CSEC Mathematics

January 2011 – Paper 2

Solutions

SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Calculate the exact value of

(i) $(5.8^2 + 1.02) \times 2.5$ [2]

Using a calculator,

$(5.8^2 + 1.02) \times 2.5 = 86.65$

(ii) $\frac{2\frac{4}{9}}{4\frac{2}{3}} - \frac{3}{7}$ [3]

$$\frac{2\frac{4}{9}}{4\frac{2}{3}} - \frac{3}{7} = \left(2\frac{4}{9} \div 4\frac{2}{3}\right) - \frac{3}{7}$$

$$= \left(\frac{22}{9} \div \frac{14}{3}\right) - \frac{3}{7}$$

$$= \left(\frac{22}{9} \times \frac{3}{14}\right) - \frac{3}{7}$$

$$= \frac{11}{21} - \frac{3}{7}$$

$$= \frac{11-9}{21}$$

$$= \frac{2}{21}$$

$\therefore \frac{2\frac{4}{9}}{4\frac{2}{3}} - \frac{3}{7} = \frac{2}{21}$

(b) A company pays its employees a basic wage of $9.50 per hour for a 40-hour week.

(i) Calculate the basic weekly wage for ONE employee. [1]

Basic weekly wage = Basic hourly rate × Number of basic hours

= $9.50 × 40

= $380

∴ The basic weekly wage for one employee is $380.

Overtime is paid at a rate of time and a half.

(ii) Calculate the overtime wage for an employee who works 6 hours overtime in a certain week. [2]

Overtime rate = 1.5 × Basic rate

= 1.5 × $9.50

= $14.25

Now,

Overtime wage = Overtime hourly rate × Number of overtime hours

= $14.25 × 6

= $85.50

∴ The overtime weekly wage for one employee is $85.50.

In a certain week, the company paid its 30 employees a total of $12 084.00 in basic and overtime wages. Calculate for that week:

(iii) The TOTAL paid in overtime wages [2]

Basic wage for 1 employee = $380

Basic wage for 30 employees = $380 × 30

= $11 400

Overtime wages = Total wages − Basic wages

= $12 084 − $11 400

= $684

∴ The total paid in overtime wages is $684.

(iv) The TOTAL number of overtime hours worked by employees [1]

$$\text{Overtime hours} = \frac{\text{Overtime wages}}{\text{Overtime rate}}$$

$$= \frac{684}{14.25}$$

$$= 48 \text{ hours}$$

∴ The total number of overtime hours worked by employees is 48 hours.

Total: 11 marks

2. (a) Simplify

$$\frac{2x}{5} - \frac{x}{3}$$

expressing your answer as a single fraction. [2]

$$\frac{2x}{5} - \frac{x}{3}$$

$$= \frac{3(2x) - 5(x)}{15}$$

$$= \frac{6x - 5x}{15}$$

$$= \frac{x}{15}$$

(b) Factorise completely

$a^2b + 2ab$ [1]

$a^2b + 2ab = ab(a + 2)$

(c) Express p as the subject of the formula

$q = \frac{p^2 - r}{t}$ [3]

$$q = \frac{p^2 - r}{t}$$

$$qt = p^2 - r$$

$$qt + r = p^2$$

$$\sqrt{qt + r} = p$$

$\therefore p = \sqrt{qt + r}$

(d) The students in a class sell donuts to raise money for their school project. The donuts are sold in small and large boxes. The number of donuts in EACH type of box is given in the table below:

Type of Box	Number of Donuts per Box
Small box	x
Large box	$2x + 3$

The students sold 8 small boxes and 5 large boxes in all.

(i) Write an expression in terms of x to represent the TOTAL number of donuts sold. [2]

Total number of donuts sold $= 8(x) + 5(2x + 3)$

$= 8x + 10x + 15$

$= 18x + 15$

$\therefore$ An expression to represent the TOTAL number of donuts sold is $18x + 15$.

(ii) The total number of donuts sold was 195. Calculate the number of donuts in a

(a) small box

Total number of donuts sold $= 18x + 15$

$= 195$

So, we have,

$18x + 15 = 195$

$18x = 195 - 15$

$18x = 180$

$x = \frac{180}{18}$

$x = 10$

$\therefore$ The number of donuts in a small box is 10 donuts.

(b) large box [4]

Number of donuts in a large box $= 2(10) + 3$

$= 20 + 3$

$= 23$

$\therefore$ The number of donuts in a large box is 23 donuts.

Total: 12 marks

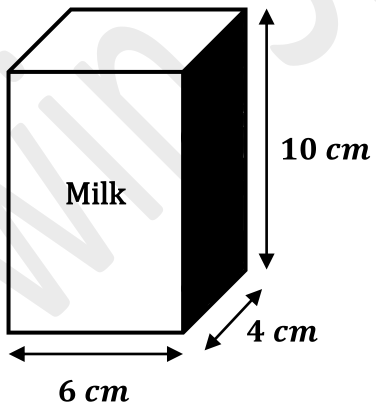
3. (a) Simplify the expression

$7p^5q^3 \times 2p^2q$ [2]

$7p^5q^3 \times 2p^2q = 14p^{5+2}q^{3+1}$

$= 14p^7q^4$

(b) Fresh Farms Dairy sells milk in cartons in the shape of a cuboid with internal dimensions 6 cm by 4 cm by 10 cm.



(i) Calculate, in cm^3, the volume of milk in EACH carton. [2]

Volume of milk $= l \times b \times h$

$= 6 \times 4 \times 10$

$= 240\ cm^3$

$\therefore$ The volume of milk in each carton is 240 cm^3.

(ii) A recipe for making ice-cream requires 3 litres of milk. How many cartons of milk should be bought to make the ice-cream? [3]

$1 \text{ litre} = 1000\ cm^3$

$3 \text{ litres} = 3 \times 1000$

$= 3000\ cm^3$

$\text{Number of cartons} = \frac{3000}{240}$

$= 12.5 \text{ cartons}$

We need to round up since we cannot buy 0.5 of a carton.

∴ The number of cartons of milk that should be bought to make the ice-cream is 13 cartons.

(iii) One carton of milk is poured into a cylindrical cup of internal diameter 5 cm. What is the height of milk in the cup? Give your answer to 3 significant figures.

[Use $\pi = 3.14$] [4]

Let h be the height of the milk in the cup.

The volume of milk in each carton is 240 cm^3.

Recall that $V = \pi r^2 h$.

So, we have,

$$V = \pi r^2 h$$

$$240 = 3.14 \times (2.5)^2 \times h$$

$$h = \frac{240}{3.14 \times (2.5)^2}$$

$$h = 12.2\ cm \qquad \text{(to 3 significant figures)}$$

∴ The height of the milk in the cup is 12.2 cm.

Total: 11 marks

4. (a) The Universal set, U, is given as

$U = \{\text{Whole numbers from 1 to 12}\}$

H is a subset of U, such that $H = \{\text{Odd numbers between 4 and 12}\}$.

(i) List the members of the set H. [1]

$H = \{\text{Odd numbers between 4 and 12}\}$

$= \{5, 7, 9, 11\}$

$\therefore$ The members of the set H are $\{5, 7, 9, 11\}$.

J is a subset of U, such that $J = \{\text{Prime numbers}\}$.

(ii) List the members of the set J. [1]

$U = \{\text{Whole numbers from 1 to 12}\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

J is a subset of U.

$J = \{\text{Prime numbers}\}$

$= \{2, 3, 5, 7, 11\}$

∴ The members of the set J are $\{2, 3, 5, 7, 11\}$.

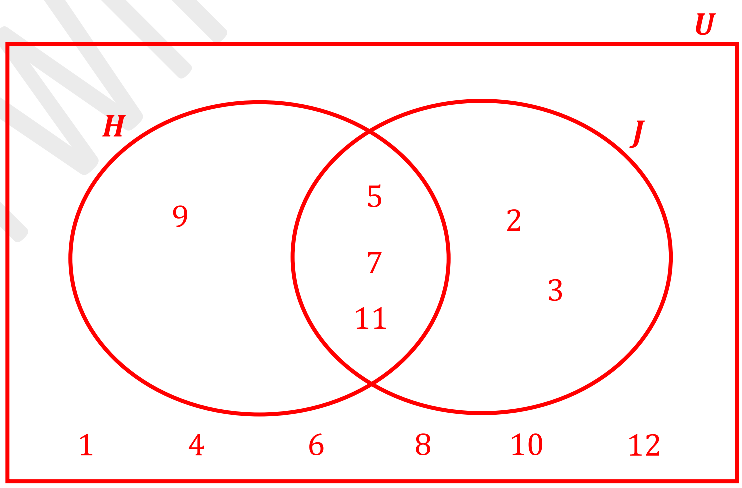
(iii) Draw a Venn diagram to represent the sets U, H and J, showing ALL the elements in the subsets. [3]

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$H = \{5, 7, 9, 11\}$

$J = \{2, 3, 5, 7, 11\}$

A Venn diagram to represent the sets U, H and J, showing all the elements in the subsets is shown below:

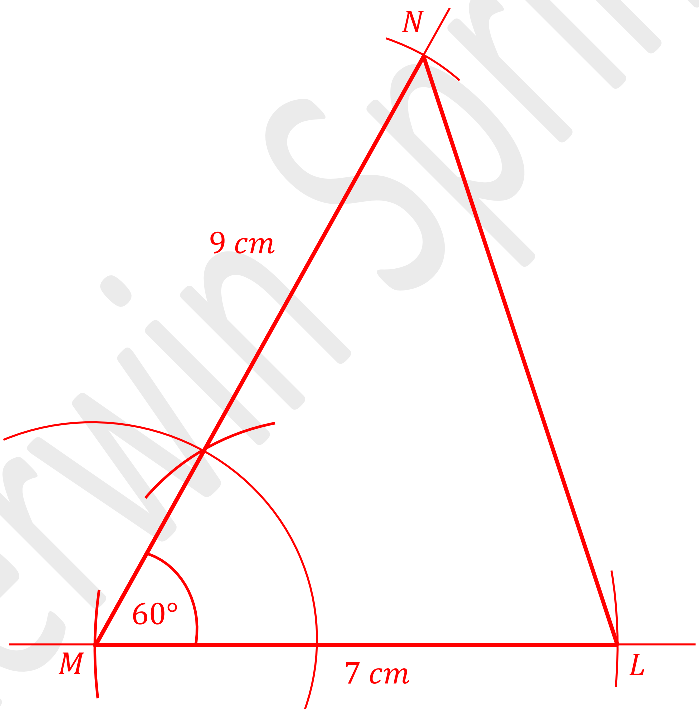


(b)(i) **Using a a ruler, a pencil and a pair of compasses**, construct triangle LMN

with angle $LMN = 60°, MN = 9\ cm$ and $LM = 7\ cm$.

ALL construction lines MUST be clearly shown. [4]

The construction of triangle LMN is shown below:

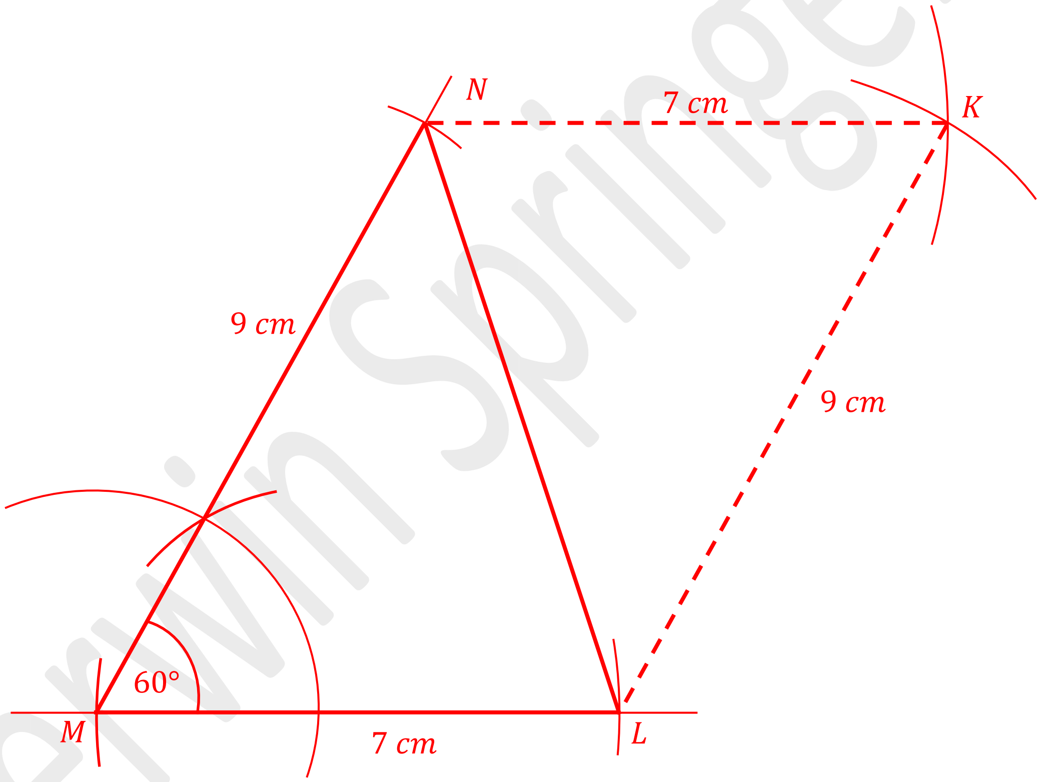


(ii) Measure and state the size of $\angle MNL$. [1]

Using a protractor, $\angle MNL = 48°$.

(iii) On the diagram, show the point, K, such that $KLMN$ is a parallelogram. [2]

The construction of parallelogram $KLMN$ is shown below:



Total: 12 marks

5. (a) The equation of a straight line is given by:

$3y = 2x - 6$

Determine

(i) the gradient of the line [2]

$3y = 2x - 6$

$(\div 3)$

$y = \frac{2}{3}x - 2$ which is in the form $y = mx + c$,

where $m = \frac{2}{3}$ and $c = -2$.

$\therefore$ The gradient of the line is $m = \frac{2}{3}$.

(ii) the equation of the line which is perpendicular to $3y = 2x - 6$, and passes through the point $(4, 7)$. [3]

Gradient of line $= \frac{2}{3}$

Gradient of perpendicular line $= -1 \div \frac{2}{3}$

$= -1 \times \frac{3}{2}$

$= -\frac{3}{2}$

Substituting $m = -\frac{3}{2}$ and point $(4, 7)$ into $y - y_1 = m(x - x_1)$ gives:

$y - 7 = -\frac{3}{2}(x - 4)$

$y - 7 = -\frac{3}{2}x + 6$

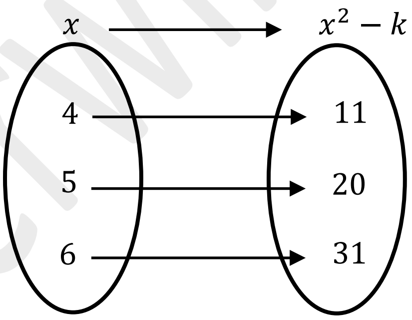
$y = -\frac{3}{2}x + 6 + 7$

$y = -\frac{3}{2}x + 13$

$\therefore$ The gradient of the perpendicular line is $y = -\frac{3}{2}x + 13$.

(b) The arrow diagram shown below represents the relation $f: x \rightarrow x^2 - k$,

where $x \in \{3, 4, 5, 6, 7, 8, 9, 10\}$.



Calculate the value of

(i) k [2]

Let $f(x) = x^2 - k$.

From the arrow diagram, when $f(4) = 11$.

$f(4) = (4)^2 - k$

$11 = 16 - k$

$k = 16 - 11$

$k = 5$

(ii) $f(3)$ [2]

$f(x) = x^2 - 5$

When $x = 3$,

$f(3) = (3)^2 - 5$

$= 9 - 5$

$= 4$

(iii) x when $f(x) = 95$ [2]

When $f(x) = 95$,

$x^2 - 5 = 95$

$x^2 = 95 + 5$

$x^2 - 100 = 0$

$(x + 10)(x - 10) = 0$

Either $x + 10 = 0$ or $x - 10 = 0$

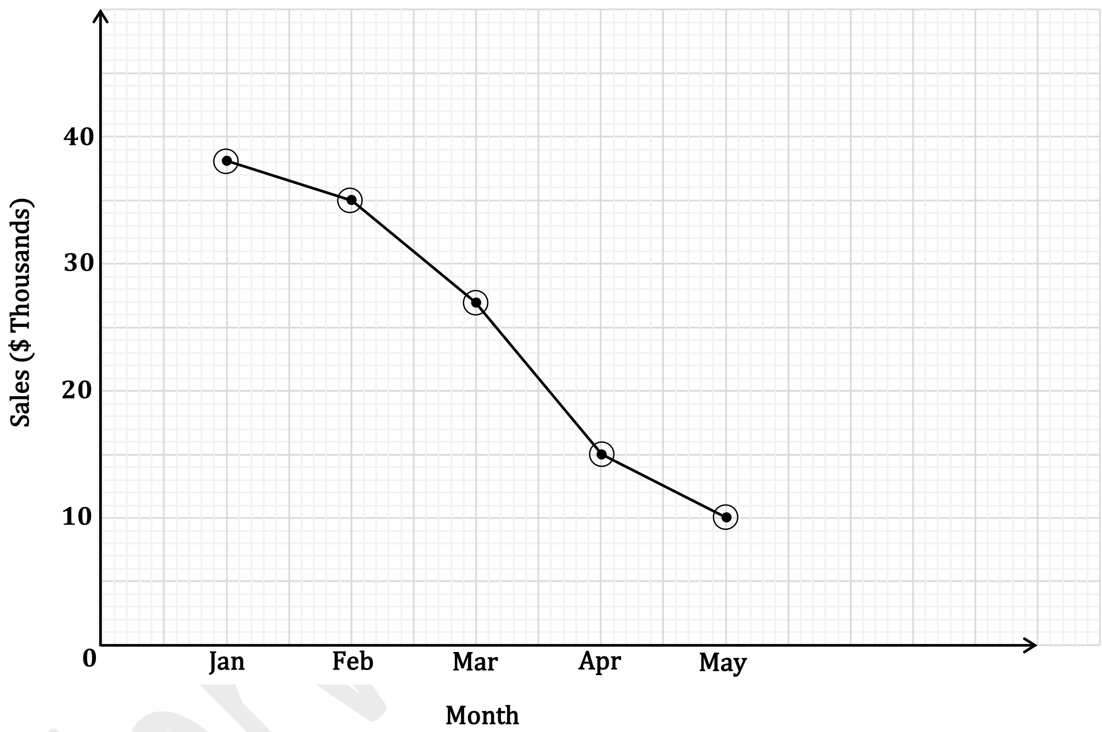
$x = -10$ $x = 10$

Since $x \in \{3, 4, 5, 6, 7, 8, 9, 10\}$, then $x = 10$.

$\therefore$ The value of x when $f(x) = 95$ is $x = 10$.

Total: 11 marks

6. The line graph below shows the monthly sales, in thousands of dollars, at a school cafeteria for the period January to May 2010.



(i) Copy and complete the table below to show the sales for EACH month. [3]

Month	Jan	Feb	Mar	Apr	May
Sales in $Thousands	38	35	27	15	10

From graph,

Sales in $Thousands for February = 35

Sales in $Thousands for April = 15

Sales in $Thousands for May = 10

(ii) Between which TWO consecutive months was there the GREATEST **decrease** in sales? [1]

Difference in sales between Jan and Feb = $38 000 − $35 000

= $3 000

Difference in sales between Feb and Mar = $35 000 − $27 000

= $8 000

Difference in sales between Mar and Apr = $27 000 − $15 000

= $12 000

Difference in sales between Apr and May = $15 000 − $10 000

= $5 000

Alternatively, from the graph, the steepes gradient occurs between March and April.

∴ The greatest decrease in sales occurred between the months of March and April.

(iii) Calculate the mean monthly sales for the period January to May 2010. [3]

Mean monthly sales $= \frac{\sum x}{n}$

$= \frac{\$38\ 000+\$35\ 000+\$27\ 000+\$15\ 000+\$10\ 000}{5}$

$= \frac{\$125\ 000}{5}$

$= \$25\ 000$

$\therefore$ The mean monthly sales for the period January to May 2010 is $25 000.

(iv) The TOTAL sales for the period January to June was $150 000. Calculate the sales, in dollars, for the month of June. [2]

Total sales in June = Sales from Jan to June − Sales from Jan to May

$= \$150\ 000 - \$125\ 000$

$= \$25\ 000$

$\therefore$ The sales for the month of June is $25 000.

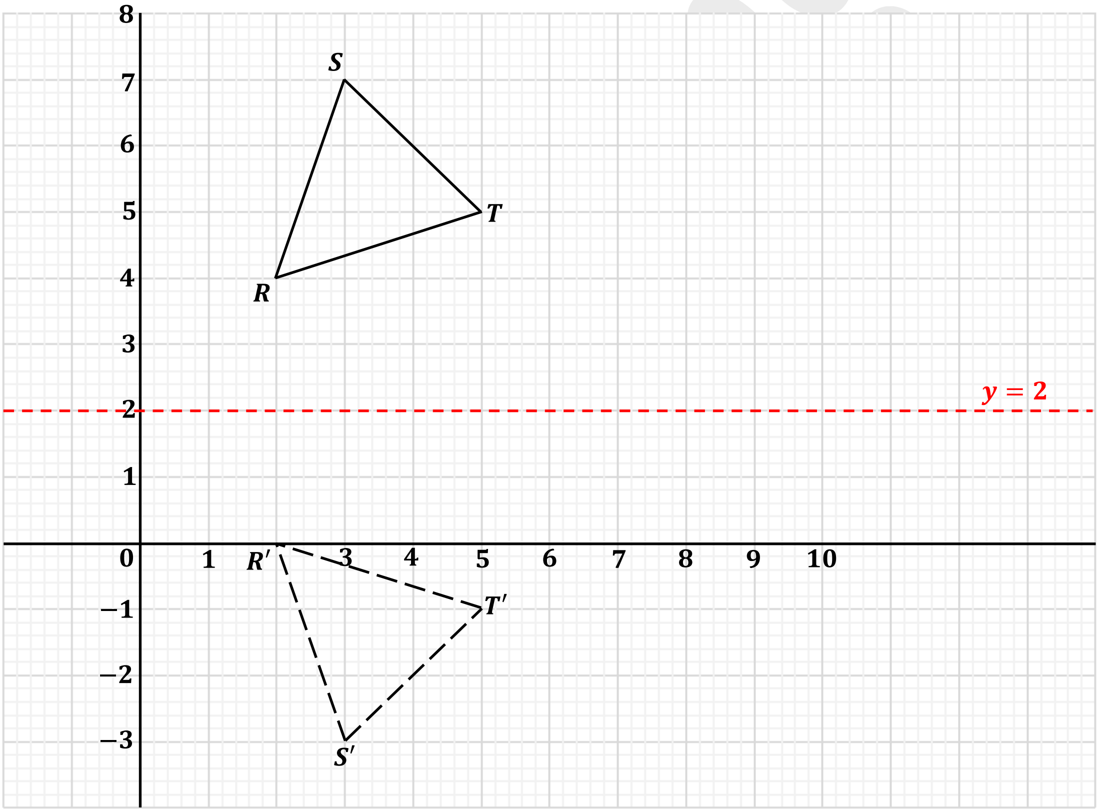
(v) Comment on how the sales in June compared with the sales in the previous five months. [2]

The sales in June shows a significant increase from April and May, but still fell below than the sales in January, February and March.

Total: 11 marks

7. **An answer sheet is provided for this question.**

The diagram below shows triangle RST and its image $R'S'T'$ after a transformation.



(i) Write down the coordinates of R and R'. [2]

From graph.

Coordinates of $R = (2, 4)$

Coordinates of $R' = (2, 0)$

(ii) Describe completely the transformation which maps triangle RST onto triangle $R'S'T'$. [3]

The line $y = 2$ is shown in the diagram above.

The transformation which maps triangle RST onto triangle $R'S'T'$ is a reflection in the line $y = 2$.

(iii) RST undergoes an enlargement, centre, $(0, 4)$, scale factor, 3. [7]

(a) **On your answer sheet**, draw triangle $R"S"T"$, the image of triangle RST under the enlargement.

The coordinates of triangle RST are $R(2, 4)$, $S(3, 7)$ and $T(5, 5)$.

The centre of rotation is $(0, 4)$.

To make the centre of rotation, $O(0, 0)$, we need to translate the vertices of triangle RST under the translation $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$.

So, we have,

$$R = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 2 + 0 \\ 4 + (-4) \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$S = \begin{pmatrix} 3 \\ 7 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 3 + 0 \\ 7 + (-4) \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

$$T = \begin{pmatrix} 5 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 5 + 0 \\ 5 + (-4) \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

The scale factor is 3 which is represented by the matrix $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.

$$R \text{ becomes} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} (3 \times 2) + (0 \times 0) \\ (0 \times 2) + (3 \times 0) \end{pmatrix}$$

$$= \begin{pmatrix} 6 + 0 \\ 0 + 0 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$

$$S \text{ becomes} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} (3 \times 3) + (0 \times 3) \\ (0 \times 3) + (3 \times 3) \end{pmatrix}$$

$$= \begin{pmatrix} 9 + 0 \\ 0 + 9 \end{pmatrix}$$

$$= \begin{pmatrix} 9 \\ 9 \end{pmatrix}$$

$$T \text{ becomes} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} (3 \times 5) + (0 \times 1) \\ (0 \times 5) + (3 \times 1) \end{pmatrix}$$

$$= \begin{pmatrix} 15 + 0 \\ 0 + 3 \end{pmatrix}$$

$$= \begin{pmatrix} 15 \\ 3 \end{pmatrix}$$

To get triangle $R"S"T"$, we need to translate these vertices under $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$.

$$R" = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 6 + 0 \\ 0 + 4 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 4 \end{pmatrix}$$

$$S" = \begin{pmatrix} 9 \\ 9 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 9 + 0 \\ 9 + 4 \end{pmatrix}$$

$$= \begin{pmatrix} 9 \\ 13 \end{pmatrix}$$

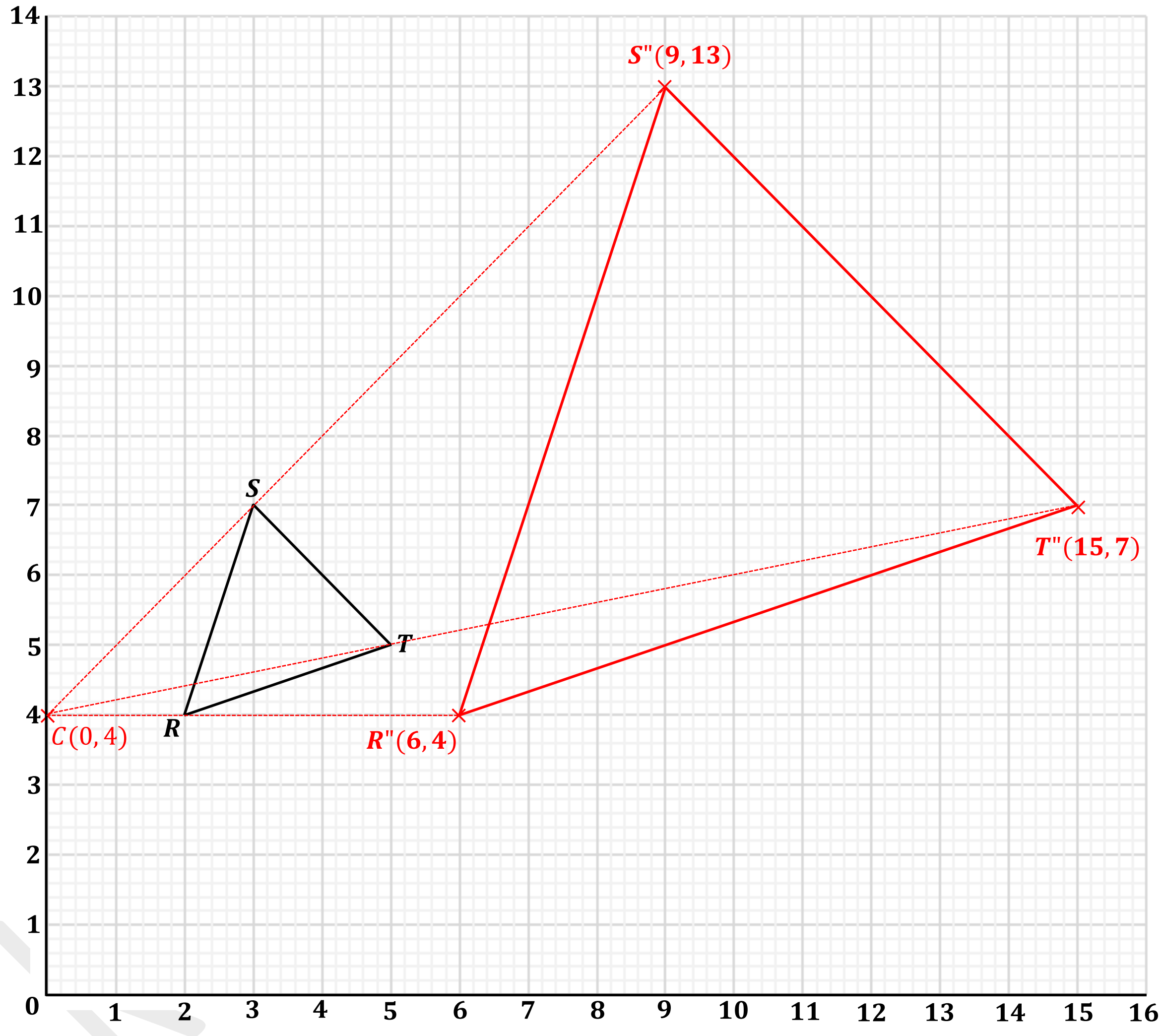
$$T" = \begin{pmatrix} 15 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 15 + 0 \\ 3 + 4 \end{pmatrix}$$

$$= \begin{pmatrix} 15 \\ 7 \end{pmatrix}$$

The triangle $R"S"T"$ has coordinates $R"(6, 4)$, $S"(9, 13)$ and $T"(15, 7)$.

The triangle $R"S"T"$ is shown below:



(b) Given that the area of triangle RST is 4 units, calculate the area of triangle $R''S''T''$.

Area of triangle $RST = 4$ units

Under an enlargement of scale factor 3, the area of image increases by the square of the scale factor.

Hence,

Area of triangle $R"S"T" = (3)^2 \times 4$

$= 36$ square units

(c) State TWO geometrical relationships between triangles RST and $R"S"T"$.

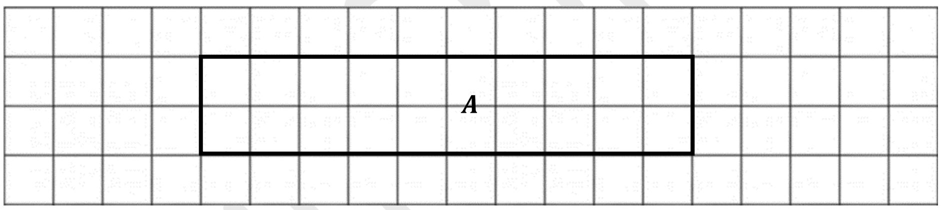
Two geometrical relationships between triangles RST and $R"S"T"$ are:

1. Triangle RST is similar to triangle $R"S"T"$. That is, $\hat{R} = \widehat{R"}$, $\hat{S} = \widehat{S"}$ and $\hat{T} = \widehat{T"}$.
2. The ratios of the corresponding sides of image and object are all the same. For example, $\frac{R"S"}{RS} = \frac{S"T"}{ST} = \frac{R"T"}{RT} = 3$ which is the scale factor.

Total: 12 marks

8. **An answer sheet is provided for this question.**

The answer sheet shows a rectangle, A, of area 20 square units and perimeter 24 units.



Use the information below to complete the table on the answer sheet, which shows the length, width, area and perimeter of rectangles *B*, *C*, *D* and *E*.

(a) **On your answer sheet,**

(i) draw and label [4]

(a) rectangle B of area 27 square units and perimeter 24 units.

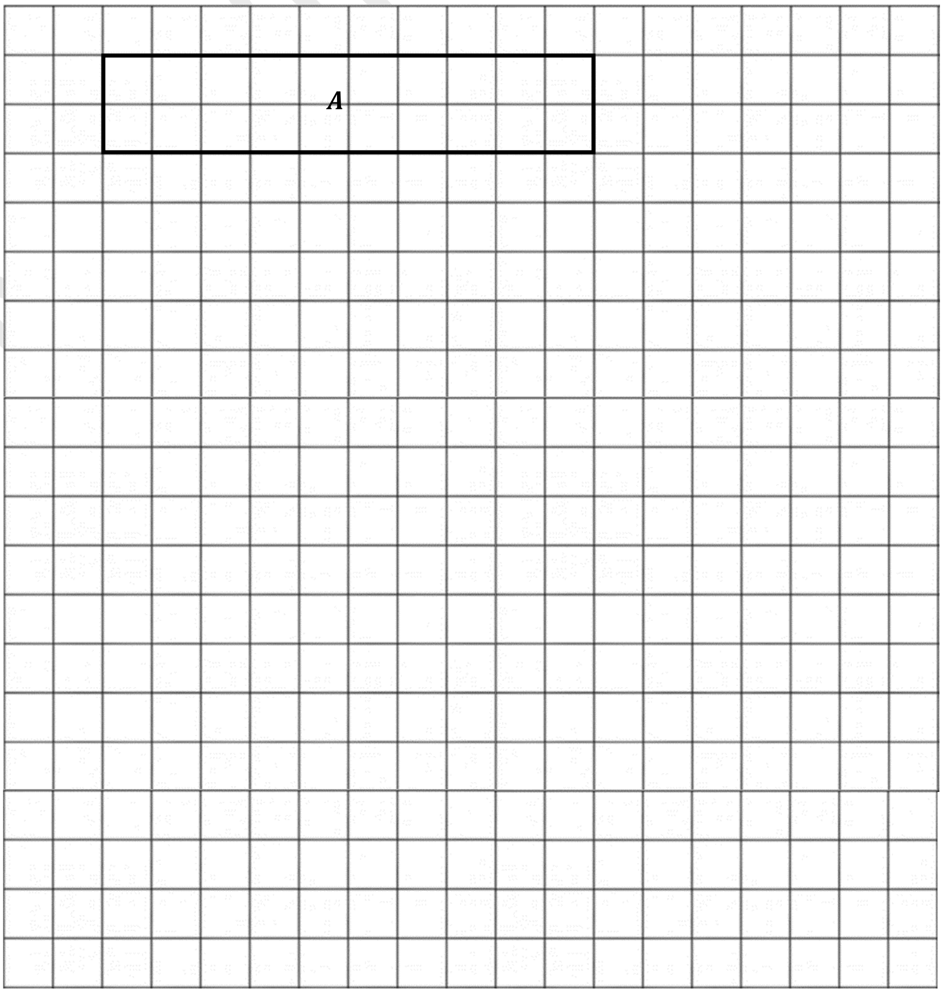
(b) rectangle C of area 32 square units and perimeter 24 units.

(ii) Complete the table to show the dimensions of rectangles B and C, drawn at (a)(i) above. [2]

(b) Rectangle D has a perimeter of 24 cm, with length, l, and width, w. If the area of the rectangle D is to be as large as possible, determine the values of l and w. In the table, write the values of l, w and the area of rectangle D. [2]

(c) Rectangle E has a perimeter of 36 cm, length, l, and width, w. If the area of the rectangle E is to be as large as possible, determine the values of l and w. In the table, write the values of l, w and the area of the rectangle E. [2]

The answer sheet is provided below:



Rectangle	Length	Width	Area (square units)	Perimeter (units)
A	10	2	20	24
B	________	________	27	24
C	________	________	32	24
D	________	________	________	24
E	________	________	________	36

Total: 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) The functions $f(x)$ and $g(x)$ are defined as:

$f(x) = \frac{2x-7}{x}$ and $g(x) = \sqrt{x+3}$

(i) Evaluate $f(5)$. [1]

(ii) Write expressions in x for

(a) $f^{-1}(x)$

(b) $gf(x)$ [6]

(b)(i) Express the quadratic function $1 - 6x - x^2$, in the form $k - a(x + h)^2$,

where a, h and k are constants. [3]

(ii) Hence state

(a) the maximum value of $1 - 6x - x^2$

(b) the equation of the axis of symmetry of the quadratic function. [2]

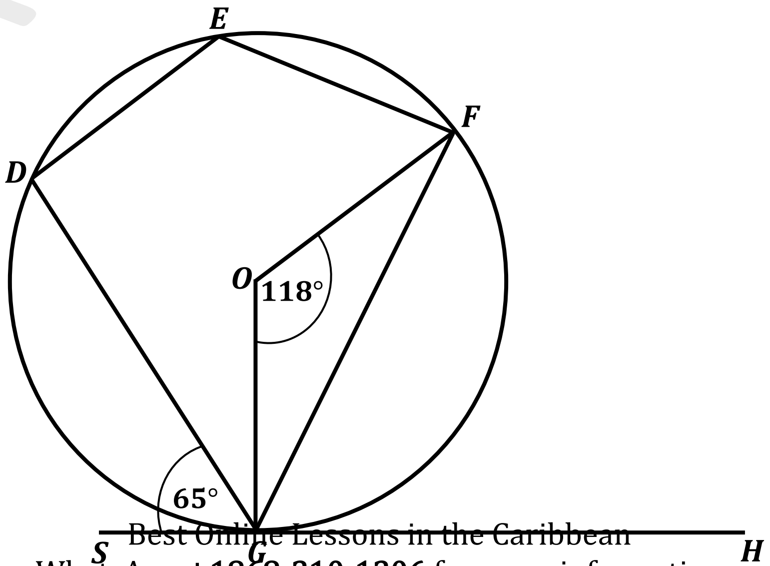
(iii) Determine the roots of $1 - 6x - x^2 = 0$, giving your answers to 2 decimal places. [3]

Total: 15 marks

MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) The diagram, below, **not drawn to scale**, shows a circle, centre, O.

SGH is a tangent to the circle, $\angle FOG = 118°$ and $\angle DGS = 65°$.



Calculate, **giving reasons for EACH step of your answer**, the measure of:

(i) $\angle OGF$ [2]

(ii) $\angle DEF$ [3]

(b) J, K and L are three sea ports. A ship began its journey at J, sailed to K, then to L and returned to J.

The bearing of K from J is 054° and L is due east of K.

$JK = 122\ m$ and $KL = 60\ km$.

(i) Draw a clearly labelled diagram to represent the above information.

Show on the diagram

(a) the north/south direction

(b) the bearing 054°

(c) the distance 122 km and 60 km [3]

(ii) Calculate

(a) the measure of angle JKL

(b) the distance JL

(c) the bearing of J from L [7]

Total: 15 marks

VECTORS AND MATRICES

11. (a) Under a matrix transformation, $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the points, V and W are mapped onto V' and W' such that:

$V(3,5) \quad \rightarrow \quad V'(5,-3)$

$W(7,2) \quad \rightarrow \quad W'(2,-7)$

(i) Determine the values of a, b, c and d. [3]

(ii) State the coordinates of Z such that $Z(x,y) \rightarrow Z'(5,1)$ under the transformation, M. [2]

(iii) Describe FULLY the geometric transformation, M. [2]

(b) $\overrightarrow{OP}$ and $\overrightarrow{OR}$ are position vectors with respect to the origin, O.

P is the point $(2, 7)$ and $\overrightarrow{PR} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

(i) Write in the form $\begin{pmatrix} a \\ b \end{pmatrix}$ the vectors:

(a) $\overrightarrow{OP}$

(b) $\overrightarrow{OR}$ [3]

(ii) A point S has coordinates $(14, -2)$.

(a) Find $\overrightarrow{RS}$.

(b) Show that P, R and S are collinear. [5]

Total: 15 marks

END OF TEST