Viewing paradox through the lens of general relativity.

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Abstract

In this letter, we continue our discussions regarding the limits to both zero and infinity in a system defined by measurement by defining the frames of reference embedded in paradoxes and assigning an observer to each frame of reference. We demonstrate that paradoxes often rely on including unstated, and relative, observer frames of reference within the paradox. When any observer, in a relative frame of reference, is asked to violate their measurement minimums, or the rules of general relativity regarding the simultaneous measurement of relative frames of reference – the result is the appearance of paradox. We break down observational frames of reference embedded in the Liar paradox, the Card paradox, the Barber paradox, the Grandfather paradox, and Schrodinger's Cat paradox. We then proceed to demonstrate how the rules of general relativity force boundary conditions on all relative sets embedded within The Russell paradox – including the set of all sets. We finish with a short discussion of paradox related to the forcing of relative rules on the set of natural numbers and the Peano axioms.

1.00 Introduction

The importance of paradox to theoretical mathematics and number theory, as well as quantum physics, is hard to overestimate. The principles of uncertainty and superposition, Kurt Gödel's theories of incompleteness[1] and Einstein's theory of General Relativity[2] are all limited by our understanding of paradox. As Einstein has demonstrated, any measurement taken by an observer must take into account the *relative* nature of the observation in context with all other observer frames of reference. He also tells us that there can be no simultaneous measurements between different frames of reference. Here, we examine the various observer frames of reference embedded in paradoxes and demonstrate that each observer is held to measurement minimums and to the rules of general relativity, regarding simultaneous measurement of relative frames of reference. We demonstrate how paradoxes often depend on an observer being asked to take a simultaneous measurement between relative frames of reference in violation of the rules of general relativity and their minimum for measurement. To understand how these boundaries affect paradox, let us begin by identifying all observer reference frames embedded in " The Liars Paradox".

1.01 - The Liar's Paradox and unstated frames of reference

Perhaps the simplest presentation of a paradox is the Liar's paradox. It simply asks the reader to evaluate the following sentence:

"This sentence is false."

The reader is, immediately, caught in a logic circle that asks what is false about a sentence which refers to itself. The trick to diagnosing this paradox is that there are, actually, three different frames of reference embedded in the paradox –two of which are unstated. Because every false statement implies the existence of truth, there is an unstated, and imaginary, frame of reference that sees a true statement. Another unstated observer frame of reference is the reader. The reader understands the difference between true and false and can "see" all the other frames of reference in the paradox. All frames of reference have different minimums for measurement:

Observer 1- only measures this sentence as false
Observer 2 - only measures this sentence as true
Observer 3 - reader knows that true and false are *relative* measurements

Observer one presents the reference frame of the problem. Observer two is an unstated frame of reference that is implied by the use of the word "false" in the presentation of the problem. Observer three, (the reader) brings to the problem the knowledge of the *relative* difference between *true* and *false*. The reader can observe the relative measurements made from two other frames of reference – one stated and the other unstated. This makes the frame of reference for Observer three a *relative frame of reference*. According to the rules of general relativity, *there can be no simultaneous measurement between relative observer frames of reference.* Einstein explained this concept using the measurements made by a man on a train and a man on the ground.. Only a third observer can truly "see" the relative differences between the two other observations. Any observer in a relative frame of reference can measure either of the other two frames of reference – just not simultaneously. The Liar paradox, like all the paradoxes we will discuss, depends heavily on including the reader as a relative observer.

As we hope to demonstrate, it is Einstein's limitation on the simultaneous measurement of relative frames of reference that causes the appearance of paradox. We can expand on this further by examining the relative frames of reference embedded in "The Card Paradox".

1.02 - The Card Paradox has two solutions and three observers

The Card paradox is the similar to the Liar's paradox. It presents us with a card with statements printed on each side.

One side of the card claims: *"The statement on the other side of this card is true."*
While the other side claims: *"The statement on the other side of this card is false."*

The comparison to the Liar's paradox is fairly obvious, but there are some subtle differences that will actually allow us to solve this paradox in two different ways. One easy resolution of the paradox is that the problem isn't a paradox – it's simply a card with words printed on it. This resolution to the problem happens in real space and meets all measurement minimums for every frame of reference. The action of turning the card has no affect on any observer frame of reference. However, this solution seems a little like cheating. Instead, we will pretend that we have a magic card that turns itself and the face of the card determines what is real. Like the Liar's paradox, we can define measurement minimums for the problem by identifying observer frames of reference:

Observer 1- measures other side of card as true
Observer 2 - measures other side of card as false
Observer 3 - reader can see both sides of card, but not simultaneously

In this paradox, the card is the minimum measurement for all observers. Only observer three sees the actual relationship between the two contradictory sides. In order for observer 3 to do this they must be able to remember both frames of reference. Therefore, observer three's frame of reference will always be composed of *real* and an *imaginary* components. For both of the other observers, their only frame of reference is the side of the card that they see. Observer 1 sees a true statement. They must assume that what they see is true and the other side is also true. Their frame of reference only sees true statements. Observer 2 must also assume that what they observe is the truth. The other side is false. The difference between observers one and two is that observer two understands the nature of a false statement and can imagine the other side of the card as false. Observer 3 has the additional advantage of being able to see both sides of the card. What this paradox allows us to discuss is the nature of imaginary potential related to paradox. The side of the card that observer 2 doesn't see only can be imagined. Observer three also must use their imagination to measure both statements, but for a different reason. For observers one and two, what is real is not determined by the flipping of the card because each frame of reference can never see the other side of the card. The reader can see both sides of the card at the same time because they exist in a *relative* frame of reference. As we have discussed, all relative frames are held to the limits of general relativity when it comes to simultaneous measurement . In this problem, the reader can only measure conflicting statements in sequence. Unlike the other two reference frames , the reader can only observe a looped cycle of sequential and conflicting measurements that have the appearance of an unsolvable paradox. In actuality, the reader is adhering the the minimums for measurement within their frame of reference. One of the measurents is always real, the other always imaginary. We will expand on this relationship further in later sections. In the next section, we show how sequential actions, and reference frame measurement minimums in real space, can provide a resolution to the Barber paradox.

1.03 - The Barber Paradox and minimums for measurement.

This paradox was first proposed by Bertrand Russell as a way to understand the Russell paradox[3], which we discuss later. The Barber paradox can be stated as follows:

"A barber must shave everyone who does not shave themselves. Does the barber shave himself?"

In this paradox we have three observers; the barber, and two groups of observers. Each observer has different minimums of measurement. The resolution of any paradox begins by defining these minimums:

Observer 1- Barber (person who shaves people that do not shave themselves)

Observer 2- People who shave themselves

Observer 3- People who not shave themselves

To establish the minimum measurement for each observer frame of reference, let us imagine we have placed observers one and two in separate buildings. On the front of each building is a single door. Next to the door is a sign. One sign says "Barber needed" the other says " No Barbers Allowed". Observers in each building do not interact with each other and can only interact with the barber. The secret to this paradox is defining the minimum measurement for each observer and taking a sequential approach to a relative problem, based on minimum available action. The minimum measurement in this paradox is not "shaving" it is "barber". To maintain his minimum measurement of " "barber" the barber has only one action available to him. His minimum of action to stay a barber is to enter the building of the observers who do not shave themselves and get to work. The barber enters the building and shaves everyone who does not shave himself. *Because he has shaved everyone who does not shave himself he can no longer be measured as a" barber"*. By shaving everyone, he has moved from the observer 3 frame and into the observer 2 frame. This is not just a clever way of eliminating the word "barber" from the problem. A minimum sequential action allows the barber to change his observational reference frame. Because he can no longer shave anyone else his minimum for measurement is still to take action. That minimum action is to enter the other building to shave himself. The problem does not state that any observers have hair that grows back, therefore; the observer 1 reference frame is eliminated and the paradox is resolved. To see how relative frames of reference can deny a minimum of action we can examine the measurement minimums for each observer in "The Grandfather paradox".

1.04 The Grandfather Paradox must obey the rules of general relativity

The Grandfather paradox asks the question:

"Can you go back in a time machine and kill your grandfather?"

Because we are discussing a paradox that is directly related to the rules of general relativity, we will demonstrate how Einstein has already resolved this paradox by limiting the actions that can be taken in all reference frames. To demonstrate how this works we, again, begin by assigning frames of reference and measurement minimums to all the observers within the paradox.

Observer 1- grandson exists in the present and has a grandfather

Observer 2 - grandson exists in the past and has a grandfather

Observer 3 - grandfather exists in the past and has no grandson

The existence of the grandfather is a minimum measurement for all observers in this paradox. Any actions that change this minimum for measurement will result in a simultaneous change for the other frames of reference. The rules of general relativity forbid any observations that can be measured simultaneously by all observers. The resolution of this paradox has to do with the minimum actions that are allowed in all reference frames. You cannot kill your grandfather because you are taking an action in one frame of reference that can be seen by the other observers simultaneously. *You cannot go back in time and take any action that changes measurement minimums for any other observer frame of reference simultaneously.* As we discussed in the introduction, Einstein has already explained why simultaneous measurement in all reference frames is impossible. To understand how minimums for measurement, and the rules of relativity, can be applied to a quantum measurement; we can move on to Dr. Schrodinger's infamous cat.

1.05 - Schrodinger's Cat

Since it's birth, in a letter to Einstein, Schrodinger's cat has been abused by theorists and layman alike to discuss the paradoxes associated with superposition principles and quantum measurement. With his original thought experiment[4], Dr. Schrodinger hoped to demonstrate to Einstein the absurdity of a cat being in "alive" and "dead" states at the same time. To save a little time, we will assume the reader is familiar with the specifics of this famous paradox. In this discussion, we will attempt to demonstrate that it is the box that defines *all* minimum measurement boundaries in the experiment – not the cat. To begin, we identify all observer frames of reference in the paradox:

Observer 1- outside of box- sees dead cat

Observer 2- outside of box - sees live cat

Observer 3- Sees inside of unopened box

All observers share a common minimum measurements of "box" and "cat". Like the card paradox, opening the box determines the difference between real and imagined frames of reference. The mistake made in most interpretations this paradox is that a dead cat is thought of as a zero state. The box exists in time whether the cat is alive or dead, therefore the cat can't disappear. For all our observers there is a cat state. The life or death state is determined once a measurement has been made in real space by observer one or observer two. As in the other paradoxes these observers do not interact. Any measurement made by observer three simultaneously determines the outcome for observers one and two. As we showed earlier, this is not allowed under the rules of general relativity. *Therefore, observer three can take no measurements in real space.* Dr. Schrodinger's cat can be imagined as alive and dead inside of the box because no real measurements can be taken by observer three. In fact, when the box is opened there is only one frame of reference. All the other frames of reference no longer exist. The solution to this paradox is forced upon us by the rules of general relativity. *No measurement can be taken inside of the box. The opening of the box determines the real frame of reference.* As we discussed in the previous sections, any relative frame of reference is forced to contain a real and imaginary component to adhere to the rules regarding simultaneous measurement. In the next section, we will discuss how these requirements for all relative sets can provide the *set forcing* needed to resolve the Russell Paradox.

1.06 The Russell paradox.

Bertrand Russell framed his famous paradox in the following manner:

"The comprehensive class we are considering, which is to embrace everything, must embrace itself as one of its members. In other words, if there is such a thing as "everything," then, "everything" is something, and is a member of the class "everything." But normally a class is not a member of itself. Mankind, for example, is not a man. Form now the assemblage of all classes which are not members of themselves. This is a class: is it a member of itself or not? If it is, it is one of those classes that are not members of themselves, i.e., it is not a member of itself. If it is not, it is not one of those classes that are not members of themselves, i.e. it is a member of itself. Thus of the two hypotheses – that it is, and that it is not, a member of itself – each implies its contradictory. This is a contradiction." [3]

For this paradox, the minimums for measurement for all reference frames are dependent on the definitions in basic set theory. We begin by breaking the problem into the various observers within the paradox and their minimum for measurement. For the Russell paradox we can associate each observer with a set:

Observer 1- sets that are members of themselves

Observer 2- sets that are not members of themselves

Observer 3- set of all sets

Because the common minimum measurement is a "set", all observers are held to the measurement minimums as defined by basic set theory. A*ll sets are forced to comply with measurement minimums and the rules of general relativity for any relative set – including the set of all sets.* Because the set of all sets is a relative set, it can contain conflicting reference frames. However, conflicting frames of reference can only be measured sequentially. *The result is that the set of all sets can be both a member of itself and not a member of itself.* Because the set of all sets is a relative frame of reference, it is *forced* to measure these members sequentially. This creates not only the forcing of the set of all sets to conform to the rules of general relativity, but also results in the forcing of any *relative* set to contain at least two members – one real and one imaginary. Any observer in this frame of reference will be forced to measure these members in sequential time. As we hope we have demonstrated in this short discussion, The Liar's paradox, The Card paradox, The Barber Paradox, The Grandfather Paradox, The Schrodinger's Cat Paradox and even the Russell Paradox are all simply complying with the rules of general relativity. In our next section we take the ideas we have discussed regarding paradox and measurement and apply them to a short discussion of how Einstein's rules impact Gödel's theory of incompleteness.

1.07 Einstein and Gödel

Kurt Gödel was a good friend of Einstein's. They would often walk to and from work together during Gödel's time at Princeton. Gödel used the paradoxes that he found in the Peano axioms to form the basis of his second theory of incompleteness[1]. Ironically, in this discussion of paradox, we will be using Einstein's limits[2] to force boundary conditions on Gödel's theories. Gödel's presentation of the paradox associated with his second theory of incompleteness can be stated in the following manner :

"*If P is consistent, then Con(P) is not provable from P*."[1]

We can use the same technique that we used earlier to identify all observers in the paradox - including Dr. Gödel:

Observer 1- sets of P

Observer 2- sets that are not of P

Observer 3- Kurt Gödel

In this instance, we have identified an unstated observer that can observe both P and not P. Dr. Gödel has neglected to include his frame of reference in this problem. His measurement minimums include all the same minimums as the other two frames of reference , but he brings with him the understanding of the concept of "provability" and the relative nature of "P" and "Con P". The other two frames of reference do not observe a paradox. From the frame of reference of observer one, all sets are of P and true. From observer two's frame of reference all sets are not P. Observer two adds a minimum measurement of an understanding of true and false. Let us summarize the measurement minimums for each frame of reference:

Observer 1- sets of P (held to minimum measurements of {set} and the definition of (P))
Observer 2- sets that are not of P (held to minimum measurements of {set} and the definition of (P) and Con(P))
Observer 3- Kurt Gödel (held to minimum measurements of {set}, the definition of (P) and Con(P), and the definitions of (provable) and (not provable))

Because he can observe the relative nature of both of the other frames of reference, Dr. Gödel is forced to comply with the rules of general relativity regarding simultaneous measurement. He has the same problem as any observer in a relative frame of reference. By trying to measure two conflicting statements simultaneously he is violating his set forcing rules. The result is the appearance of paradox. All relative sets are forced to contain a minimum of a real and a complex measureable state. To describe this requirement in set notation we can say that:

$$\{\, i\,, R_m\} \;=\{S_R\} \qquad (01)$$

Where $\{S_R\}$ represents any relative set and R_m represents the measured frame of reference. Obviously, this subject deserves a more extensive and formal investigation, related to the roots of set and number theory, that is beyond the scope of this letter. However, in our final sections, we will have a short discussion of how relative set forcing can establish minimums for the measurement of the the unit integral and set real boundaries for the Peano axioms.

1.08 Relativity and the unit integral

In order to be of any use to science, theoretical principles must, at some point, succumb to measurement in real space. As we have shown earlier, all relative sets can be forced to comply with the rules of general relativity and held to minimums for measurement. We have shown that one of the repercussions is that the set of all sets can be forced to follow the limits of general relativity regarding the simultaneous measurement of relative members of the set. In our final two sections, we will attempt to demonstrate that, *in any set bounded by real measurement both zero and infinity are not measurable states.* This simple and logical premise invalidates all smooth approaches to zero in a system defined by measurement and establishes a measurement boundary that is greater than zero for any real measurement. To demonstrate how these limits affect basic number theory, we will exclude zero from the set of natural numbers and replace it with a measurement boundary dividing real space from complex potential. We can represent this as a hard measurement boundary between the real and complex potentials within any relative set:

$$\{\, i\,, R_m\} \;= \{S_R\} = 1 \qquad (02)$$

In which R_m represents the measurement of complex potential in real space. To understand how this affects number theory, we need to define minimums for the measurement of the unit integral. All relative sets are required to contain an imaginary component and a real component that represents the measurement of complex potential in real space. Therefore, we can define the unit integer using a measurement boundary and an imaginary zero:

$$0i| \leftarrow 1 \qquad (03)$$

The boundary we have placed based on minimums for the measurement of any relative set will always prevent the establishment of a zero in real space. As we have demonstrated, *all zeros are imaginary under these set forcing boundaries*. To represent this geometrically we can think of any line with a real point at one end and an imaginary zero at the other:

$$0i| \leftarrow - - - - - - - - - - - - - - - - - - 1 \qquad (04)$$

This establishes the real part of our function converging discretely on an imaginary zero. In effect we have a line that has real point on one end and imaginary on the other. *It only becomes a real measurement when both ends of the line are established through measurement.*

$$R_m \leftarrow - - - - - - - - - - - - - - - - - -1 \quad (05)$$

$$R_m = 1 \quad (06)$$

In which R_m represents the stopping measurement of the converging function as well as a barrier to the measurement of any null or empty sets. There is no way to measure a purely imaginary number in real space. The minimum measurement is always a real number, therefore *real arguments will always outweigh any imaginary arguments in a real measurement.* Requiring that any lower boundary for real measurement to be greater than the sum of complex arguments:

$$\sum_{m=0_i}^{m=\infty} i_m < {}^{R_m}/_2 \quad (07)$$

Establishing an *real* boundary for measurements in the complex plane at $< \left(\frac{1}{2}\right)$ therefore:

$$\left\{ 0i \mid \tfrac{1}{2} \right\} = \{ i_m , R_m \} = \{S_R\} = 1 \quad (08)$$

Which leads us to the conclusion that *any imaginary zero must have a real part of one half to be a real measurement.* Unfortunately, a formal mathematical discussion of the how we establish limits to number probabilities must wait for a later investigation and a more specialized audience. In our final section, we discuss how defining minimums for measuring the unit integral in real space affects the Peano axioms and Gödel's theory of incompleteness.

1.09 Measuring the Peano axioms

Since it's adoption in the latter part of the 19th century, Peano arithmetic[5] has served as the foundation for mathematical theory and practice. It is based on the simple idea that inductive reasoning can provide rules for logical formulation of the number sets. It was intended as a *complete* description of the basic operations of arithmetic. Kurt Gödel later demonstrated that the Peano axioms were incomplete because they allowed for infinite sets which could exceed the set of natural numbers[1]. This established a formalized view of infinite sets and the use of zero as one of the members of the set of natural numbers. The Peano axioms can be stated as follows:

1. 0 *is included in the set of natural numbers*

2. *If (a) is a number then it's successor,(b), is a number*

3. *Two numbers whose successors are equal are also equal*

4. *If a set of S contains both* 0 *and every successor of every number in S, then every number is in S*

It turns out there is a very simple solution for this that will allow us to complete the Peano axioms. The solution is to consider the Peano axioms as measurements. This requires that zero only exist in the complex plane and not be included in the set of measureable natural numbers. The Peano axioms and the set of natural numbers can be now re-stated as follows:

1. *For any natural number there exists forced set requirements*

2. 0 *is not a natural number in any frame of reference because it has no measureable minimum*

3. *any number(a) must be measured to establish its frame of reference.*

4. *If a set of S contains both* $0i$ *and the measurement definition of (a) then the set of S is complete within that frame of reference.*

5. *If (a) is a measured number then it's successor,(b), is a number*

6. *Two numbers whose successors are equal are also equal*

7. *If a set of S contains both* $0i$ *and every successor of every number in S, then every number is in S*

The *measured* Peano axioms now establish a minimum real measurement of 1 for the set of natural numbers using the same minimums for measurement that we established for the resolution of paradox. This has the effect of constraining all infinite sets to the complex plane. Our adjustment to the Peano axioms have made it impossible to make a measurement in real space using infinities. Therefore, infinite sets, in a system defined by measurement, are invalid in real space. Gödel's proof of incompleteness relies completely on the existence of infinite sets and on the inconsistencies within the Peano axioms. By limiting all infinite sets to the complex plane we have invalided Gödel's proof in measurable space. This allows us to use the adjusted Peano axioms to define the boundaries for all numbers used as measurements in real space.

Because:

$$\left\{ 0i \mid \tfrac{1}{2} \right\} = \{ i_m , R_m \} = \{S_R\} = 1 \qquad (09)$$

All zeros and empty sets are eliminated from a system of mathematics defined by measurement. Therefore, the unit integral for set of natural numbers becomes bounded by measurement as well:

$$\{ 0,1,2,3,4,5\ldots \} = \{N\} \qquad (10)$$

becomes

$$\{0i ,1,2,3,4,5\ldots \} = \{N_n\} \qquad (11)$$

Where $\{N_n\}$ represents the *adjusted set of natural numbers*. For the adjusted set of naturals, all zeros and infinities are held to the complex plane. *This simple and intuitive solution results in a complete theory of mathematics for measurement.*

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