CSEC Mathematics

June 2014 – Paper 2

Solutions

SECTION I

Answer ALL questions in this section.

All working must be clearly shown.

1. (a) Using a calculator, or otherwise, determine the value of:

   (i) $5.25 \div 0.015$ [1]

   Using a calculator,

   $5.25 \div 0.015 = 350$

   (ii) $\sqrt{6.5025}$ [1]

   Using a calculator,

   $\sqrt{6.5025} = 2.55$

   (iii) 3.142×2.236^2 (correct to 3 significant figures)   [2]

   Using a calculator,

   $3.142 \times 2.236^2 = 15.7$ (to 3 significant figures)

(b) Concrete tiles are made using buckets of cement, sand and gravel mixed in the ratio 1 : 4 : 6.

(i)   How many buckets of gravel are needed for 4 buckets of cement?   [2]

Cement : Sand : Gravel

1   :   4   :   6

(× 4)

4   :   16   :   24

∴ 4 buckets of cement would require 24 buckets of gravel.

(ii)   If 20 buckets of sand are used, how many buckets of EACH of the following will be needed?

(a) Cement

Cement : Sand : Gravel

1   :   4   :   6

(× 5)

5   :   20   :   30

∴ 5 buckets of cement will be needed.

(b) Gravel   [3]

∴ 30 buckets of gravel will be needed.

(c) The cash price of a laptop is $1 299. It can be bought on hire purchase by making a deposit of $350 and 10 monthly payments of $120 each.

(i) What is the TOTAL hire purchase price of the laptop? [2]

Hire Purchase Price = Deposit + Instalments

$= \$350 + (10 \times \$120)$

$= \$350 + \$1200$

$= \$1550$

∴ The total hire purchase price of the laptop is $1550.

(ii) How much is saved by buying the laptop for cash? [1]

Amount saved = Hire Purchase Price − Cash Price

$= \$1550 - \$1299$

$= \$251$

∴ $251 is saved by buying the laptop for cash.

Total: 12 marks

2. (a) Write as a single fraction in its LOWEST terms

$$\frac{x-2}{3} + \frac{x+1}{4}$$ [3]

$$\frac{x-2}{3} + \frac{x+1}{4}$$

$$= \frac{4(x-2)+3(x+1)}{12}$$

$$= \frac{4x-8+3x+3}{12}$$

$$= \frac{7x-5}{12}$$

(b) Write an equation in x to represent EACH statement below. Do NOT solve the equation.

(i) When 4 is added to a certain number the result is the same as halving the number and adding 10.   [1]
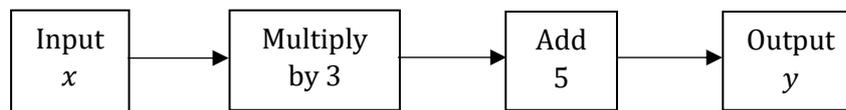
Let the unknown number be x.

An equation to represent the statement is: $4 + x = \frac{1}{2}x + 10$

(ii) Squaring a number and subtracting 6 gives the same result as doubling the number and adding 9.   [1]

Let the unknown number be x.

An equation to represent the statement is: $x^2 - 6 = 2x + 9$

(c) John drew the diagram below to show what he was thinking.

| Input x | → | Multiply by 3 | → | Add 5 | → | Output y |

(i)   Use information from the diagram to write a formula for y in

terms of x.   [1]

A formula for y in terms of x is $y = 3x + 5$.

(ii)   If the number 4 is the input, what number would be the output?   [1]

When $x = 4$,

$y = 3(4) + 5$

$= 12 + 5$

$= 17$

(iii)   If the number 8 was the output, what number was the input?   [1]

When $y = 8$,

$3x + 5 = 8$

$3x = 8 - 5$

$3x = 3$

$x = \frac{3}{3}$

$x = 1$

(iv)   Reverse the formula written at (c)(i) above to write x in terms

of y.   [1]

$y = 3x + 5$

$3x = y - 5$

$x = \frac{y-5}{3}$

(d) Solve the following simultaneous equations:

   $2x + 3y = 9$

   $3x - y = 8$ [3]

$2x + 3y = 9$ → Equation 1

$3x - y = 8$ → Equation 2

Multiplying Equation 2 by 3 gives,

$9x - 3y = 24$ → Equation 3

$2x + 3y = 9$ → Equation 1

Adding Equation 1 and Equation 3 gives,

$11x = 33$

$$x = \frac{33}{11}$$

$$x = 3$$

Substituting $x = 3$ into Equation 1 gives,

$$2(3) + 3y = 9$$

$$6 + 3y = 9$$

$$3y = 9 - 6$$

$$3y = 3$$

$$y = \frac{3}{3}$$

$$y = 1$$

$$\therefore x = 3 \text{ and } y = 1$$

Total: 12 marks

3. (a) The universal set, U, is defined as the set of integers between 11 and 26.

   A and B are subsets of U such that:

   $A = \{$even numbers$\}$

   $B = \{$multiples of 3$\}$

   (i) How many members are in the universal set, U? [1]

   $U = \{$set of integers between 11 and 26$\}$

   $\quad = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26\}$

   Now,

   $n(U) = 16$

   $\therefore$ 16 members are in the universal set, U.

   (ii) List the members of the subset A. [1]

   $A = \{$even numbers$\}$

   $\quad = \{12, 14, 16, 18, 20, 22, 24, 26\}$
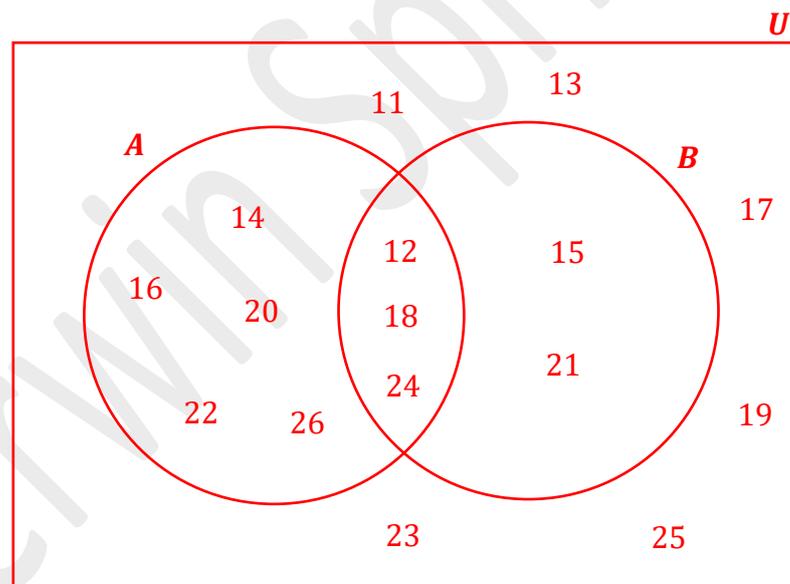
(iii)   List the members of the subset B.   [1]

$B = \{\text{multiples of 3}\}$

$= \{12, 15, 18, 21, 24\}$

(iv)   Draw a Venn diagram to represent the relationships among A, B

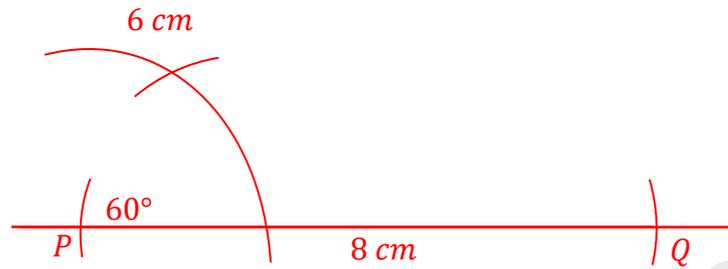and U.   [3]

The Venn diagram is shown below:

(b) (i) **Using a ruler, a pencil** and **a pair of compasses**, construct

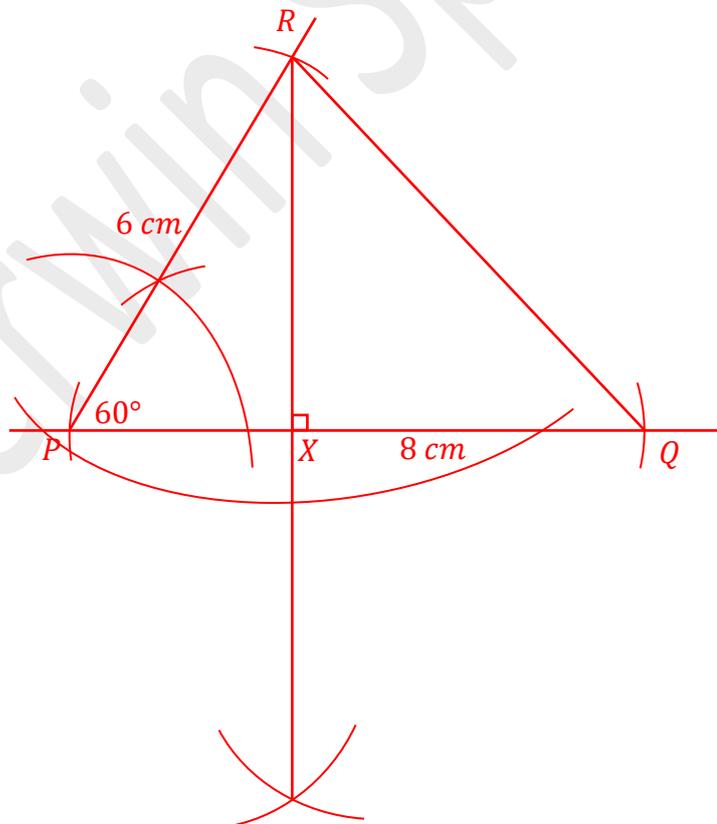(a) a triangle PQR in which $PQ = 8$ cm, $PR = 6$ cm and angle $P = 60°$ [3]

The construction diagram is shown below:

6 cm

60°

P   8 cm   Q

(b) the line segment RX which is perpendicular to PQ and meets PQ

at X.   [2]

The construction diagram is shown below:

R

6 cm

60°

P   X   8 cm   Q
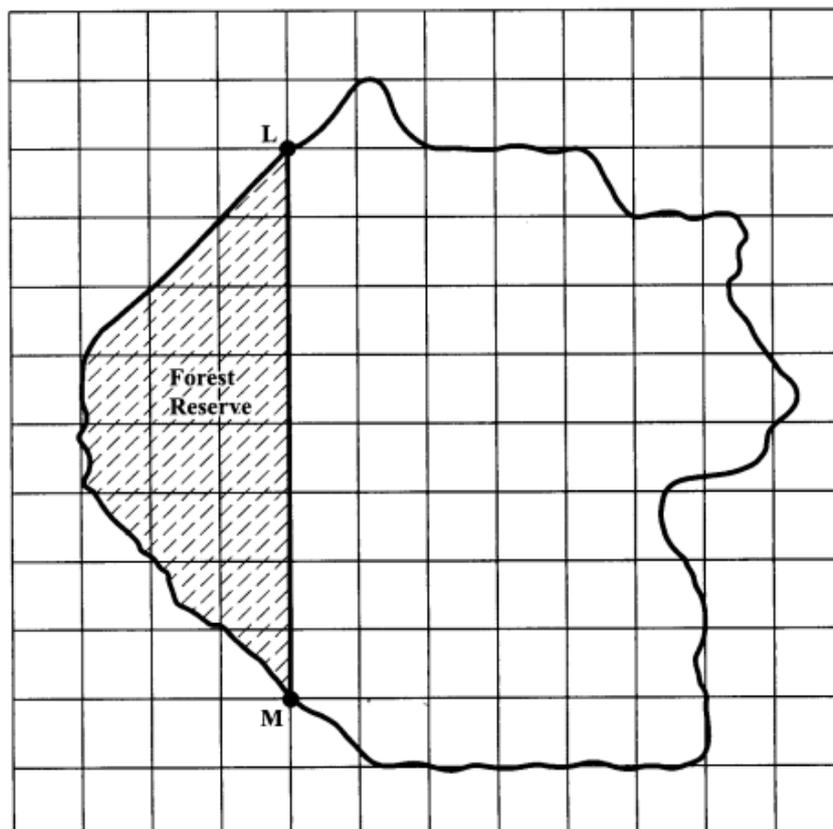
(ii) Measure and state the size of angle QRX. [1]

With the use of a protractor, the size of $\angle QRX = 44°$.

Total: 12 marks

4. The diagram below shows a map of an island drawn on a grid of 1-cm squares.

The map is drawn to a scale of 1: 50 000.

(a) Copy and complete EACH of the following sentences:

(i) 1 cm on the map represents _____ cm on the island. [1]

(ii) An area of 1 cm^2 on the map represents an area of _____

cm^2 on the island. [1]

(iii)   Given that 1 km $= 100\,000\ cm$, a distance of 1 cm on the map

represents a distance of _____ km on the island.   [1]

(b) (i) L and M are two tracking stations. State, in centimetres, the distance LM

on the map.   [1]

(ii) Calculate the ACTUAL distance, in kilometres, from L to M on the

island.   [2]

(c) (i) The area shaded on the map is a forest reserve. By counting squares

estimate, in cm^2, the area of the forest reserve as shown on the map.   [2]

(ii) Calculate, in km^2, the ACTUAL area of the forest reserve.   [2]

Total: 10 marks

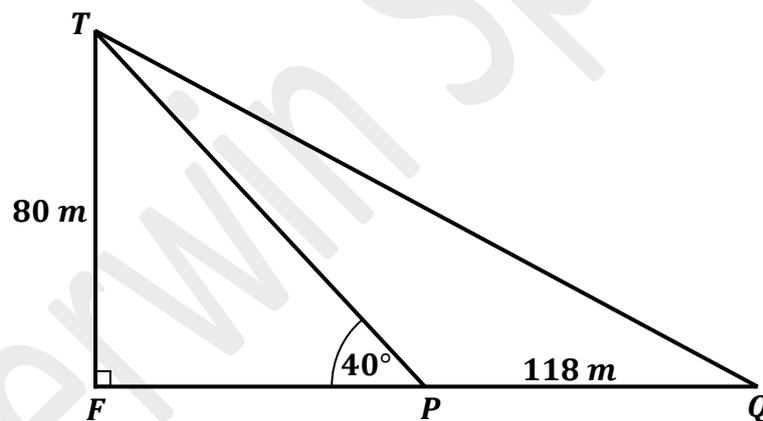5. An answer sheet is provided for this question.

(a) Triangle ABC has coordinates $A(1, 2)$, $B(4, 2)$ and $C(1, 0)$.

(i) **On the answer sheet provided**, draw triangle $A'B'C'$, the image of triangle ABC, under an enlargement, centre O and scale factor 2.   [3]

(ii) Triangle $A"B"C"$ is the image of triangle ABC, under a transformation, M. Describe **completely** the transformation, M.   [3]
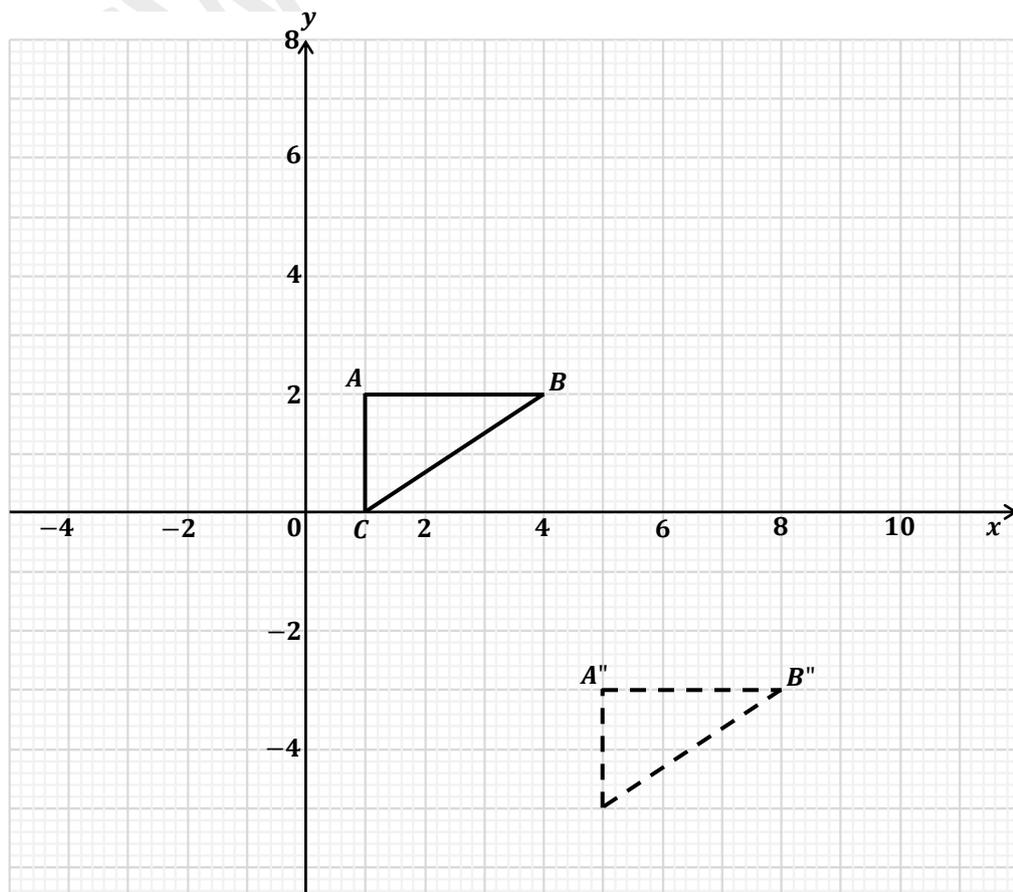
(b) The diagram below, not drawn to scale, shows the position of two ships, P and Q, at anchor. FT is the vertical face of a cliff jutting out of the water. P and Q are $118\ m$ apart. $FT = 80\ m$ and $\angle FPT = 40°$.

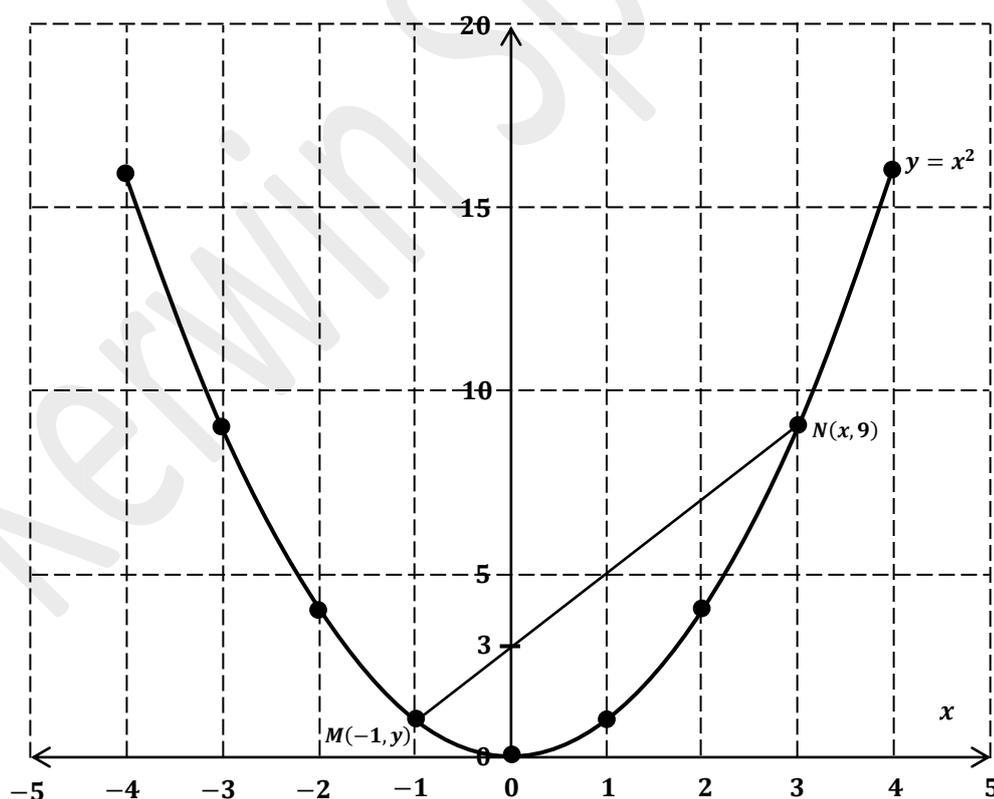

Determine

(i)   the angle of elevation of T from P [1]

(ii)   the length of FP [2]

(iii)   the angle of elevation of T from Q [3]

Total: 12 marks

6. **An answer sheet is provided for this question.**

The graph of the quadratic function $y = x^2$ for $-4 \leq x \leq 4$ is shown below.

(a) The coordinates of the points M and N are $(-1, y)$ and $(x, 9)$ respectively.

Determine the value of

(i) x [1]

(ii) y [1]

(b) Determine

    (i)   the gradient of the line MN   [1]

    (ii)   the equation of the line MN   [2]

    (iii)   the equation of the line parallel to MN, and passing through the

         origin.   [2]

(c) **On the answer sheet provided**, carefully, draw the tangent line to the graph

    $y = x^2$ at the point $(2, 4)$.   [2]

(d) Estimate the gradient of the tangent to the curve at $(2, 4)$.   [2]

Total: 11 marks

7. A class of 30 students counted the number of books in their bags on a certain day. The number of books in EACH bag is shown below.

5	4	6	3	2	1	7	4	5	3
6	5	4	3	7	6	2	5	4	5
5	7	5	4	3	2	1	6	3	4

(a) Copy and complete the frequency table for the data shown above.   [4]

Number of Books (x)	Tally	Frequency (f)	$f \times x$
1	||	2	2
2	|||	3	6
3	||||	5	15
4	ℍ |	6	24
5	ℍ ||	7	35

| 6 | \|\|\|\| | 4 | 24 |
| 7 | \|\|\| | 3 | 21 |

(b) State the modal number of books in the bags of the sample of students.   [1]

The modal number of books is 5 books.

(c) Using the table in (a) above, or otherwise, calculate

    (i)   the TOTAL number of books   [2]

$$\sum fx = 2 + 6 + 15 + 24 + 35 + 24 + 21$$

$$= 127$$

$\therefore$ The total number of books is 127 books.

    (ii)   the mean number of books per bag.   [2]

$$\overline{x} = \frac{\sum fx}{\sum f}$$

$$= \frac{127}{30}$$

$$= 4.23 \text{ books} \quad \text{(to 3 significant figures)}$$

$\therefore$ The mean number of books per bag is 4.23 books.

(d) Determine the probability that a student chosen at random has LESS THAN 4

books in his/her bag. [2]

$$\text{Probability} = \frac{\textit{Number of students who have less than 4 books}}{\textit{Total number of students}}$$

$$= \frac{2+3+5}{30}$$

$$= \frac{10}{30}$$

$$= \frac{1}{3}$$

Total: 11 marks

8. **An answer sheet is provided for this question.**

A number sequence may be formed by counting the number of dots used to draw

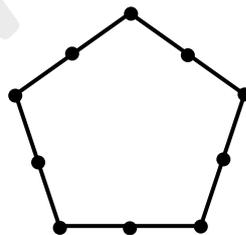each set of geometric figures. The first three figures are shown below.
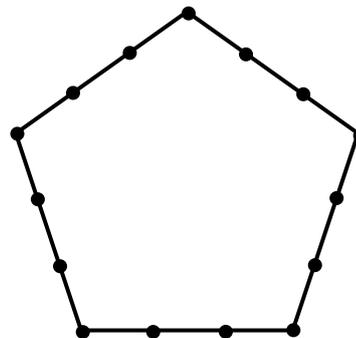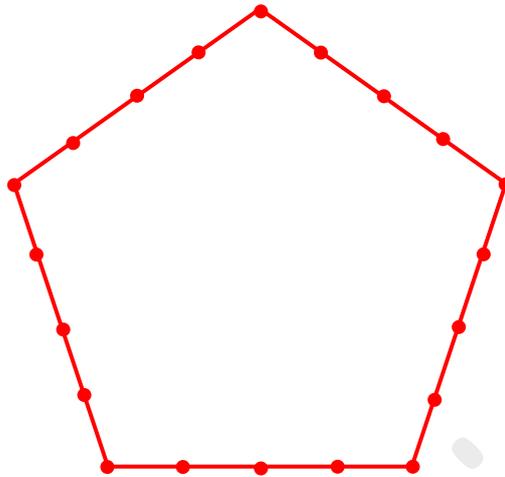

Figure 1   Figure 2   Figure 3

On the answer sheet provided,

(a) Draw Figure 4, the next figure in the sequence above. [2]

Figure 4 is shown below:

(b) Complete the table below by inserting the missing information at the rows

numbered (i) and (ii). [4]

	Figure	Total Number of Dots	
	(f)	Formula	Number (n)
	1	$5 \times 2 - 5$	5
	2	$5 \times 3 - 5$	10
	3	$5 \times 4 - 5$	15
	4	Not required	Not required
(i)	5	$5 \times (5 + 1) - 5$	25
(ii)	6	$5 \times (6 + 1) - 5$	30

(c) Write an expression in f for the number (n) of dots used in drawing the fth figure. [2]

The number of dots, n, in the fth figure is given by: $\quad 5f = n$

(d) Which figure in the sequence contains 145 dots? [2]

$145 = 5f$

$f = \dfrac{145}{5}$

$f = 29$

Hence, there are 145 dots in the 29th figure.

Total: 10 marks

SECTION II

Answer TWO questions in this section.

ALGEBRA AND RELATIONS, FUNCTIONS AND GRAPHS

9. (a) Two functions are defined as follows:

$g(x) = 4x + 3$

$f(x) = \dfrac{2x+7}{x+1}$

(i) State the value of x for which $f(x)$ is undefined. [1]

$f(x) = \dfrac{2x+7}{x+1}$

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A function is undefined when the denominator is equal to zero.

When $x + 1 = 0$,

$$x = -1$$

$\therefore f(x)$ is undefined when $x = -1$.

(ii) Calculate the value of $gf(5)$.   [3]

Firstly,

$f(5) = \frac{2(5)+7}{5+1}$

$= \frac{17}{6}$

Now,

$gf(5) = g\left(\frac{17}{6}\right)$

$= 4\left(\frac{17}{6}\right) + 3$

$= \frac{34}{3} + 3$

$= \frac{43}{3}$

(iii) Find $f^{-1}(x)$.   [3]

$f(x) = \frac{2x+7}{x+1}$

(b) A ball is thrown vertically upwards. Its height, h metres, above the ground after t seconds is shown in the table below.

t (s)	0	1	2	3	4	5	6
h (m)	0	50	80	90	80	50	0

(i)   Using **2 cm to represent 1 second** on the x-axis and **1 cm to represent 10 metres** on the y-axis, plot a graph to show the height of the ball during the first 6 seconds.   [4]
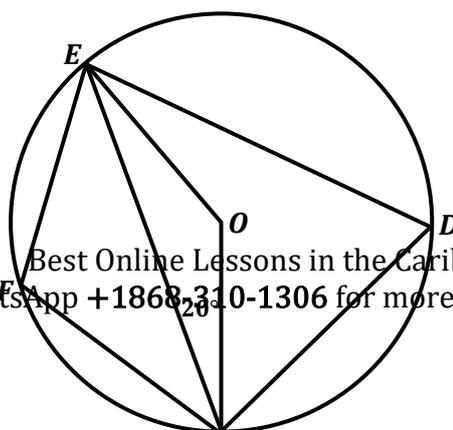
(ii)   **Using your graph**, determine

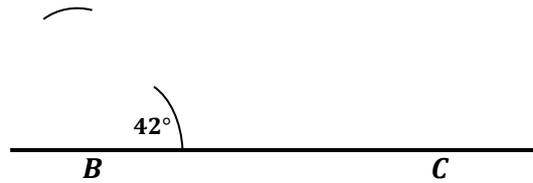(a) the average speed of the ball during the first 2 seconds   [2]

(b) the speed of the ball when $t = 3$ seconds.   [2]

Total: 15 marks

MEASUREMENT, GEOMETRY AND TRIGONOMETRY

10. (a) The diagram below, **not drawn to scale**, shows a circle, centre O.The line BC

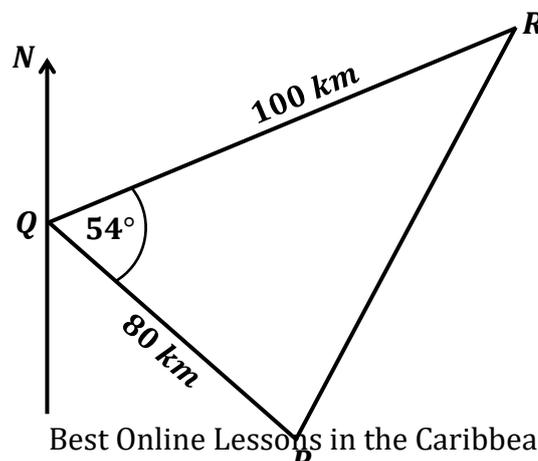is a tangent to the circle at B. Angle $CBD = 42°$ and angle $OBE = 20°$.

42°

B   C

Calculate, giving a reason for EACH step of your answer, the measure of:

(i)   ∠BOE   [2]

(ii)   ∠OED   [2]

(iii) ∠BFE   [3]

(b) The diagram below, **not drawn to scale**, shows the positions of three ports, P,

Q and R.

Q is 80 km from P.

R is 100 km from Q on a bearing of 066°.

$\angle PQR = 54°$.

Calculate

(i)   the bearing of P from Q [2]

(ii) the distance PR correct to 2 decimal places   [3]

(iii) the measure of $\angle QPR$ to the **nearest** degree   [3]

Total: 15 marks

VECTORS AND MATRICES

11. (a) The matrix M is defined as

$$M = \begin{pmatrix} 7 & 2 \\ p & -1 \end{pmatrix}$$

Determine the value of p for which the matrix M does NOT have an

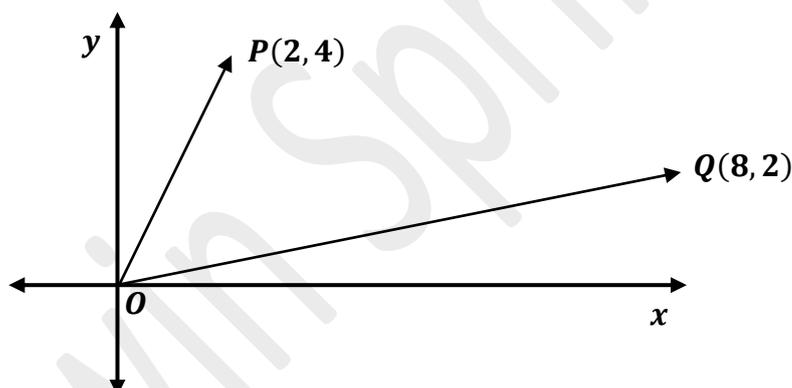inverse.   [2]

(b) Express the equations

$$4x - 2y = 0$$

$$2x + 3y = 4$$

in the form $AX = B$, where A, X and B are matrices. [2]

(c) In the diagram below, the coordinates of P and Q are $(2, 4)$ and $(8, 2)$ respectively. The line segment joining the origin $(0, 0)$ to the point P may be written as $\overrightarrow{OP}$.

(i)   What terms is used to describe $\overrightarrow{OP}$? [2]

(ii) Write EACH of the following in the form : $\begin{pmatrix} a \\ b \end{pmatrix}$

   (a) $\overrightarrow{OP}$ [1]

   (b) $\overrightarrow{OQ}$ [1]

   (c) $\overrightarrow{PQ}$ [2]

(iii) Given that $\overrightarrow{OP} = \overrightarrow{RQ}$, determine the coordinates of the point, R. [3]

(iv)   State the type of quadrilateral formed by *PQRO*.

Justify your answer.   [2]

Total: 15 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.