# CSEC Mathematics

# January 2025 - Paper 2

# Solutions

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## SECTION I

**Answer ALL questions.**

**ALL working must be clearly shown.**

1. (a) Using a calculator or otherwise, determine the value of

(i) $\frac{2}{3}$ of $\left[\frac{1}{8}+\frac{5}{12}\div\frac{1}{9}\right]$, giving your answer in EXACT form. [2]

Method 1:

Using a calculator,

$\frac{2}{3}$ of $\left[\frac{1}{8}+\frac{5}{12}\div\frac{1}{9}\right]$

$$=\frac{2}{3}\times\left[\frac{1}{8}+\frac{5}{12}\div\frac{1}{9}\right]$$

$$=\frac{2}{3}\times\left[\frac{31}{8}\right]$$

$$=\frac{31}{12}\quad or\quad 2\frac{7}{12}\quad \text{(in exact form)}$$

Method 2:

$\frac{2}{3}$ of $\left[\frac{1}{8}+\frac{5}{12}\div\frac{1}{9}\right]$

$$=\frac{2}{3}\times\left[\frac{1}{8}+\left(\frac{5}{12}\div\frac{1}{9}\right)\right]$$

$$=\frac{2}{3}\times\left[\frac{1}{8}+\left(\frac{5}{\cancel{12}_{4}}\times\frac{\cancel{9}^{3}}{1}\right)\right]$$

$$=\frac{2}{3}\times\left[\frac{1}{8}+\left(\frac{15}{4}\right)\right]$$

$$=\frac{2}{3}\times\left[\frac{1+30}{8}\right]$$

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$$= \frac{2}{3} \times \left[\frac{31}{8}\right]$$

$$= \frac{\overset{1}{\cancel{2}}}{3} \times \frac{31}{\underset{4}{\cancel{8}}}$$

$$= \frac{31}{12} \quad or \quad 2\frac{7}{12} \quad \text{(in exact form)}$$

(ii) $314.2 - \frac{26082}{52164}$, giving your answer in standard form. [2]

Using a calculator,

$$314.2 - \frac{26082}{52164}$$

$$= 314.2 - \frac{1}{2}$$

$$= 314.2 - 0.5$$

$$= 313.7$$

$$= 3.137 \times 10^2 \quad \text{(in standard form)}$$

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(b) Jim packed several cases of fruit juice for sale. Each case contained 24 boxes of juice in 3 different varieties, apple, orange and pineapple, in the ratio 2:5:1 respectively.

(i) How many boxes of **pineapple** juice were packed in each case? [1]

Ratio of boxes of juices according to variety is:

Apple : Orange : Pineapple

2 : 5 : 1

$$\text{Total parts} = 2 + 5 + 1$$
$$= 8 \text{ parts}$$

$$8 \text{ parts} = 24 \text{ boxes of juice}$$

$$1 \text{ part} = \frac{24}{8}$$
$$= 3 \text{ boxes of juice}$$

Since Pineapple = 1 part, then there were 3 boxes of pineapple juice in each case.

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(ii) The profit gained from selling ALL of the boxes of pineapple juice is $35.64. Each box of pineapple juice was sold at $3.34.

a) Show that the cost price of a box of pineapple juice is $2.35. [3]

Since the question did not give the number of cases of juice sold, it becomes unsolvable.

b) Calculate the percentage profit made of the sale of the boxes of pineapple juice. [1]

$$\text{Profit} = \text{Selling Price} - \text{Cost Price}$$

$$= \$3.34 - \$2.35$$

$$= \$0.99$$

$$\text{Percentage Profit} = \frac{Profit}{Cost\ Price} \times \frac{100}{1}$$

$$= \frac{0.99}{2.35} \times \frac{100}{1}$$

$$= 42.1\% \quad \text{(to 3 significant figures)}$$

**Total: 15 marks**

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2. (a) (i) Factorize EACH of the following algebraic expressions.

a. $x^2 - 49$

$x^2 - 49$

$= x^2 - 7^2$ [Difference of two squares]

$= (x + 7)(x - 7)$

b. $x^2 + 2x - 35$

$x^2 + 2x - 35$

$= x^2 + 7x - 5x - 35$

$= x(x + 7) - 5(x + 7)$

$= (x - 5)(x + 7)$

[2]

(ii) Hence, simplify the expression

$$\frac{x^2 - 49}{x^2 + 2x - 35}$$

[1]

$$\frac{x^2 - 49}{x^2 + 2x - 35}$$

$$= \frac{\cancel{(x + 7)}(x - 7)}{(x - 5)\cancel{(x + 7)}}$$

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$$= \frac{(x-7)}{(x-5)}$$

b) Rearrange the formula shown below to make $m$ the subject.

$$s = k - m^2$$

[2]

$$s = k - m^2$$

$$m^2 = k - s$$

$$m = \sqrt{k - s}$$

(c) Lisa has $56 to buy a total of **no more than** 70 red balloons and green balloons for her party.

She buys more green balloons than red balloons but must buy at least 15 red balloons. Each red balloon costs $0.75 and each green balloon costs $0.50.

Let $x$ and $y$ represent the number of red balloons and the number of green balloons respectively. Write TWO inequalities in $x$ and $y$, other than $x \geq 0$ and $y \geq 0$, to represent the information above. [2]

Let $x$ represent the number of red balloons.

Let $y$ represent the number of green balloons.

The total number of red balloons and green balloons is no more than 70 (i.e. $\leq 70$)

Therefore, we can say that $x + y \leq 70$.

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She buys more green balloons than red balloons.

Therefore, we can say that $y > x$.

She must buy at least 15 red balloons.

Therefore $x \geq 15$.

Each red balloon costs \$0.75 and each green balloon costs \$0.50. Lisa only has \$56.

Therefore, $0.75x + 0.50y \leq 56$.

There are four possible inequalities so students can list any two:

$x + y \leq 70$

$y > x$

$x \geq 15$

$0.75x + 0.50y \leq 56$

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(d) Given that $y$ is inversely proportional to $(x-2)$ and $x=11$ when $y=9$, find the value of $y$ when $x=29$. [2]

$y$ is inversely proportional to $(x-2)$

$$y \propto \frac{1}{x-2}$$

$$y = \frac{k}{x-2}$$ where k is the constant of proportionality

When $x=11, y=9$.

$$(9) = \frac{k}{(11)-2}$$

$$9 = \frac{k}{9}$$

$$9 \times 9 = k$$

$$81 = k$$

$$k = 81$$

$$\therefore y = \frac{81}{x-2}$$

When $x = 29$,

$$y = \frac{81}{(29)-2}$$

$$y = \frac{81}{27}$$

$$y = 3$$

$\therefore$ When $x=29$ the value of $y=3$.

**Total: 9 marks**

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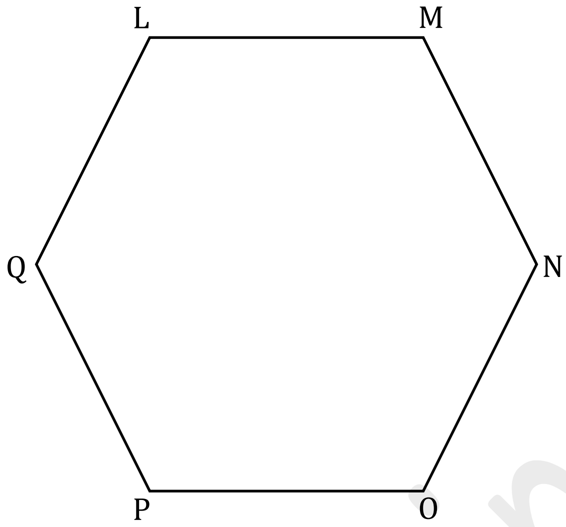
3. (a) The diagram below shows a **regular** hexagon, $LMNOPQ$, whose side is 8 $cm$.




(i) Show that the value of Angle $PQL$ is 120°. [2]

Angle $PQL$ is an interior angle.

The sum of the interior angles of a polygon with $n$ sides

$= (n - 2) \times 180°$

In a regular hexagon, $n = 6$. Therefore, the sum of interior angles in a hexagon

$= ((6) - 2) \times 180°$

$= (4) \times 180°$

$= 720°$

In a regular hexagon, the interior angles are equal.

$\therefore Angle\ PQL = \frac{720°}{6}$

$= 120°$

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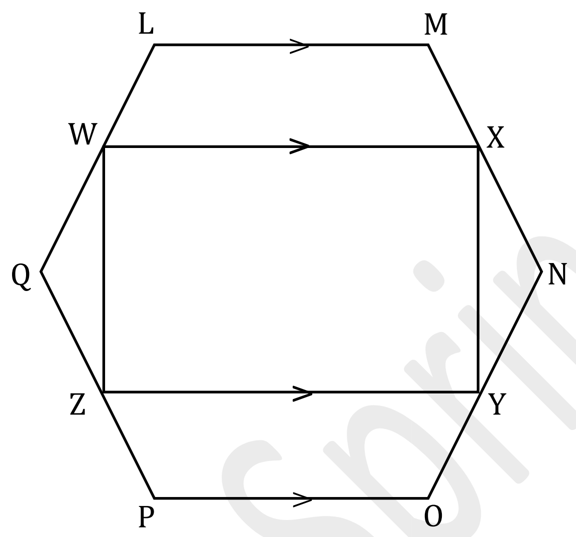
Q.E.D.

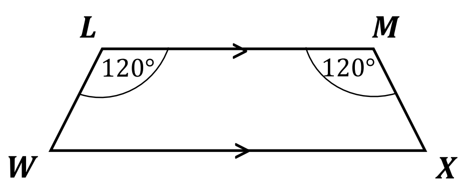
(ii) The vertices of a rectangle, $WXYZ$, touch the sides $PQ$, $QL$, $MN$ and $NO$ of the hexagon in Part (a). $ZY$ and $WX$ are parallel to $LM$ and $PO$.




Calculate the value of Angle $LWX$. [2]

In a regular hexagon, the interior angles are equal with each angle being 120°.

Looking closer, we can see that $LWXM$ is a trapezium with $M\hat{L}W = 120°$ and $X\hat{M}L = 120°$.



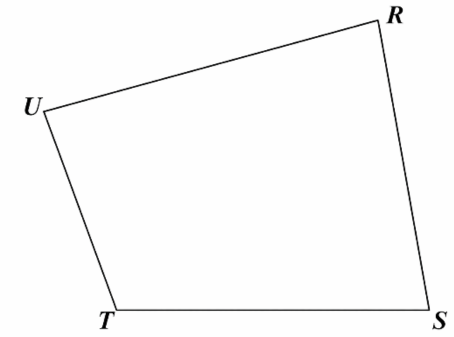

Since $M\hat{L}W$ and $L\hat{W}X$ are co-interior angles that are supplementary, then

$L\widehat{W}X = 180° - 120°$

$= 60°$

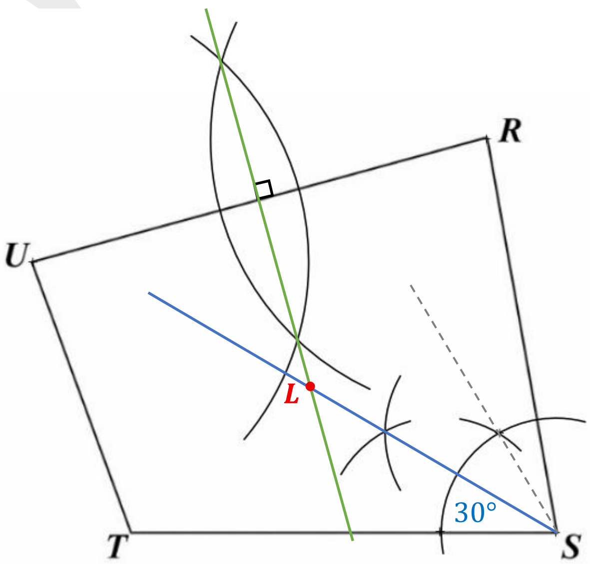
(b) **In the following parts, show all your construction lines where required.**

The field of a sports club is in the shape of a quadrilateral, $RSTU$. A scaled diagram of this field is shown below.




A lamppost is to be erected on the field at a point marked $L$, so that floodlights can be installed. The point $L$ should be located in such a way that $L$ lies on the perpendicular bisector of the line $UR$ and Angle $LST$ equals 30°.

Using a ruler and compasses only, locate the point $L$ on the field. [5]

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Total: 9 marks

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4. A line segment joins the points $C(-5, 6)$ and $D(7,2)$.

(a) Calculate the midpoint of the line segment $CD$. [2]

The points given were:

$C(-5, 6)$ and $D(7, 2)$

$x_1, y_1$ $x_2, y_2$

$$\text{Midpoint of CD} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-5 + 7}{2}, \frac{6 + 2}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{8}{2}\right)$$

$$= (1, 4)$$

(b) Find the gradient of the line segment $CD$. [2]

The points given were:

$C(-5, 6)$ and $D(7, 2)$

$x_1, y_1$ $x_2, y_2$

$$\text{Gradient of CD} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - 6}{7 - (-5)}$$

$$= \frac{-4}{12}$$

$$= -\frac{1}{3}$$

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(c) Determine the equation of the perpendicular bisector of $CD$. [3]

Gradient of CD $= -\frac{1}{3}$

Gradient of perpendicular bisector of CD $= -1 \div -\frac{1}{3}$

$= -1 \times -3$

$= 3$

Midpoint of CD $= \underset{x_1\,,\,y_1}{(1, 4)}$

We can substitute $m = 3$ and the midpoint $(1, 4)$ into $y - y_1 = m(x - x_1)$.

$y - (4) = 3(x - (1))$

$y - 4 = 3(x - 1)$

$y - 4 = 3x - 3$

$y = 3x - 3 + 4$

$y = 3x + 1$

$\therefore$ The equation of the perpendicular bisector of $CD$ is $y = 3x + 1$.

(d) Another line, $AB$, is parallel to $CD$ and passes through the point $(0, 1)$.

Write down the equation of the line $AB$. [2]

Gradient of CD $= -\frac{1}{3}$

Gradient of the parallel line AB $= -\frac{1}{3}$

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We can substitute $m = -\frac{1}{3}$ and $c = 1$ into $y = mx + c$.

$$y = -\frac{1}{3}x + 1$$

$\therefore$ The equation of the line AB is $y = -\frac{1}{3}x + 1$.

**Total: 9 marks**

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5. (a) The table below shows the marks, out of 10, that 40 students in a class gained on an essay writing test.

| Marks ($x$) | Number of Students ($f$) |
|---|---|
| 4 | 3 |
| 6 | 9 |
| 7 | 8 |
| 8 | 7 |
| 9 | 8 |
| 10 | 5 |

(i) Calculate the students' mean score on the test. [2]

$$Mean, \bar{x} = \frac{\sum fx}{\sum x}$$

$$= \frac{(4\times3)+(6\times9)+(7\times8)+(8\times7)+(9\times8)+(10\times5)}{4+6+7+8+9+10}$$

$$= \frac{12+54+56+56+72+50}{40}$$

$$= \frac{300}{40}$$

$$= 7.5 \ \ or \ \ 7\frac{1}{2} \ \ \text{marks}$$

(ii) Determine the

a) Modal mark [1]

The modal mark is 6 because the largest number of students scored 6 marks.



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b) Median mark [1]

The median mark will occur at $\frac{n+1}{2}$

$= \frac{40+1}{2}$

$= \frac{41}{2}$

$= 20.5^{th}$ value

Reading off the table, the $20^{th}$ mark is 7 and the $21^{st}$ mark is 8.

Median $= \frac{7+8}{2}$

$= \frac{15}{2}$

$= 7.5$ or $7\frac{1}{2}$ marks

(iii) Using the information in the table below, a pie chart is constructed to represent the marks students gained. [2]

| Marks $(x)$ | Number of Students $(f)$ |
|---|---|
| $3 \leq x \leq 4$ | 3 |
| $5 \leq x \leq 6$ | 9 |
| $7 \leq x \leq 8$ | 15 |
| $9 \leq x \leq 10$ | 13 |

Calculate the angle for the sector representing the interval marks,



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$5 \leq x \leq 6$, in the pie chart. [1]

Number of students who got $5 \leq x \leq 6$ marks $= 9$

Total number of students $= 3 + 9 + 15 + 13$

$= 40$

$\therefore$ The angle for the sector representing $5 \leq x \leq 6$ marks on a pie chart

$= \frac{9}{40} \times 360°$

$= 81°$

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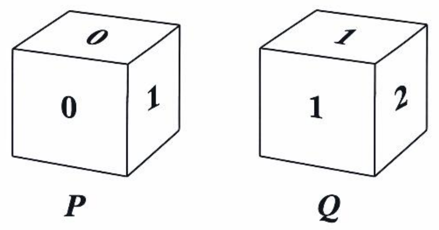
(b) The diagram below shows two fair six-sided dice, $P$ and $Q$.




The six number of Die $P$ are 0, 0, 1, 1, 2, 3

The six number of Die $Q$ are 1, 1, 1, 2, 2, 3.

When a die is rolled, the score is the number on the top face.

(i) Di $P$ is rolled once. What is the probability that the score is NOT 2? [1]

There is a total of 6 outcomes with Die $P$: 0, 0, 1, 1, 2, 3.

If the score is not 2, then the other possible 5 outcomes are: 0, 0, 1, 1, 3

Therefore,

$$\text{Probability (score is not 2)} = \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{5}{6}$$

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(ii) Die $Q$ is rolled **twice.** What is the probability that the score is 1 both times? [1]

There is a total of 6 outcomes with Die $Q$: 1, 1, 1, 2, 2, 3.

There are 3 outcomes of obtaining a 1: 1, 1, 1, 2, 2, 3.

Therefore,

$$\text{Probability (score is 1)} = \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

Rolling Die $Q$ two times are independent events.

$$\therefore \text{Probability (score is 1 both times)} = \text{P(score is 1)} \times \text{P(score is 1)}$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4}$$

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(iii) Die $Q$ is rolled 72 times. Calculate an estimate of the number of times the score is 3. [1]

There is a total of 6 outcomes with Die $Q$: 1, 1, 1, 2, 2, 3.

There is 1 outcome of obtaining a 3: 1, 1, 1, 2, 2, 3.

Therefore,

$$\text{Probability (score is 3)} = \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{1}{6}$$

If Die $Q$ is rolled 72 times, the number of times 3 is expected would be:

$$\text{Probability (score is 3 for 72 rolls)} = \frac{1}{6} \times 72$$

$$= 12 \text{ times}$$

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(iv) Each die is rolled once. The product of the scores is recorded. The sample space diagram is shown below.

| | | | | | | |
|---|---|---|---|---|---|---|
| **3** | 0 | 0 | 3 | 3 | 6 | 9 |
| **2** | 0 | 0 | 2 | 2 | 4 | 6 |
| **2** | 0 | 0 | 2 | 2 | 4 | 6 |
| **1** | 0 | 0 | 1 | 1 | 2 | 3 |
| **1** | 0 | 0 | 1 | 1 | 2 | 3 |
| **1** | 0 | 0 | 1 | 1 | 2 | 3 |
| × | **0** | **0** | **1** | **1** | **2** | **3** |

Find the probability that the product of the scores is 2 OR 3. [1]

The total number of outcomes $= 36$

Number of outcomes that result in a score of 2 or 3 $= 12$

$$\therefore \text{Probability (product is 2 OR 3)} = \frac{\text{Number of desired outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{12}{36}$$

$$= \frac{1}{3}$$

**Total: 9 marks**

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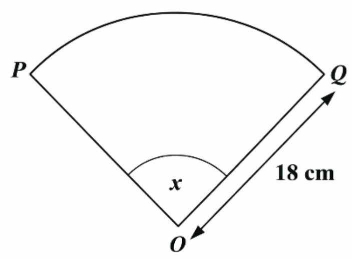
6. [In this question, use $\pi = \frac{22}{7}$]

(a) A piece of wire, 61 cm long, is bent to form a sector, as shown in the diagram below. The sector of the circle, $OPQ$, has centre $O$ and a radius of 18 $cm$.




(i) Show that the value of $x$ is approximately 80° [3]

Since the piece of wire was bent to form a sector made up of 2 radii and an arc, we can say that

Perimeter of the sector = 61 $cm$.

Length of arc, $PQ$ = Perimeter of the sector − (2 × radius)

$= 61 - (2 \times 18)$

$= 61 - (36)$

$= 25\ cm$

Substituting Length of arc, $PQ = 25\ cm$, $\pi = \frac{22}{7}$ and $r = 18\ cm$ into:

Length of arc, $PQ = \frac{x}{360°} \times 2\pi r$

$$25 = \frac{x}{360°} \times 2\left(\frac{22}{7}\right)(18)$$

$$25 = \frac{x}{360°} \times \frac{792}{7}$$

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$$25 \div \frac{792}{7} = \frac{x}{360°}$$

$$\frac{175}{792} = \frac{x}{360°}$$

$$\frac{175}{792} \times 360 = x$$

$$\frac{875}{11} = x$$

$$79.54 = x$$

$$x = 79.54$$

$$x \approx 80° \text{ (to the nearest degree)}$$

(ii) Calculate the area enclosed by the wire. [2]

Substituting $x = 80°$, $\pi = \frac{22}{7}$ and $r = 18\ cm$ into:

Area of a sector $= \frac{x}{360°} \times \pi r^2$

$$= \frac{80}{360°} \times \left(\frac{22}{7}\right)(18)^2$$

$$= \frac{2}{9} \times \left(\frac{22}{7}\right) 324$$

$$= \frac{2}{9} \times \frac{7128}{7}$$

$$= \frac{1584}{7}$$

$$\approx 226\ cm^2 \text{ (to the nearest whole number)}$$

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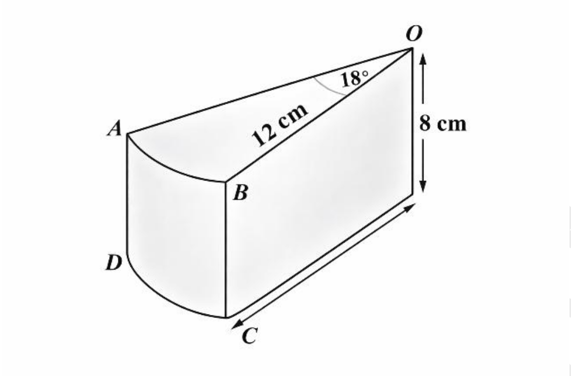
(b) A cylindrical block of cheese has a radius of 12cm and a height of 8cm. The cheese is divided into equal slices. The uniform cross-section of a slice of the cheese is a sector whose angle is 18°, as shown in the diagram below.




(i) Calculate the length of the arc $AB$. [1]

Substituting $\theta = 18°$, $\pi = \frac{22}{7}$ and $r = 12\ cm$ into

$$Length\ of\ arc\ AB = \frac{\theta}{360°} \times 2\pi r$$

$$= \frac{18°}{360°} \times 2\left(\frac{22}{7}\right)(12)$$

$$= \frac{18°}{360°} \times \frac{528}{7}$$

$$= \frac{132}{35}$$

$\approx 3.77$ cm (correct to 3 significant figures)

(ii) Determine the area of the curved face, $ABCD$. [1]

Area of $ABCD = length\ of\ arc\ AB \times height\ of\ BC$

$= 3.77 \times 8$

$= 30.16\ cm^2$

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(iii) Given that the area of $OAB$ is 22.6 $cm^2$, calculate the volume of the ENTIRE block of cheese. [2]

Volume of the given slice $= Area\ of\ OAB \times height$

$= 22.6 \times 8$

$= 180.8\ cm^3$

Number of slices in the block of cheese $= \frac{360°}{18°}$

$= 20$ slices

$\therefore$ Volume of the entire block of cheese $= 180.8 \times 20$

$= 3616\ cm^3$

**Total: 9 marks**

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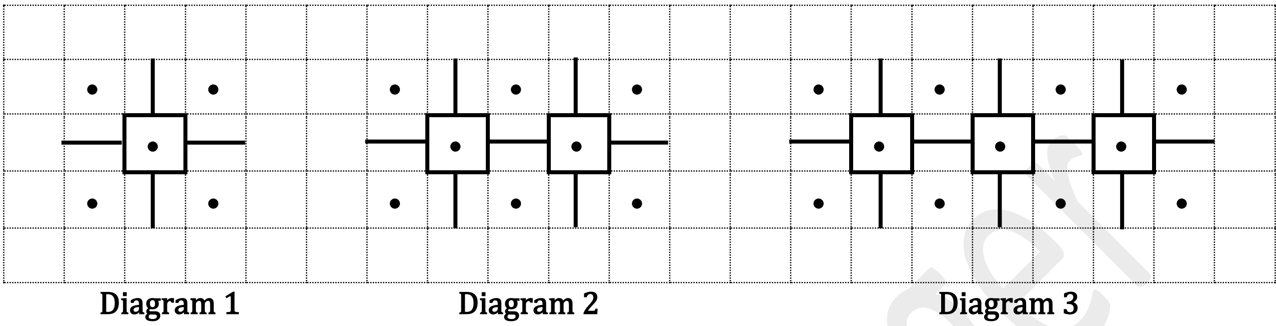
7. A sequence of patterns is made of dots and lines of unit length. Some of these lines form squares. The first three diagrams in the sequence are shown below.




(a) Add more lines and dots to the diagram below to shown Diagram 4 of the sequence. [2]

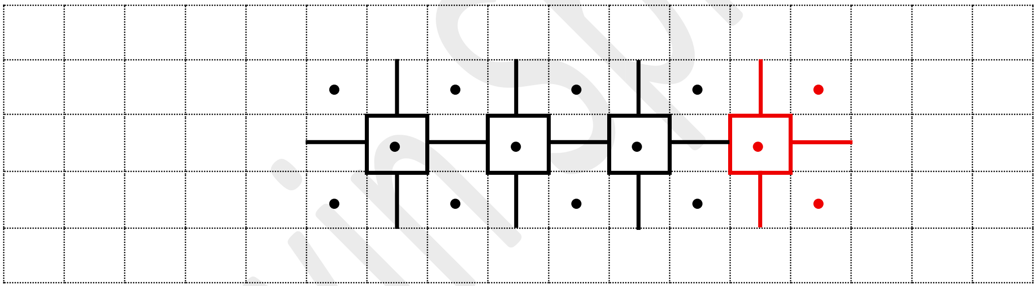


Diagram 4

Diagram 4 has been completed above.

(b) The number of dots, $D$, and the number of unit lines that form each diagram, $L$, form a pattern. The values for $D$ and $L$ for the first 3 diagrams are written in the table below. Study the pattern of numbers in each row of the table.

Complete the rows numbered (i), (ii) and (iii).

| | Diagram | Number of Dots ($D$) | Number of Lines ($L$) | |
|---|---|---|---|---|
| | 1 | 5 | 8 | |
| | 2 | 8 | 15 | |
| | 3 | 11 | 22 | |
| (i) | 4 | 14 | 29 | [2] |
| | ⋮ | ⋮ | ⋮ | |
| (ii) | 19 | 59 | 134 | [2] |
| | ⋮ | ⋮ | ⋮ | |
| (iii) | $n$ | $3n + 2$ | $7n + 1$ | [2] |

(iii) Looking at Diagrams 1, 2 and 3, we can deduce that

Number of Dots, $D = 3n + 2$

Number of Lines, $L = 7n + 1$

Explanation for $D$:

| Diagram | Number of Dots ($D$) | Number of Lines ($L$) |
|---|---|---|
| 1 | 5 | 8 |
| 2 | 8 | 15 |
| 3 | 11 | 22 |

The number of dots increase by 3.

$\therefore$ In terms of $n$, we can deduce that $D = 3n + k$ (where $k$ is a constant)

Substituting $n = 1$ and $D = 5$ into:

$D = 3n + k$

$5 = 3(1) + k$

$5 = 3 + k$

$5 - 3 = k$

$2 = k$

$k = 2$

Substituting $n = 2$ and $D = 8$ into:

$D = 3n + k$

$8 = 3(2) + k$

$8 = 6 + k$

$8 - 6 = k$

$2 = k$

$k = 2$

Substituting $n = 3$ and $D = 11$ into:

$D = 3n + k$

$11 = 3(3) + k$

$11 = 9 + k$

$11 - 9 = k$

$2 = k$

$k = 2$

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$\therefore D = 3n + 2$

Explanation for $L$:

| Diagram | Number of Dots $(D)$ | Number of Lines $(L)$ |
|---|---|---|
| 1 | 5 | 8 |
| 2 | 8 | 15 |
| 3 | 11 | 22 |

The number of lines increase by 7.

$\therefore$ In terms of $n$, we can deduce that $L = 7n + k$ (where $k$ is a constant)

Substituting $n = 1$ and $L = 8$ into:

$L = 7n + k$

$8 = 7(1) + k$

$8 = 7 + k$

$8 - 7 = k$

$1 = k$

$k = 1$

Substituting $n = 2$ and $L = 15$ into:

$L = 7n + k$

$15 = 7(2) + k$

$15 = 14 + k$

$15 - 14 = k$

$1 = k$

$k = 1$

Substituting $n = 3$ and $L = 22$ into:

$L = 7n + k$

$22 = 7(3) + k$

$22 = 21 + k$

$22 - 21 = k$

$1 = k$

$k = 1$

$\therefore\ L = 7n + 1$

(i) When $n = 4$,

$D = 3(4) + 2$

$D = 12 + 2$

$D = 14$

When $n = 4$,

$L = 7(4) + 1$

$L = 28 + 1$

$L = 29$

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(ii) When $D = 59,$

$3n + 2 = 59$

$3n = 59 - 2$

$3n = 57$

$n = \frac{57}{3}$

$n = 19$

When $n = 19,$

$L = 7(19) + 1$

$L = 133 + 1$

$L = 134$

(c) One of the diagrams in the sequence has 148 **lines**. Calculate the number of **dots** in this diagram. [2]

Substituting $L = 148$ into

$L = 7n + 1$

$148 = 7n + 1$

$148 - 1 = 7n$

$147 = 7n$



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$$\frac{147}{7} = n$$

$$21 = n$$

$$n = 21$$

When $n = 21$,

$$D = 3(21) + 2$$

$$D = 63 + 2$$

$$D = 65$$

$\therefore$ The number of dots in this diagram would be 65.

**Total: 10 marks**

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SECTION II

**Answer ALL questions.**

**ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS.**

8. (a) The functions $f$ and $g$ are defined as follows

$$f: x \to \frac{1+3x}{x-1}, \quad x \neq 1$$

$$g: x \to 5 - x$$

(i) Calculate the value of $f(-2)$. [1]

$$f(x) = \frac{1+3x}{x-1}$$

$$f(-2) = \frac{1+3(-2)}{(-2)-1}$$

$$= \frac{1-6}{-2-1}$$

$$= \frac{-5}{-3}$$

$$= \frac{5}{3}$$

(ii) Determine a simplified expression for $fg(x)$. [2]

$f(x) = \frac{1+3x}{x-1}$ $\qquad$ $g(x) = 5 - x$

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$$fg(x) = f[g(x)]$$

$$= f(5-x)$$

$$= \frac{1+3(5-x)}{(5-x)-1}$$

$$= \frac{1+15-3x}{5-x-1}$$

$$= \frac{16-3x}{5-1-x}$$

$$= \frac{16-3x}{4-x}$$

$$\therefore fg(x) = \frac{16-3x}{4-x} \quad , \quad x \neq 4$$

(iii) Derive an expression in terms of $x$ for the inverse function, $f^{-1}(x)$. [3]

$$f(x) = \frac{1+3x}{x-1}$$

Let $y = f(x)$

$$y = \frac{1+3x}{x-1}$$

Make $x$ the subject of the formula:

$$y(x-1) = 1+3x$$

$$xy - y = 1+3x$$

$$xy - 3x = 1+y$$

$$x(y-3) = 1+y$$



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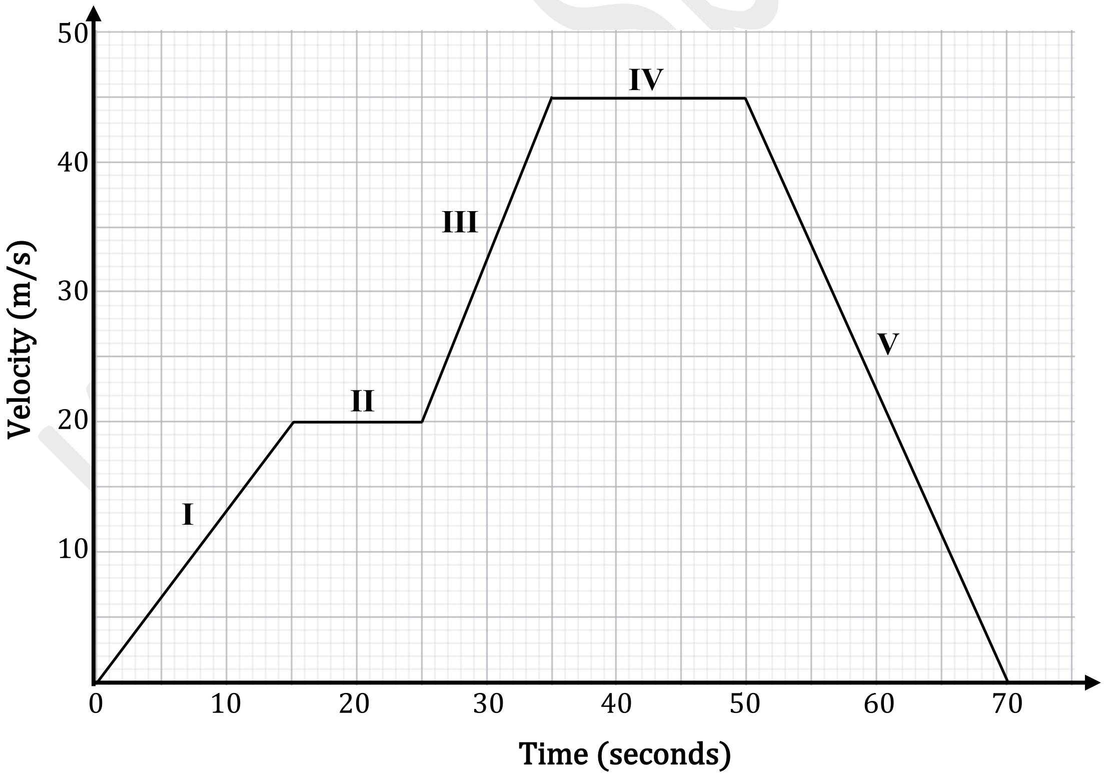
$$x = \frac{1+y}{y-3}$$

Interchange $x$ and $y$:

$$y = \frac{1+x}{x-3}$$

$$\therefore f(x)^{-1} = \frac{1+x}{x-3}$$

(b) The velocity-time graph below describes the journey of a car over a period of 70 seconds. The journey is represented in 5 stages labelled I to V.




(i) Complete the following statement.

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During Stage IV, the car is travelling at ______45______ m/s with an acceleration of ______0______ $m/s^2$. [2]

From the graph given, the $y$-axis represents the velocity therefore, reading off the graph at Stage IV the car is travelling at 45 m/s.

At Stage IV, the line is horizontal which means that the gradient $= 0$.

Therefore, acceleration at Stage IV will be $0\ m/s^2$.

(ii) Determine the MAXIMUM acceleration of the car during the 70 seconds. [2]

Stage I

Using the points $(0, 0)$ and $(15, 20)$,

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{20 - 0}{15 - 0}$$

$$= \frac{20}{15}$$

$$= \frac{4}{3} \; or \; 1\frac{1}{3} \; ms^{-2}$$

Stage II

This stage is represented by a horizontal line which means that its gradient $= 0$.

$\therefore$ Acceleration at Stage II will be $0\ m/s^2$.



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Stage III

Using the points $(25, 20)$ and $(35, 45)$,

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{45 - 20}{35 - 25}$$

$$= \frac{25}{10}$$

$$= 2.5\ ms^{-2}$$

Stage IV

This stage is represented by a horizontal line which means that its gradient $= 0$.

$\therefore$ Acceleration at Stage IV will be $0\ \text{m/s}^2$.

Stage V

Using the points $(50, 45)$ and $(70, 0)$,

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - 45}{70 - 50}$$

$$= \frac{-45}{20}$$

$$= -\frac{9}{4} \quad or \quad -2.25\ ms^{-2}$$

$\therefore$ The maximum acceleration of the car during the 70 seconds was 2.5 $ms^{-2}$.

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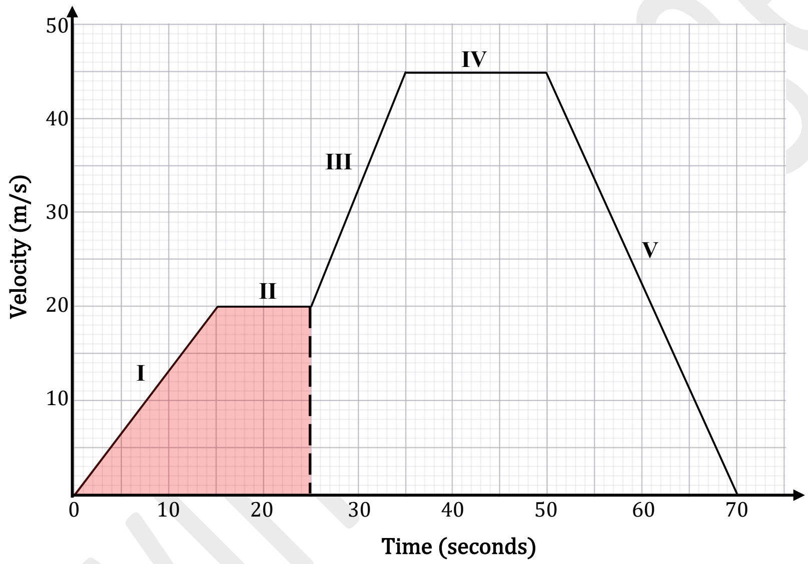
(iii) Calculate the distance travelled by the car during the **first 25 seconds** of its journey. [2]

The distance travelled can be found by finding the area of the trapezium.




$$\text{Distance travelled} = \frac{1}{2}(a+b)h$$

$$= \frac{1}{2}(10+25)(20)$$

$$= \frac{1}{2}(35)(20)$$

$$= \frac{1}{2}(700)$$

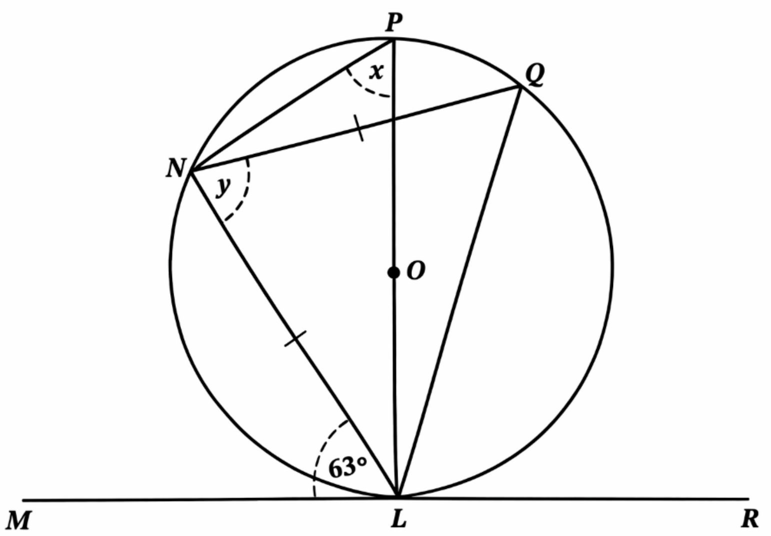
$$= 350\ m$$

∴ The distance travelled by the car during the first 25 seconds of its journey was 350 $m$.

Total: 12 marks

## GEOMETRY AND TRIGONOMETRY

9. (a) The diagram below shows a circle with its centre $O$ and the points $P, Q, L$ and $N$ lying on its circumference. $LN = NQ$ and $RM$ is a tangent to the circle at $L$. Angle $MLN = 63°$.




(i) Explain why Angle $x$ and Angle $NQL$ are equal. [1]

Angle $x$ and angle $NQL$ are equal because they are angles subtended by the same chord $NL$ at the circumference of the circle. Angles standing on the same chord and in the same segment of a circle are equal.

(ii) Determine the value of EACH of the following angles. Show detailed working where possible and give a **reason** for your answer.

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a) Angle $x$

The angle between a tangent and a chord is equal to the angle in the alternate segment.

$\therefore$ Angle $x = 63°$

[2]

b) Angle $y$

Angle $NQL$ = Angle $x = 63°$

Since triangle $NQL$ is isosceles, then the base angles would be equal.

$\therefore N\hat{Q}L = Q\hat{L}N = 63°$

Since the sum of interior angles in a triangle $= 180°$,

Angle $y = 180° - (63° + 63°)$

$= 180° - 126°$

$= 54°$

[2]

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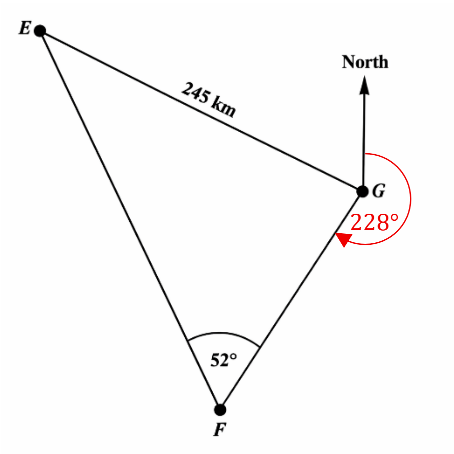
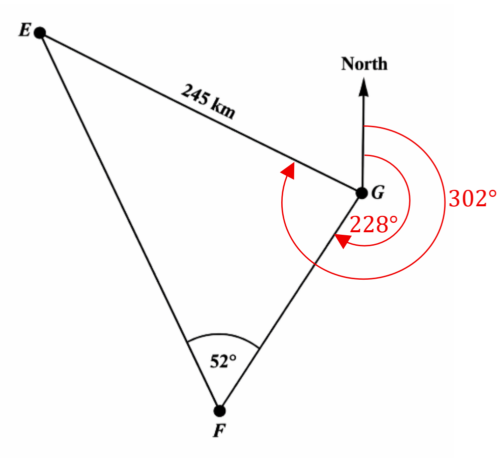
(b) Two ports, $E$ and $G$, are on level ground, 245 km apart. The bearing of $E$ from $G$ is 302°. A ship is anchored at $F$, some distance away from $G$, on a bearing of 228°. Angle $EFG = 52°$. This information is shown on the diagram below.




(i) a) On the diagram above, insert the angle 228°, the bearing of $F$ from $G$. [1]

The angle 228° was inserted on the diagram above.

b) Determine the value of Angle $FEG$. [1]




$E\hat{G}F = 302° - 228°$

$= 74°$

Since the sum of the interior angles in a triangle is equal to 180°,

Angle $FEG = 180° - (52° + 74°)$

$= 180° - 126°$

$= 54°$

(ii) Calculate $GF$, the distance the ship is from Port $G$. [2]

Substituting $F\hat{E}G = 54°$, $EG = 245\ km$, $E\hat{F}G = 52°$ the sine rule:

$$\frac{GF}{\sin F\hat{E}G} = \frac{EG}{\sin E\hat{F}G}$$

$$\frac{GF}{\sin 54°} = \frac{245}{\sin 52°}$$

$$GF = \frac{245}{\sin 52°} \times \sin 54°$$

$GF = 251.5\ km$ (to one decimal place)

(iii) a) Indicate the point $H$ on the line $EF$, such that $GH$ is the SHORTEST distance from $G$ to $EF$. [1]

For $GH$ to be the shortest distance from $G$ to $EF$, $GH$ must be perpendicular to $EF$.

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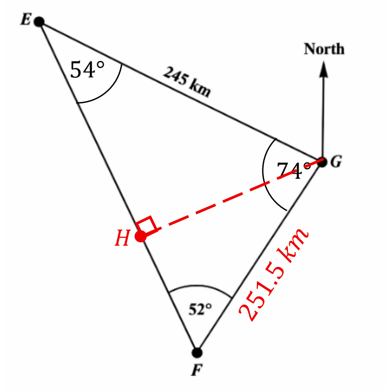


b) Determine the distance $GH$. [2]

Triangle $EGH$ is a right-angled triangle.

$$\sin\theta = \frac{opp}{hyp}$$

$$\sin\theta = \frac{GH}{EG}$$

$$\sin 54° = \frac{GH}{245}$$

$$\sin 54° \times 245 = GH$$

$$198.2 = GH$$

$GH = 198.2\ km$ (to 1 decimal place)

**Total: 12 marks**

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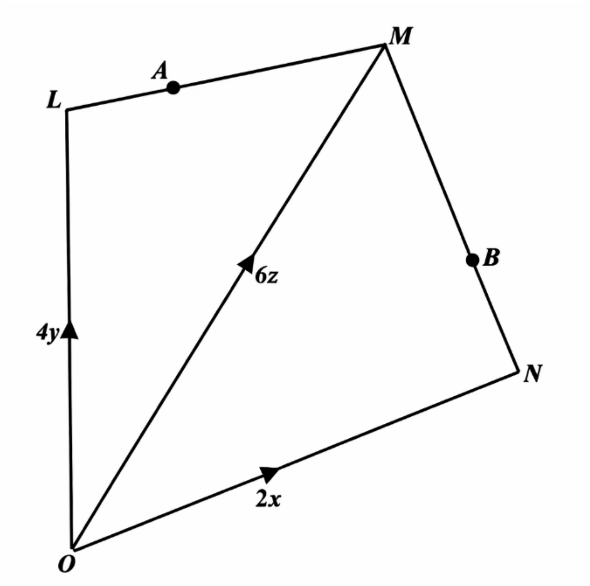
## VECTORS AND MATRICES

10. (a) The diagram below shows quadrilateral $OLMN$, in which $O$ is the origin, $\overrightarrow{OL} = 4y$, $\overrightarrow{OM} = 6z$ and $\overrightarrow{ON} = 2x$. The point $A$ lies on $LM$ such that $LA: AM = 1: 2$ and the point $B$ on $MN$ such that $MB: BN = 2: 1$.




(i) Express, in its simplest form, $\overrightarrow{MN}$ in terms of $x$ and $z$. [1]

Using the triangle law,

$\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM}$

$= 2x - 6z$

(ii) a) Find, in terms of $x$ and $y$, in its simplest form, an expression for $\overrightarrow{LN}$.

[1]

Using the triangle law,

$\overrightarrow{LN} = \overrightarrow{ON} - \overrightarrow{OL}$

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$= 2x - 4y$

b) Show that $\overrightarrow{AB}$ equal $\frac{2}{3}(2x - 4y)$. [2]

Using the triangle law,

$\overrightarrow{LM} = \overrightarrow{OM} - \overrightarrow{OL}$

$= 6z - 4y$

From the Question, $LA: AM = 1: 2$

$\therefore \overrightarrow{AM} = \frac{2}{3}\overrightarrow{LM}$

$= \frac{2}{3}(6z - 4y)$

$= 4z - \frac{8}{3}y$

From the Question, $MB: BN = 2: 1$

$\therefore \overrightarrow{MB} = \frac{2}{3}\overrightarrow{MN}$

$= \frac{2}{3}(2x - 6z)$

$= \frac{4}{3}x - 4z$

Using the triangle law,

$\overrightarrow{AB} = \overrightarrow{AM} + \overrightarrow{MB}$

$= \left(4z - \frac{8}{3}y\right) + \left(\frac{4}{3}x - 4z\right)$

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$$= 4z - 4z - \frac{8}{3}y + \frac{4}{3}x$$

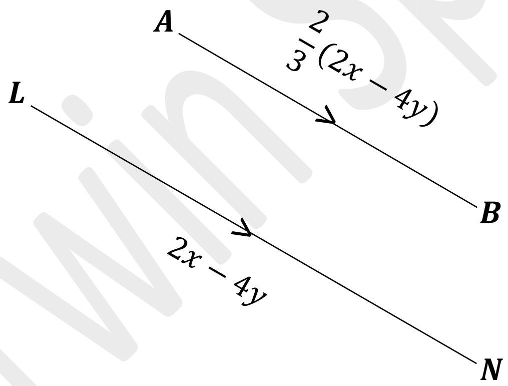
$$= -\frac{8}{3}y + \frac{4}{3}x$$

$$= \frac{2}{3}(-4y + 2x)$$

$$= \frac{2}{3}(2x - 4y)$$

Q.E.D.

(iii) Based on your results in Part (ii), state TWO geometric properties relating $LN$ to $AB$. [2]




The two geometric properties relating $\overrightarrow{LN}$ to $\overrightarrow{AB}$ are:

1. $|\overrightarrow{AB}| = \frac{2}{3}|\overrightarrow{LN}|$

2. $\overrightarrow{LN}$ is parallel to $\overrightarrow{AB}$

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(b) Determine the values of the unknowns in EACH of the matric equations below.

(i) $\begin{pmatrix} 4 & 0 \\ -2 & 5 \end{pmatrix} + \begin{pmatrix} x & 2 \\ 8 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ y & 4 \end{pmatrix}$

[2]

$$\begin{pmatrix} 4 & 0 \\ -2 & 5 \end{pmatrix} + \begin{pmatrix} x & 2 \\ 8 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ y & 4 \end{pmatrix}$$

$$\begin{pmatrix} 4+x & 0+2 \\ -2+8 & 5+(-1) \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ y & 4 \end{pmatrix}$$

$$\begin{pmatrix} 4+x & 0+2 \\ -2+8 & 5-1 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ y & 4 \end{pmatrix}$$

$$\begin{pmatrix} 4+x & 2 \\ 6 & 4 \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ y & 4 \end{pmatrix}$$

Since the matrices are equal, we can equate the corresponding entries.

Therefore,

$4 + x = -3$

$x = -3 - 4$

$x = -7$

$6 = y$

$y = 6$

$\therefore x = -7$ and $y = 6$.

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(ii) $\begin{pmatrix} 5 & -3 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} a & 2 \\ c & -1 \end{pmatrix} = \begin{pmatrix} -10 & 13 \\ 17 & 1 \end{pmatrix}$

[4]

$$\begin{pmatrix} 5 & -3 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} a & 2 \\ c & -1 \end{pmatrix} = \begin{pmatrix} -10 & 13 \\ 17 & 1 \end{pmatrix}$$

$$\begin{pmatrix} (5 \times a) + (-3 \times c) & (5 \times 2) + (-3 \times -1) \\ (2 \times a) + (3 \times c) & (2 \times 2) + (3 \times -1) \end{pmatrix} = \begin{pmatrix} -10 & 13 \\ 17 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 5a - 3c & 13 \\ 2a + 3c & 1 \end{pmatrix} = \begin{pmatrix} -10 & 13 \\ 17 & 1 \end{pmatrix}$$

Equating the corresponding entries gives:

$5a - 3c = -10$ —— Equation 1

$2a + 3c = 17$ —— Equation 2

Equation 1 + Equation 2:

$7a = 7$

$a = \frac{7}{7}$

$a = 1$

Substituting $a = 1$ into Equation 1:

$5(1) - 3c = -10$

$5 - 3c = -10$

$-3c = -10 - 5$

$-3c = -15$

$c = \frac{-15}{-3}$

$c = 5$

$\therefore$ The unknowns are $a = 1$ and $c = 5$.

**Total: 12 marks**

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**END OF TEST**

**IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.**

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