CSEC Mathematics

June 2019 – Paper 2

Solutions

SECTION I

Answer ALL questions.

All working must be clearly shown.

1. (a) Using a calculator or otherwise, evaluate EACH of the following,

(i) $\dfrac{2\frac{1}{4} - 1\frac{3}{5}}{3}$ [2]

$$\frac{2\frac{1}{4}-1\frac{3}{5}}{3} = \left(\frac{9}{4} - \frac{8}{5}\right) \div 3$$

$$= \left(\frac{5(9)-4(8)}{20}\right) \div \frac{3}{1}$$

$$= \left(\frac{45-32}{20}\right) \div \frac{3}{1}$$

$$= \frac{13}{20} \times \frac{1}{3}$$

$$= \frac{13}{60}$$

(ii) 2.14 sin 75°, giving your answer to 2 decimal places [1]

Using a calculator,

$2.14 \sin 75° = 2.07$ (to 2 decimal places)

(b) Irma's take-home pay is \$4 320 per fortnight (every two weeks). Each fortnight Irma's pay is allocated to the following table.

Item	**Amount Allocated**
Rent	$\$x$
Food	\$629
Other living expenses	$\$2x$
Savings	\$1 750
Total	**\$4 320**

(i) What is Irma's **annual** take-home pay? (Assume she works 52 weeks in any given year.) [1]

Rate of pay = \$4, 320 per fortnight (every 2 weeks)

There are 52 weeks in one year.

Number of fortnights in 1 year $= \frac{52}{2}$

$= 26$ fortnights

Now,

Irma's annual take-home pay $= 26 \times \$4320$

$= \$112\ 320$

∴ Irma's annual take-home pay is \$112 320.

(ii) Determine the amount of money that Irma allocates for rent each month. [3]

$$x + 629 + 2x + 1750 = 4320$$

$$x + 2x + 2379 = 4320$$

$$3x = 4320 - 2379$$

$$3x = 1941$$

$$x = \frac{1941}{3}$$

$$x = 647$$

The allocation for rent per fortnight = \$647

There are two fortnights per month.

So, Irma's rent per month = \$647 × 2

= \$1294

∴ Irma allocates \$1294 for rent each month.

(iii) All of Irma's savings is used to pay her son's university tuition cost, which is $150 000.

If Irma's pay remains the same and she saves the same amount each month, what is the MINIMUM number of years that she must work in order to save enough money to cover her son's tuition cost? [2]

$$\text{Tuition cost} = \$150,000$$

$$\text{Savings per fortnight} = \$1,750$$

There are 52 weeks in 1 year which is equivalent to 26 fortnights.

$$\text{Total savings per year} = \$1,750 \times 26$$

$$= \$45,500$$

$$\text{Number of years to pay off} = \frac{\$150,000}{\$45,500}$$

$$= 3.30 \quad \text{(to 2 decimal places)}$$

Since the number of years has to be a whole number, then the number of years to pay off is 4 years.

$\therefore$ She must work 4 years in order to save enough money to cover her son's tuition cost.

Total: 9 marks

2. (a) Simplify completely

(i) $3p^2 \times 4p^5$ [1]

$$3p^2 \times 4p^5 = 12p^{2+5}$$

$$= 12p^7$$

(ii) $\frac{3x}{4y^3} \div \frac{21x^2}{20y^2}$ [2]

$$\frac{3x}{4y^3} \div \frac{21x^2}{20y^2}$$

$$= \frac{3x}{4y^3} \times \frac{20y^2}{21x^2}$$

$$= \frac{60xy^2}{84x^2y^3}$$

$$= \frac{5xy^2}{7x^2y^3}$$

$$= \frac{5}{7xy}$$

(b) Solve the equation

$$\frac{3}{7x-1} + \frac{1}{x} = 0$$ [3]

$$\frac{3}{7x-1} + \frac{1}{x} = 0$$

$$\frac{3(x)+1(7x-1)}{x(7x-1)} = 0$$

$$\frac{3x+7x-1}{x(7x-1)} = 0$$

$$\frac{10x-1}{x(7x-1)} = 0$$

$$10x - 1 = 0$$

$$10x = 1$$

$$x = \frac{1}{10}$$

(c) When a number, x, is multiplied by 2, the result is squared to give a new number, y.

(i) Express y in terms of x. [1]

This can be expressed as:

$$(x \times 2)^2 = y$$

$$(2x)^2 = y$$

$$y = 4x^2$$

(ii) Determine the two values of x that satisfy the equation $y = x$ AND the equation derived in (c)(i). [2]

$y = x$ → Equation 1

$y = 4x^2$ → Equation 2

Equating both equations gives:

$$4x^2 = x$$

$$4x^2 - x = 0$$

$$x(4x - 1) = 0$$

Either $x = 0$ or $4x - 1 = 0$

$$4x = 1$$

$$x = \frac{1}{4}$$

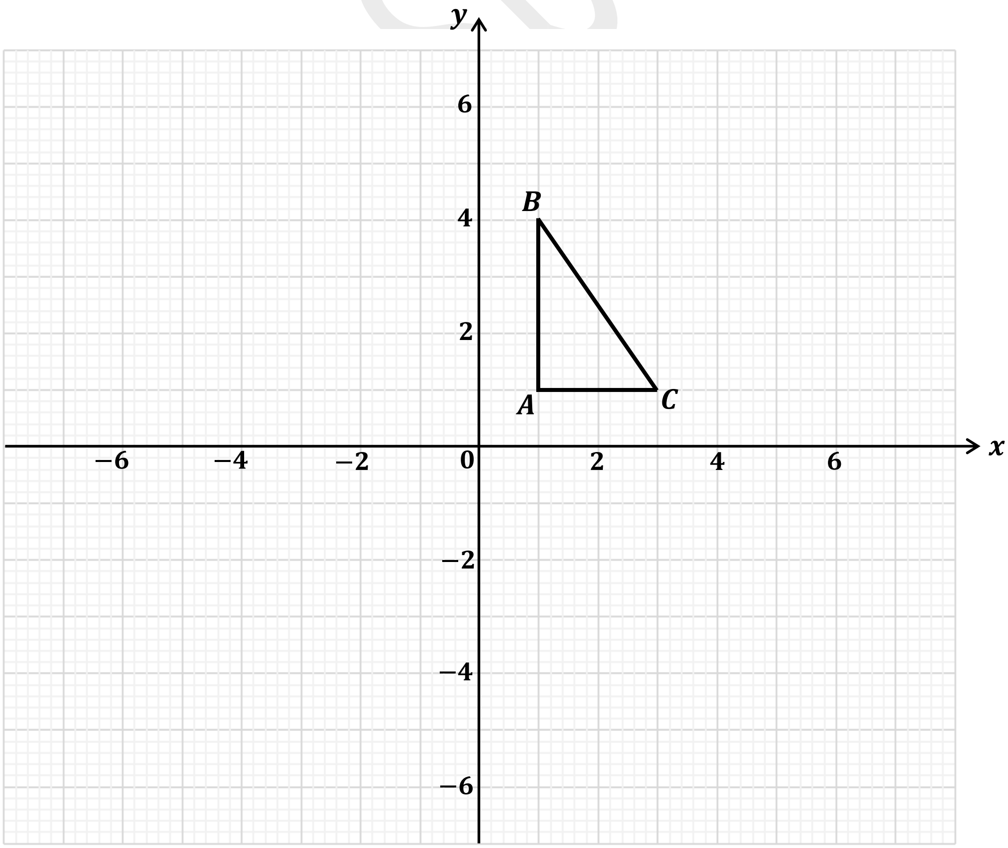
$\therefore x = 0$ or $x = \frac{1}{4}$

Total: 9 marks

3. (a) Using a ruler, a pencil and a pair of compasses, construct the triangle NLM, in which $LM = 12$ cm, $\angle MLN = 30°$ and $\angle LMN = 90°$. [4]

(Credit will be given for clearly drawn construction lines.)

(b) Triangle ABC with vertices $A(1, 1)$, $B(1, 4)$ and $C(3, 1)$ is shown on the diagram below.



ΔABC is mapped onto ΔLMN by a reflection in the x-axis followed by a reflection in the y-axis.

(i) On the diagram, draw and label ΔLMN. [2]

(ii) Describe fully a single transformation that maps ΔABC onto ΔLMN. [2]

(iii) State the 2×2 matrix for the transformation that maps ΔABC onto ΔLMN. [1]

Total: 9 marks

4. (a) The quantity P varies inversely as the square of V.

(i) Using the letters P, V and k, form an **equation** connecting the quantities P and V. [1]

P varies inversely as the square of V.

$P \propto \frac{1}{V^2}$

$\therefore P = \frac{k}{V^2}$

(ii) Given that $V = 3$ when $P = 4$, determine the positive value of V when $P = 1$. [2]

$P = \frac{k}{V^2}$

Substituting $V = 3$ and $P = 4$ gives,

$4 = \frac{k}{3^2}$

$4 = \frac{k}{9}$

$k = 9 \times 4$

$k = 36$

So, we have $P = \frac{36}{V^2}$.

When $P = 1$,

$1 = \frac{36}{V^2}$

$V^2 = 36$

$V = \sqrt{36}$

$V = 6$

(b) (i) Given that x is a real number, solve the inequality [2]

$$-7 < 3x + 5 \leq 7$$

$-7 < 3x + 5 \leq 7$

$-7 - 5 < 3x \leq 7 - 5$

$-12 < 3x \leq 2$

$\frac{-12}{3} < x \leq \frac{2}{3}$

$-4 < x \leq \frac{2}{3}$

(ii) Represent your answer in (b)(i) on the number line shown below. [1]

Inequality: $-4 < x \leq \frac{2}{3}$

The inequality is represented on the number line below:

-4 $\frac{2}{3}$

(c) The equation of a straight line is given as

$$\frac{x}{3} + \frac{y}{7} = 1$$

This line crosses the y-axis at Q.

(i) Determine the coordinates of Q. [1]

$\frac{x}{3} + \frac{y}{7} = 1$

The line crosses the y-axis at $x = 0$.

When $x = 0$,

$\frac{0}{3} + \frac{y}{7} = 1$

$0 + \frac{y}{7} = 1$

$\frac{y}{7} = 1$

$y = 1 \times 7$

$y = 7$

$\therefore$ Coordinates of $Q = (0,7)$

(ii) What is the gradient of this line? [2]

$\frac{x}{3} + \frac{y}{7} = 1$

Multiplying by the LCM $= 21$ of the denominators gives:

$7x + 3y = 21$

$3y = -7x + 21$

$y = -\frac{7}{3}x + \frac{21}{3}$

$y = -\frac{7}{3}x + 7$ which is in the form $y = mx + c$,

where $m = -\frac{7}{3}$ and $c = 7$.

$\therefore$ The gradient of the line is $m = -\frac{7}{3}$.

Total: 9 marks

5. The cumulative frequency distribution of the volume of petrol needed to fill the tanks of 150 different vehicles is shown below.

Volume (litres)	Cumulative Frequency
11 – 20	24
21 – 30	59
31 – 40	101
41 – 50	129
51 – 60	150

(a) For the class 21 – 30, determine the

(i) lower class boundary [1]

The lower class boundary for 21 – 30 is 20.5.

(ii) class width [1]

Class width = Upper Class Boundary − Lower Class Boundary

$= 30.5 - 20.5$

$= 10$ vehicles

(b) How many vehicles were recorded in the class 31 − 40? [1]

Number of vehicles recorded $= 101 - 59$

$= 42$ vehicles

(c) A vehicles is chosen at random from the 150 vehicles. What is the probability that the volume of petrol needed to fill its tank is **more** than 50.5 litres? **Leave your answer as a fraction.** [2]

$$P(need > 50.5\ litres\ to\ fill\ tank) = \frac{Number\ of\ Desired\ Outcomes}{Total\ Number\ of\ Outcomes}$$

$$= \frac{150-129}{150}$$

$$= \frac{21}{150}$$

$$= \frac{7}{50}$$

(d) Byron estimates the median amount of petrol to be 43.5 litres. Explain why Byron's estimate is INCORRECT. [1]

Byron estimates the median amount of petrol to be 43.5 litres.

Bryon's estimate is wrong because the median would be given at the 75th vehicle (half the cumulative frequency) which would be found in the interval 31-40.

(e) On the partially labelled grid below construct a histogram to represent the distribution of the volume of petrol needed to fill the tanks of the 150 vehicles. [3]

Total: 9 marks

6. (a) The scale on a map is 1: 25 000.

(i) Determine the actual distance, in km, represented by 0.5 cm on the map. [2]

The scale is 1 : 25 000.

1 cm on the map represents 25 000 cm on the ground.

$\therefore 1\ cm = 25\ 000\ cm$

$0.5\ cm = 0.5 \times 25\ 000$

$= 12\ 500\ cm$

Now,

$100\ 000\ cm = 1\ km$

$1\ cm = \frac{1}{100\ 000}\ km$

$12\ 500\ cm = \frac{1}{100\ 000} \times 12\ 500$

$= 0.125\ km$

$\therefore$ The actual distance represented by $0.5\ cm$ on the map is $0.125\ km$.

(ii) Calculate the actual area, in km^2, represented by $2.25\ cm^2$ on the map. [3]

$1\ cm$ on the map $= 25\ 000\ cm$ on the ground

$1\ cm^2$ on the map $= (25\ 000)^2\ cm^2$

$= 25\ 000 \times 25\ 000$

$= 625\ 000\ 000\ cm^2$ on the ground

We need to represent $2.25\ cm^2$ on the map.

$2.25\ cm^2$ on the map $= 2.25 \times 625\ 000\ 000\ cm^2$

$= 1\ 406\ 250\ 000\ cm^2$ on the ground

Now, we need to represent this value in km^2.

$1\ km = 100\ 000\ cm$

$1\ km^2 = (100\ 000)^2\ cm^2$

$$(100\ 000)^2\ cm^2 = 1\ km^2$$

$$1\ cm^2 = \frac{1}{(100\ 000)^2}\ km^2$$

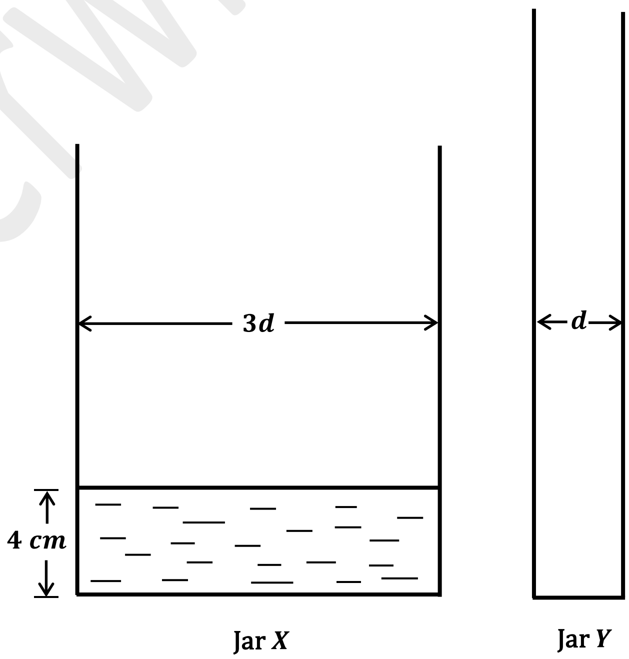
$$1\ 406\ 250\ 000\ cm^2 = 1\ 406\ 250\ 000 \times \frac{1}{(100\ 000)^2}$$

$$= 0.140625\ km^2$$

$\therefore$ The actual area represented by 2.25 cm^2 on the map is 0.140625 km^2.

(b) The diagram below (**not drawn to scale**) shows the cross-section of two cylindrical jars, Jar X and Jar Y. The diameters of Jar X and Jar Y are $3d\ cm$ and $d\ cm$ respectively.

Initially, Jar Y is empty and Jar X contains water to a height (depth) of 4 cm.



(i) Determine, in terms of π and d, the volume of water in Jar X. [2]

Volume of water in $X = \pi r^2 h$

Volume of water in $X = \pi \times \left(\frac{3d}{2}\right)^2 \times 4$

Volume of water in $X = \pi \times \frac{9d^2}{4} \times 4$

Volume of water in $X = 9\pi d^2\ cm^3$

$\therefore$ The volume of water in Jar X is $9\pi d^2\ cm^3$.

(ii) If all the water from Jar X is now poured into Jar Y, calculate the height it will reach. [2]

Volume of water in $Y = \pi r^2 h$

$= \pi \times \left(\frac{d}{2}\right)^2 \times h$

$= \pi \times \frac{d^2}{4} \times h$

$= \frac{\pi d^2 h}{4}\ cm^3$

Hence,

$9\pi d^2 = \frac{\pi d^2 h}{4}$

$9\pi d^2 = \frac{\pi d^2 h}{4}$

$9 = \frac{h}{4}$

$h = 9 \times 4$

$h = 36\ cm$

$\therefore$ The height of the water in Jar Y is $36\ cm$.

Total: 9 marks

7. (a) The nth term, T_n, of a sequence is given by

$T_n = 3n^2 - 2$

(i) Show that the first term of the sequence is 1. [1]

$T_n = 3n^2 - 2$

When $n = 1$,

$T_1 = 3(1)^2 - 2$

$= 3 - 2$

$= 1$

$\therefore$ The first term of the sequence is 1.

(ii) What is the third term of the sequence? [1]

$T_n = 3n^2 - 2$

When $n = 3$,

$T_3 = 3(3)^2 - 2$

$= 3(9) - 2$

$= 27 - 2$

$= 25$

$\therefore$ The third term of the sequence is 25.

(iii) Given that $T_n = 145$, what is the value of n? [3]

$T_n = 145$

So, we have

$3n^2 - 2 = 145$

$3n^2 = 145 + 2$

$3n^2 = 147$

$n^2 = \frac{147}{3}$

$n^2 = 49$

$n = 7$ since n is a positive integer

$\therefore$ The value of $n = 7$.

(b) The first 8 terms of another sequence with n^{th} term, $U(n)$, are

1, 1, 2, 3, 5, 8, 13, 21

where

$U(1) = 1$

$U(2) = 1$ and

$U(n) = U(n-1) + U(n-2)$ for $n \geq 3$.

For example, the fifth and seventh terms are

$U(5) = U(4) + U(3) = 3 + 2 = 5$

$U(7) = U(6) + U(5) = 8 + 5 = 13$

(i) Write down the next two terms in the sequence, that is, $U(9)$ and $U(10)$. [2]

The first 8 terms of the sequence are 1, 1, 2, 3, 5, 8, 13, 21.

$U(1) = 1$

$U(2) = 1$

$U(3) = 2$

$U(4) = 3$

$U(5) = 5$

$U(6) = 8$

$U(7) = 13$

$U(8) = 21$

Each term is the sum of the two terms that came before it (from the 3rd term onwards).

This is Fibonacci's sequence.

$U(9) = U(8) + U(7)$

$= 21 + 13$

$= 34$

$U(10) = U(9) + U(8)$

$= 34 + 21$

$= 55$

(ii) Which term in the sequence is the sum of $U(18)$ and $U(19)$? [1]

The term in the sequence which is the sum of $U(18)$ and $U(19)$ is $U(20)$.

Any term is the sum of the two terms that came immediately before it.

(iii) Show that $U(20) - U(19) = U(19) - U(17)$. [2]

$U(n) = U(n-1) + U(n-2)$

So,

$U(20) - U(19) = U(18)$ and $U(19) - U(17) = U(18)$

$\therefore U(20) - U(19) = U(19) - U(17).$

Q.E.D.

Total: 10 marks

SECTION II

Answer ALL questions.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. (a) The functions f and g are defined by

$f(x) = \frac{9}{2x+1}$ and $g(x) = x - 3.$

(i) State a value of x that CANNOT be in the domain of f. [1]

This is the value of x when the denominator is equal to zero.

$2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

$\therefore$ The value of x that cannot be in the domain of f is $x = -\frac{1}{2}$.

(ii) Find, in its simplest form, expressions for

(a) $fg(x)$ [2]

$$fg(x) = f[g(x)]$$

$$= \frac{9}{2(x-3)+1}$$

$$= \frac{9}{2x-6+1}$$

$$= \frac{9}{2x-5}$$

(b) $f^{-1}(x)$ [2]

$f(x) = \frac{9}{2x+1}$

Let $y = f(x)$.

$y = \frac{9}{2x+1}$

Interchange the variables x and y.

$x = \frac{9}{2y+1}$

Make y the subject.

$x(2y + 1) = 9$

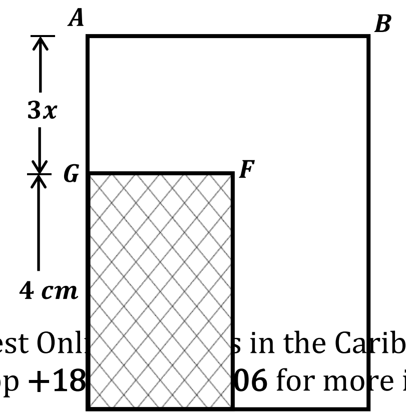
$2xy + x = 9$

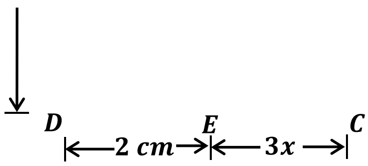
$2xy = 9 - x$

$y = \frac{9-x}{2x}$

$\therefore f^{-1}(x) = \frac{9-x}{2x}$

(b) The diagram below shows two rectangles, $ABCD$ and $GFED$. $ABCD$ has an area of $44\ cm^2$. $GFED$ has sides $4\ cm$ and $2\ cm$. $AG = EC = 3x\ cm$.





(i) By writing an expression for the area of rectangle $ABCD$, show that $x^2 + 2x - 4 = 0$. [3]

Length of DC, $l = 4 + 3x$

Length of AD, $b = 2 + 3x$

Area of $ABCD = l \times b$

$= (4 + 3x)(2 + 3x)$

$= 4(2 + 3x) + 3x(2 + 3x)$

$= 8 + 12x + 6x + 9x^2$

$= 9x^2 + 18x + 8$

Given that area is $44\ cm^2$, then

$9x^2 + 18x + 8 = 44$

$9x^2 + 18x - 36 = 0$

$x^2 + 2x - 4 = 0$

Q.E.D.

(ii) Calculate, to 3 decimal places, the value of x. [2]

$x^2 + 2x - 4 = 0$ which is in the form $ax^2 + bx + c = 0$,

where $a = 1, b = 2, c = -4$.

Using the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4+16}}{2}$$

$$= \frac{-2 \pm \sqrt{20}}{2}$$

Either $x = \frac{-2-\sqrt{20}}{2}$ or $x = \frac{-2-\sqrt{20}}{2}$

$= -3.236$ (to 3 d.p.) $= 1.236$ (to 3 d.p.)

$\therefore x = -3.236$ and $x = 1.236$ (to 3 decimal places)

(iii) Calculate the perimeter of the UNSHADED region. [2]

Perimeter $= (3x) + (3x + 2) + (3x + 4) + (3x) + (4) + (2)$

$= 12x + 12$

$= 12(1.23605) + 12$ (since x is positive)

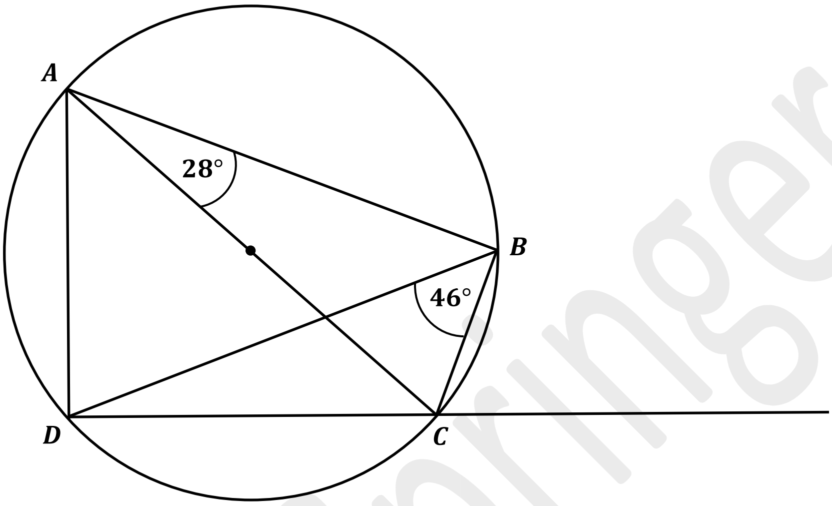
$= 26.8326$

$= 26.833\ cm$ (correct to 3 decimal places)

Total: 12 marks

GEOMETRY AND TRIGONOMETRY

9. (a) The diagram below shows a circle where AC is a diameter. B and D are two other points on the circle and DCE is a straight line. Angle $CAB = 28°$ and $\angle DBC = 46°$.



Calculate the value of each of the following angles. Show detailed working where necessary and give a reason to support your answers.

(i) $\angle DBA$ [2]

Reason:

The angle in a semi-circle is right-angle.

So, $A\hat{B}C = 90°$.

$A\hat{B}D = A\hat{B}C - D\hat{B}C$

$= 90° - 46°$

$= 44°$

(ii) $\angle DAC$ [2]

Reason:

The angles subtended by a chord (DC) at the circumference of a circle

$(D\hat{B}C$ and $D\hat{A}C)$ and standing on the same arc are equal.

$\therefore D\hat{A}C = D\hat{B}C$

$= 46°$

(iii) $\angle BCE$ [2]

Reason:

The angles subtended by a chord (BC) at the circumference of a circle

$(B\hat{A}C$ and $B\widehat{D}C)$ and standing on the same arc are equal.

So, we have,

$B\widehat{D}C = B\hat{A}C$

$= 28°$

The exterior angle of a triangle is equal to the sum of the interior

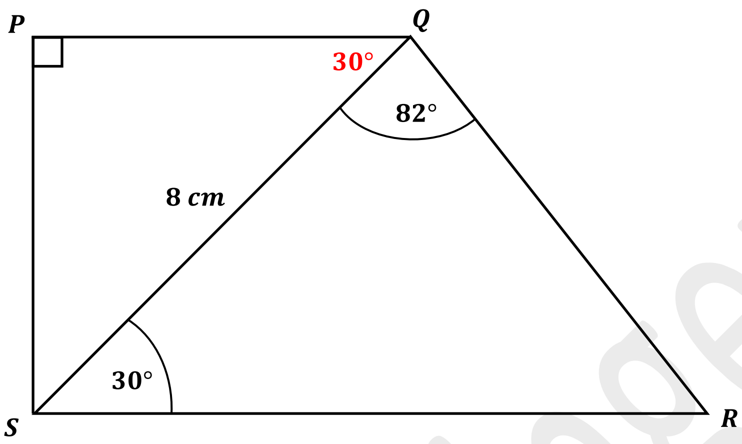
opposite angles.

$B\hat{C}E = 28° + 46°$

$= 74°$

(b) The diagram below shows a quadrilateral $PQRS$ where PQ and SR are parallel,

$SQ = 8$ cm, $\angle SPQ = 90°$, $\angle SQR = 82°$ and $\angle QSR = 30°$.



Determine

(i) the length PS [2]

Note that $\angle PQS$ is equal to $\angle RSQ$ since they are alternating angles.

$\angle PQS = \angle RSQ$

$= 30°$

Now,

$\sin PQS = \frac{PS}{QS}$

$\sin 30° = \frac{PS}{8}$

$PS = 8 \sin 30°$

$PS = 4\ cm$

$\therefore$ The length of $PS = 4\ cm$.

(ii) the length PQ [1]

Using Pythagoras' Theorem,

$QS^2 = PQ^2 + PS^2$

$\therefore PQ^2 = QS^2 - PS^2$

$= 8^2 - 4^2$

$= 64 - 16$

$= 48$

$PQ = \sqrt{48}$

$= 6.93\ cm$ (to 3 significant figures)

$\therefore$ The length of PQ is $6.93\ cm$.

(iii) the area of $PQRS$ [3]

$\angle QRS = 180° - (82° + 30°)$

$\angle QRS = 180° - 112°$

$\angle QRS = 68°$

Using the sine rule to find RS:

$\frac{RS}{\sin RQS} = \frac{QS}{\sin QRS}$

$\frac{RS}{\sin 82°} = \frac{8}{\sin 68°}$

$RS = \frac{8 \times \sin 82^o}{\sin 68^o}$

$RS = 8.54\ cm$ (to 3 significant figures)

Now,

Area of a trapezium, $PQRS = \frac{1}{2}(PQ + RS) \times PS$

$= \frac{1}{2}(6.93 + 8.54) \times 4$

$= 30.94\ cm^2$

$\therefore$ The area of $PQRS$ is $30.94\ cm^2$.

Total: 12 marks

VECTORS AND MATRICES

10. (a)(i)(a) Find the matrix product

$$\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix}\begin{pmatrix} k \\ 5 \end{pmatrix}.$$ [2]

$$\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix}\begin{pmatrix} k \\ 5 \end{pmatrix} = \begin{pmatrix} (-1 \times k) + (3 \times 5) \\ (4 \times k) + (h \times 5) \end{pmatrix}$$

$$\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix}\begin{pmatrix} k \\ 5 \end{pmatrix} = \begin{pmatrix} -k + 15 \\ 4k + 5h \end{pmatrix}$$

(b) Hence, find the values of h and k that satisfy the matrix equation

$$\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix}\begin{pmatrix} k \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$ [2]

$$\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix}\begin{pmatrix} k \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -k + 15 \\ 4k + 5h \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Equating corresponding entries:

$-k + 15 = 0$

$k = 15$

$4k + 5h = 0$

$4(15) + 5h = 0$

$60 + 5h = 0$

$5h = -60$

$$h = \frac{-60}{5}$$

$$h = -12$$

$\therefore h = -12$ and $k = 15$

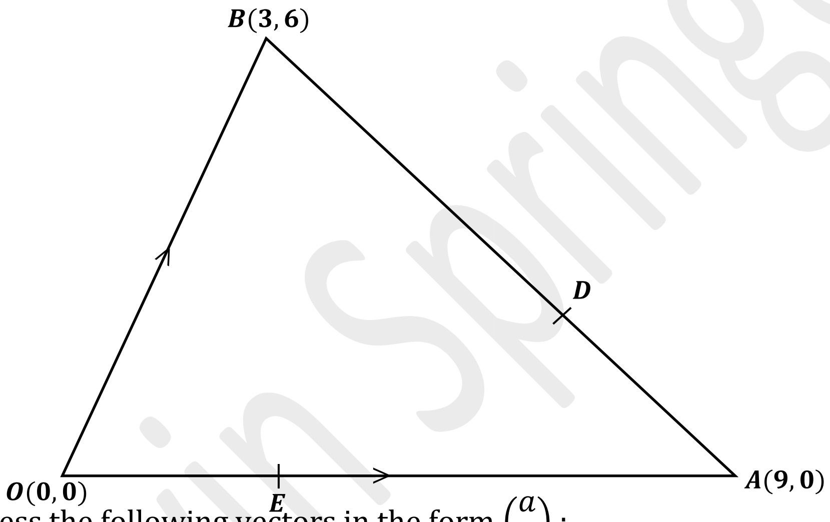
(ii) Using a matrix method, solve the simultaneous equations

$$2x + 3y = 5$$

$$-5x + y = 13$$ [3]

(b) Relative to the origin $O(0,0)$, the position vector of the points A and B are

$OA = \begin{pmatrix} 9 \\ 0 \end{pmatrix}$ and $OB = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$ respectively. The points D and E are on AB and OA respectively and are such that $AD = \frac{1}{3}AB$ and $OE = \frac{1}{3}OA$. The following diagram illustrates this information.



Express the following vectors in the form $\begin{pmatrix} a \\ b \end{pmatrix}$:

(i) $\overrightarrow{AB}$ [1]

(ii) $\overrightarrow{OD}$ [2]

(iii) $\overrightarrow{BE}$ [2]

Total: 12 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.