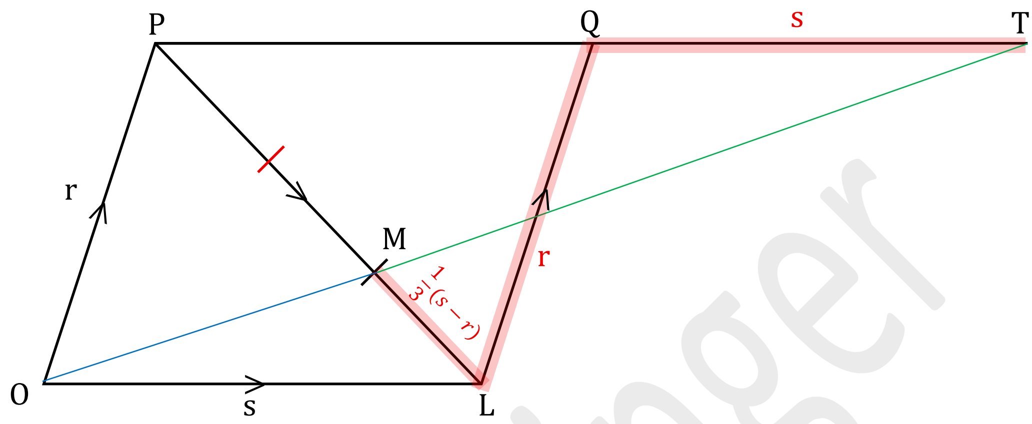
$$= \boldsymbol{s} + \left(-\frac{1}{3}\boldsymbol{s} + \frac{1}{3}\boldsymbol{r}\right)$$

$$= \boldsymbol{s} - \frac{1}{3}\boldsymbol{s} + \frac{1}{3}\boldsymbol{r}$$

$$= \frac{2}{3}\boldsymbol{s} + \frac{1}{3}\boldsymbol{r}$$

$$= \frac{1}{3}(2\boldsymbol{s} + \boldsymbol{r})$$

(ii) Prove that the points O, M and T are collinear. [3]



$$\overrightarrow{MT} = \overrightarrow{ML} + \overrightarrow{LQ} + \overrightarrow{QT}$$

$$= \frac{1}{3}(\boldsymbol{s} - \boldsymbol{r}) + \boldsymbol{r} + \boldsymbol{s}$$

$$= \frac{1}{3}\boldsymbol{s} - \frac{1}{3}\boldsymbol{r} + \boldsymbol{r} + \boldsymbol{s}$$

$$= \frac{1}{3}\boldsymbol{s} + \boldsymbol{s} + \boldsymbol{r} - \frac{1}{3}\boldsymbol{r}$$

$$= \frac{4}{3}\boldsymbol{s} + \frac{2}{3}\boldsymbol{r}$$

$$= \frac{2}{3}(2\boldsymbol{s} + \boldsymbol{r})$$

Since $\overrightarrow{OM} = \frac{1}{3}(2\boldsymbol{s} + \boldsymbol{r})$ and $\overrightarrow{MT} = \frac{2}{3}(2\boldsymbol{s} + \boldsymbol{r})$, we can say that

$2\overrightarrow{OM} = \overrightarrow{MT}$

Therefore, they are parallel with a scalar factor of 2.

Since they share a common point, M, they lie on the same line.

$\therefore$ O, M and T are collinear.

(b) Three matrices, A, B and C, are such that

$$A = \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 0 & 2 \\ 3 & -1 & 7 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} 4 & -1 & 2 \\ 7 & 3 & -5 \end{pmatrix}$$

(i) Find the matrix $AB + C$. [3]

$$AB = \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} 4 & 0 & 2 \\ 3 & -1 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 12+6 & 0+(-2) & 6+14 \\ 20+12 & 0+(-4) & 10+28 \end{pmatrix}$$

$$= \begin{pmatrix} 18 & -2 & 20 \\ 32 & -4 & 38 \end{pmatrix}$$

$$AB + C = \begin{pmatrix} 18 & -2 & 20 \\ 32 & -4 & 38 \end{pmatrix} + \begin{pmatrix} 4 & -1 & 2 \\ 7 & 3 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 22 & -3 & 22 \\ 39 & -1 & 33 \end{pmatrix}$$

(ii) Find A^{-1}, the inverse of A. [2]

$A = \begin{pmatrix} 3 & 2 \\ 5 & 4 \end{pmatrix}$ which is of the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\det A = ad - bc$

$= (3)(4) - (2)(5)$

$= 12 - 10$

$= 2$

$$adj\ A = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -2 \\ -5 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det A} \times Adj\ A$$

$$= \frac{1}{2}\begin{pmatrix} 4 & -2 \\ -5 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -1 \\ -\frac{5}{2} & \frac{3}{2} \end{pmatrix}$$

(iii) Write down the 2×2 matrix that represents the matrix product AA^{-1}.

[1]

$$AA^{-1} = I$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\therefore$ The 2×2 matrix that represents the matrix product AA^{-1} is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Total: 12 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.