Measuring the Universe Part II: Establishing the Topological Boundaries for Gravity, Dark Energy and Dark Matter in the early universe.

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Abstract:

In our paper, "Measuring the universe Part I: Dark Matter, Dark Energy, and The Big Flip"(2022), we established an equivalency between the curvatures defined in the Friedman equations and the mathematical equivalencies found in topology. We also introduced a new model for "The Big Bang" that adheres to Planck measurement standards while explaining both the nature of dark matter and dark energy. In May of 2022, we used this model to accurately predict the early appearance of black holes and massive galaxies before, they were observed by the James Webb Space Telescope (JWST). In Part II, we will demonstrate the complete scalability of this model, from quantum to galactic scales, as well as explaining the nature of DM and DE using William Thurston's topological limits on a three-sphere.

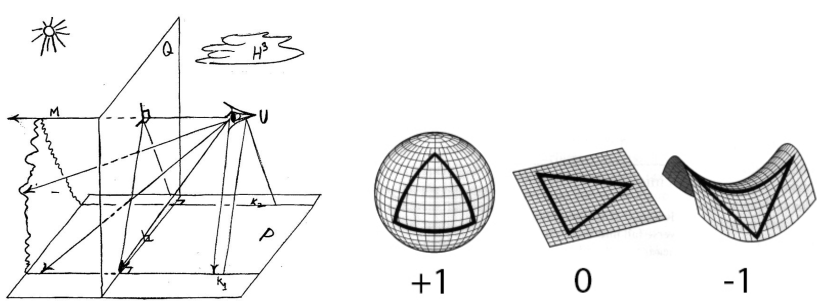
Introduction

We will assume the reader is familiar with the monumental achievements of William Thurston (1946-2012). Any discussion of topological 3-manifolds begins, and ends, with Bill Thurston's "*The Geometry and Topology of Three-Manifolds" William Thurston, (1979, AMS 2021).* His understanding of geometric and topological equivalencies will never find an equal. Let us begin our discussion with his opening statement from Chapter 2-" *Elliptical and Hyperbolic Geometry"*:

" There are three kinds of geometry which possess a notion of distance, and which look the same from any viewpoint with your head turned in any orientation: these are elliptic geometry (or spherical geometry), Euclidean or parabolic geometry, and hyperbolic or Lobachevskian geometry. The underlying spaces of these three geometries are naturally Riemannian manifolds of constant sectional curvature +1, 0, and −1, respectively."

William Thurston, *Geometry and Topology of Three-manifolds (1979, AMS 2021)*

In this paper, we hope to demonstrate how Thurston's equivalencies between topological boundaries on a three-sphere can be used to form conformal measurement frames of reference (MFR) for gravity, DM and DE. Unlike current models of the universe, which require a choice between a flat or curved universe, our use of conformal topology allows for the simultaneous measurement of curve and inverse curve as a measure of completeness. Because both curve and inverse curve potentials exist simultaneously, their associated measurement frames of reference (MFR) also exist, *simultaneously (Figure 1).*

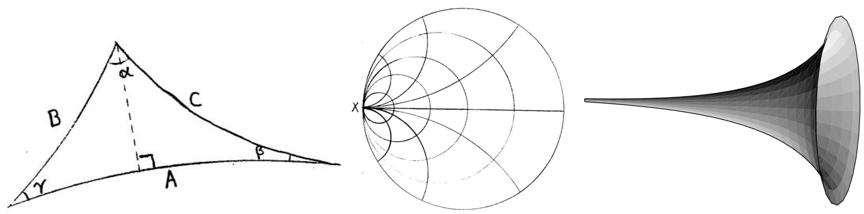


(Figure 1) *In a complete universe, all conformal observations of completeness exist simultaneously. We show Bill Thurston's drawing of simultaneous, perpendicular observations in hyperbolic 3-space next to an illustration of Riemannian manifolds of constant sectional curvature of +1,0 and -1.*

In Part I, of our continuing discussion on measurement theory, we demonstrated the direct relationship between the topological measurements of completeness and measurements associated with gravity, Dark Matter (DM), Dark Energy (DE). We discussed how Measurement Frames of Reference (MFR), *at all scales*, can be used to connect the boundaries associated with set theory and geometry to the measurement of quantum and relative probabilities in real space. In this discussion, we will formalize MFR using Bill Thurston's definitions of geometric completeness, as well as our own. We will demonstrate how topological completeness, and Lorentz invariance, allows us to equate to all measurements associated with the current ΛCDM model without the need for conformal "fields" or zero point vacua. *(Sections 1.0-1.2)* will concentrate on presenting some of Bill Thurston's concepts related to completeness in ring and knot theory as a scaffold for our interpretation of measurement theory. *(Sections 2.0-2.2)* will discuss how topological MFR, and probability theory, establishes the measurement boundaries for completeness. We discuss how topological completeness can be equated to the measurement of gravity, DM and DE at quantum and relative scales. *(Section 3.0)* demonstrates how galaxies and clusters can be modeled as conformal hyperbolic knots. We expand on the discussion regarding the current ΛCDM model and demonstrate how the principles of phase-space bifurcation can model the early ionization of the universe and the formation of early Black Holes (BH). We also demonstrate how the equivalences in ring topology and homology can be used as a framework for the expansion of the universe. Perhaps the clearest example of topological equivalency, in real space, is the equivalency we find between Thurston's use of the hyperbolic plane and the boundaries of General Relativity found in Einstein's field equations. Let us begin our discussion there.

1.0-Hyperbolic Coordinate Space

Measurements in hyperbolic space are written in (H^3 *or* $H^2 \times R$) coordinates. Hyperbolic space preserves angles, but not lengths. Unlike Euclidean space, hyperbolic space cannot admit parallel measurements *(Figure 2).*



(Figure 2) *Hyperbolic coordinates preserves angles, but not lengths. Thurston's original drawings demonstrate that the Law of Sines and Fermat's Theorem are both valid in hyperbolic space. In the center, a Thurston horosphere is illustrated using conformal closed curves. On the right, we include the 3-conformal surface tensor that can be equated to the stress tensor in Einstein's field equations.*

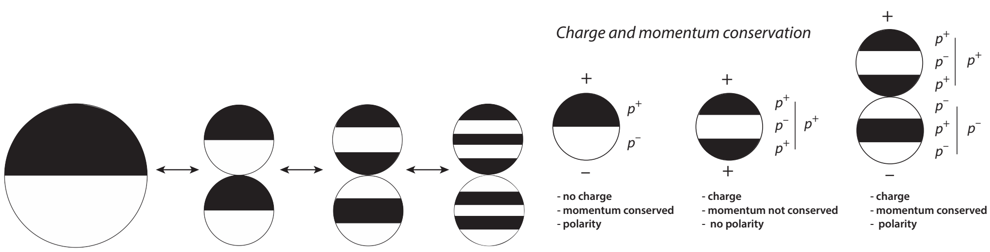
Topological descriptions of hyperbolic space are what we associate with the actions of the tensors in Einstein's field equations. The stress tensor in the Einstein field equations is a single point tensor for four-momentum and is written in hyperbolic coordinate space.

$$T_{\mu v} = R_{\mu v} - \frac{1}{2} R g_{\mu v} + \Lambda g_{\mu v}$$

In the field equations, $T_{\mu v}$ represents the stress momentum tensor, $R_{\mu v}$ the Ricci curvature tensor, $Rg_{\mu v}$ the Ricci scale tensor, and $\Lambda g_{\mu v}$, represents the actions of Einstein's cosmological constant. Both Ricci tensors are related to the deformation of Euclidean coordinate space. The metric tensor, $g_{\mu v}$ maps Euclidean lengths and angles onto conformal hyperbolic space. The equivalencies found between the field equations and Thurston's descriptions of hyperbolic space are fairly obvious. Hyperbolic MFR are limited to a single point of reference for all Lorentz invariant properties. All probabilities are, therefore, relative to the measurement frame of reference. This is the nature of gravity. Describing GR and Lorentz invariance, using topology, allows us to step away from mass and work strictly with the topological equivalencies found in Einstein's field equations. In the next section, we shall discuss how the transformation from hyperbolic to spherical, and elliptical probabilities, allows for polarity, the establishment of charge and a topological equivalency to measurements of "force" and "mass" in the standard model.

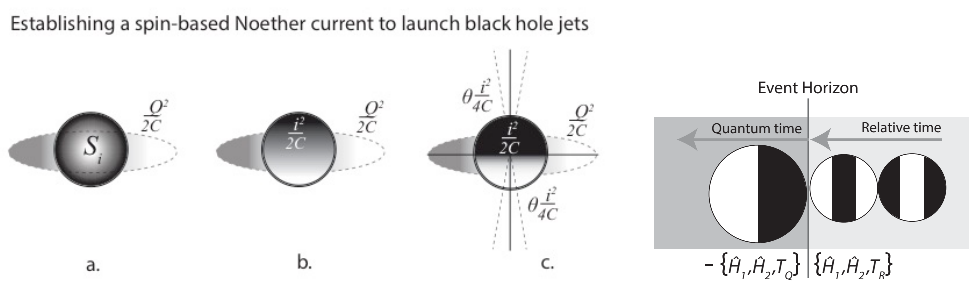
1.1-Spherical and elliptical coordinate spaces

In ring theory, spherical boundaries are represented as ($S^3 or\ S^2 \times R$), spaces. Spherical measurements with the poles identified are elliptic measurements ($E^3 or\ E^2 \times R$). Any two points measured in elliptic coordinate space define a real line. Like the limits we established for all relative sets, elliptical measurements can be equated to spherical measurements by including both real, and imaginary, components. Any 2-point correlation, or wave-based measurement, is measured in spherical or elliptic coordinate space with a real and imaginary component. In spherical space, one of the two poles, in a dipole, is always imaginary and in elliptic space the poles are defined. Spherical and elliptical probabilities are tangent to the hyperbolic imaginary plane. Thurston uses perpendicularity to the Poincare disk, D^n, to translate between hyperbolic, spherical and Euclidean coordinate-spaces. As we mentioned earlier, hyperbolic, spherical and elliptic spaces cannot admit to parallel measurements. Parallel lines can only be measured in Euclidean space. In our paper on QCD dynamics and equivalencies, we demonstrated that this geometric boundary results in all mass probabilities in real space having the limit of coupling, represented by the "strong force" in High Energy Particle Physics (HEP)*(Figure 3).*



(Figure 3) *Polarity, current, spin, angular momentum and Coulomb dynamics must be conserved for all charged particles. Conservation of these Lorentz invariant boundary conditions is accomplished through coupling.*

Topological boundaries can be used at both the quantum and relative scales to define all the probabilities currently associated with HEP. Because polarity, charge and coupling are requirements for any measurement of mass, spherical and elliptical requirements define all measurements in the standard model of particle physics. As we have discussed, a statistical MFR is the basis of both Quantum Electrodynamics (QED) and current High Energy Particle theory (HEP). Statistical MFR allow us work without the "fields" and "forces" currently written into the standard model. Like hyperbolic space, spherical coordinates do not admit to parallel lines. We can, also, use spherical coordinates to define the polarity of a black hole at the horizon. As we demonstrated in our paper, *"A Black Hole as ideal spin capacitor" (Blackwood 2019,)* the real and imaginary nature of spherical coordinates, allow us to model the interior of a BH using Noether ring spin potential to establish a dipole moment and Noether current to launch jets*. (Figure 4).*

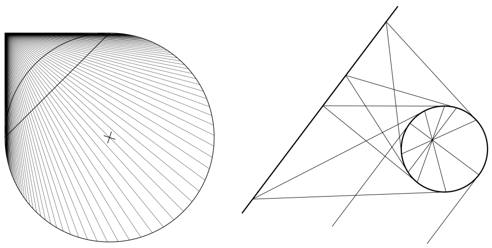


(Figure 4) *The real and imaginary nature of spherical coordinates and Noether rings, allows us to model the interior of a BH as an ideal spin capacitor.*

Our model for a black hole, as an ideal spin capacitor, is written as a topological boundary between Euclidean and Hyperbolic probability spaces and the imaginary hyperbolic plane allows us to use Noether ring theory to translate between Bayesian and Markovian MFR. We discuss this further in (*Section 2.0- Simultaneous measurement and completeness).*

1.2-Euclidean Coordinate Space

Euclidean space defines geometric lines and points. It is the space of Euclid's Geometry and Fermat's probabilities. Of all the three MFR, *Euclidean space is the only measurement frame of reference that allows for the existence of parallel lines (Figure 5).*



(Figure 5) *Hyperbolic, Spherical and Elliptic measurements cannot admit to parallel measurements. Parallel measurements, and quantum uncertainty exist in the imaginary Euclidean plane.*

In this paper, we will exploit this equivalency to create our definitions of DM and DE using the concept of topological completeness. Thurston defines completeness in Euclidean space using the isometry between curve and inverse curve. He reminds us that any sphere in Euclidean space, with radius r, has a constant curvature of $\frac{1}{r^2}$. He uses the law of Sines, as well as the relationship between curve and inverse curve, to establish the concept of geometric completeness. As observers in hyperbolic space, we are unable to measure straight-line probabilities. In both hyperbolic and spherical spaces, straight lines, points and parallel lines are all imaginary measurements. Euclidean least-time probabilities exist outside of our view and establish the appearance of invisible "forces" for all observers. The mathematical and statistical equivalency, between imaginary curve potential, and it's measurement in real space, allows us to equate the boundaries of geometric completeness to the measurement of quantum and relative probabilities in real space.

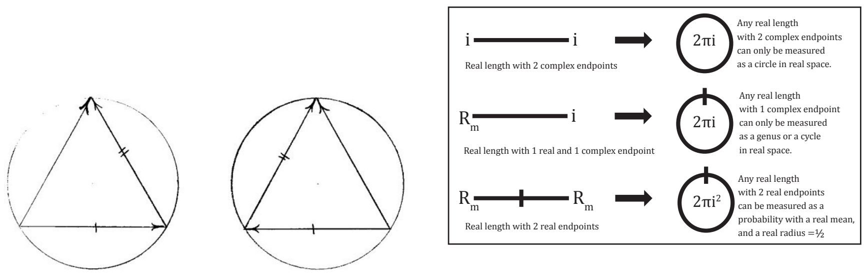
2.0-Simultaneous measurement and completeness

In our paper on the, "*Theorem of Geometric Completeness" (Blackwood 2021),* we proved that the smallest measure in real or complex space is a simple, closed curve. It is this definition, along with Bill Thurston's definition of geometric completeness, that will connect topological and geometric boundaries to their measurement in real space as gravity, DM and DE. Thurston's, proven, boundaries on a three-sphere are limited to the same real and imaginary definitions of completeness that we use in set theory to define a minimum measurement of completeness. He uses the concept of the irreducibility of curve and inverse curve, Sine and cosine, to define the boundaries of a complete hyperbolic manifold:

"Any compact, irreducible, atoroidal 3-manifold containing an incompressible surface has a complete Hyperbolic structure."

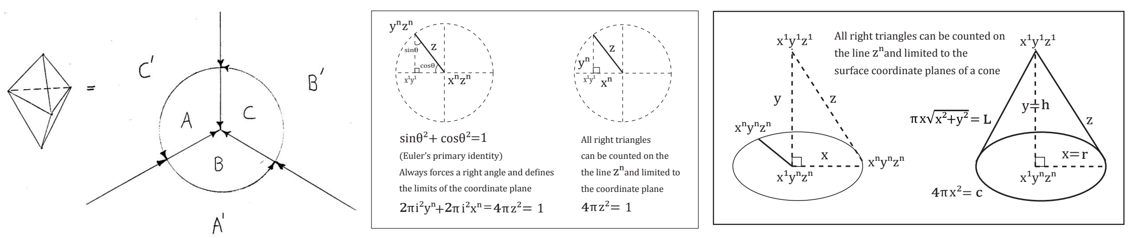
William Thurston, *Geometry and Topology of Three-manifolds (1979, AMS 2021)*

The measurement of geometric completeness, in real and complex spaces, will allow us to equate all quantum and relative "forces" to the topological boundaries found in ring and knot theory. *(Figure 6)* shows how completeness is related to geometric and topological irreducibility using a closed simple curve. Our proof on this subject is based on the same concepts used by Thurston to define geometric completeness.



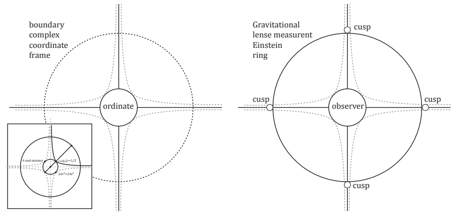
(Figure 6) *On the left Thurston demonstrates how hyperbolic completeness can be equated to Euclidean geometry. The other illustration is from our paper, "The Geometric Theorem of Completeness"(2021), where demonstrated that the smallest measurement in complex or real space is a simple closed curve.*

Everything we demonstrate depends on the simultaneous existence of all topological spaces. Because hyperbolic space preserves angles but distorts lengths, we can also define completeness using the basic proofs, established by Fermat, regarding least-time principles in Euclidean space. Least-time principles allow us to set a common time frame between relative and quantum probabilities. *(Figure -7)* shows Thurston's equivalency between geometric space and completeness in the hyperbolic plane. Next to Bill's drawing we have placed two illustrations from our paper on Fermat's theorem and geometric completeness.



(Figure 7) *On the left Thurston demonstrates how the isometry of the Law of Sines defines completeness. The other two illustrations are from our paper, "Three paths for Fermat"(2020), where we used solutions to Fermat's Theorem to define completeness.*

The advantage of using geometric completeness as a measurement limit becomes evident in any translation of boundaries between Quantum Mechanics (QM) and General Relativity (GR). General Relativity relies on a hyperbolic singularity to define the measurement of completeness. Quantum measurements must always include the concept of uncertainty. Completeness in quantum MFR, therefore, requires both a real and imaginary component. Our geometric boundaries associated with measurement theory can be equated to completeness in hyperbolic, spherical and Euclidean spaces. Because spherical and elliptical MFR allow for polarity and charge, all wave-based measurements exist in spherical or elliptic space. Parallel measurements can only be made in complex Euclidean space. The simultaneous real and complex probabilities of Quantum Electro-dynamics (QED) follow the limits of Spherical space. Spherical potentials exist in Bayesian time as waves or Markovian time as wave packets. Elliptical space defines both poles in a real measurement and can be used in the measurement of coupled probabilities. As we discussed in Part I, real and imaginary measurements of geometric completeness can, also, be equated to a complete Einstein ring *(Figure 8).*



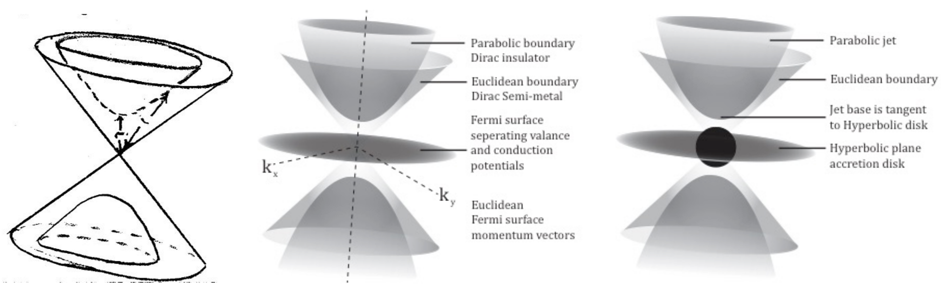
(Figure 8) *Any complete system of geometry must be able to represent both the vertex of a cone in real space, and a genus one modular form (torus) in the upper half of the coordinate plane. All modular forms and homographic groups, in any complete system of geometry, have a minimum real length* $= 2\pi i$. *Any real length or radius, with either, real or complex endpoints, can be held to the minimum measure of a simple closed curve and equated to the boundaries of an Einstein ring.*

Like our limits in set theory, spherical space has a real and imaginary part that fulfills the conditions for quantum uncertainty and allows for completeness of quantum measurements at any scale. We can add Thurston's geometric definitions of completeness to our own to define the measurement of relative and quantum probabilities as "forces" in real space. Thurston tells us that:

"A sphere in Euclidean space with radius r has a constant curvature $1/_{r^2}$*. Thus, Hyperbolic space should be a sphere of radius i. To give this a reasonable interpretation, we use an indefinite metric* $dx^2 = dx_1^2 + \cdots + dx_n^2 - dx_{n+1}^2$ in R^{n+1}*. The sphere of radius i about the origin in this metric is the hyperboloid* $x^2 = x_1^2 + \cdots + x_n^2 - x_{n+1}^2$ "

William Thurston, *Geometry and Topology of Three-manifolds (1979, AMS 2021)*

Hyperbolic space must establish a minimum measure of length to establish a metric. In our paper establishing the *"Geometric Theorem of Completeness"(Blackwood, 2021),* we proved that measurement in real or complex space contains a minimum length measure of a simple closed curve. The Hyperbolic sphere of radius *i* allows us to equate a constant inverse curvature with a sphere in Euclidean space and our limits on completeness. Geometric completeness is not limited by scale. The conformal Hyperboloid metric, described by Thurston, can be tied to the conic and parabolic components of black hole Jets in Active Galactic Nuclei, (AGN) as well as the quantum probabilities associated with measurements of topological insulators *(Figure 9)(Appendix A)*.



(Figure 9) *The conformal Hyperboloid metric, described by Thurston, can be tied to the conic and parabolic components of Black Hole Jets in AGN as well as the quantum probabilities associated with measurements of topological insulators.*

Geometric probability and completeness exist, *simultaneously*, for all observer frames of reference. Quantum and relative MFR have different time requirements. All quantum probabilities must be measured using Markov chains to preserve uncertainty. Bayesian time can be equated to waves or cycles. For both quantum and relative MFR there exists a simultaneous and measureable probability in both hyperbolic and Euclidean space. Gravity is the result of all of relative MFR being held to the same hyperbolic curve. Quantum frames of reference use Markovian time and Markov chain probabilities, but follow the same curved hyperbolic curve. The imaginary hyperbolic plane allows us to use ring theory to translate between Bayesian and Markovian MFR. Because Euclidean space allows for parallel MFR, quantum MFR are allowed parallel, Markovian, time-lines. The result is the ability of Euclidean probabilities to adhere to the principle of uncertainty while equating to measurements in relative time. In the next Section we will discuss how the completeness of curve and inverse curve manifest themselves as conformal mapping probabilities in hyperbolic space for measurements of DE and the Hubble constant.

2.1-DM as measure of topological completeness

The ability to make simultaneous equivalencies between curve and inverse curve allows us to map any conformal hyperbolic surface with a Euclidean tensor, tangent to the hyperbolic disk D^n. It is this, conformal, Euclidean tensor that we will use to model the measurements of "dark matter". A conformal hyperbolic torus allows us to model all galaxies using the same topography. In this model, galaxies become complete geometric probabilities built around the intersection of hyperbolic, spherical and Euclidean probability spaces. Galaxies can then be modeled as, both, topologically and relatively complete. As a complete Euclidean tensor acting on hyperbolic conformal space, DM boundaries allow for parallel measurements. Lorentz invariance requires any conformal, and complete, Euclidean tensor to act on all relative *and* quantum probabilities. DM is not measureable using radiation because it requires a Lorentz invariant measurement of all relative probabilities, like those found in gravitational lensing. We can associate all current measurements associated with DM with a topological equivalent. *(Table 1)* gives a quick overview of the primary measurements currently associated with the actions of DM coupled with their topological equivalents:

(Table 1)-Measurements of Dark Matter

Measurement Dark Matter	Topological Equivalent
Gravitational lensing measurements	Gravitational lensing measurements are complete because they are Lorentz invariant. Strong gravitational lensing measurements can be equated to Einstein ring topology. Gravitational wave measurements are written in spherical or elliptic coordinate spaces because of the addition of polarity to the MFR.
Galaxy rotation and structure	Hyperbolic space conforming to a Euclidean tensor equates to all actions associated with momentum measurements of star orbits in galaxies. This is because all galaxies are complete rings or knots.
Galactic clusters	Galactic clusters are the result of conformal hyperbolic rings combining into single knot. The measurements associated with galactic clusters are the same as the conformal space requirements for each of the component galaxies. In our model many knots combine to form a single knot because they all share the same unknot. (Section 3.0- *Galaxies and Cluster as knots*)
(Bullet Cluster)	Observations of The Bullet Cluster provided early evidence existence of DM. The Collision of two large clusters of galaxies demonstrated that most of the matter could not be measured using x-ray observations and could only be measured using gravitational 2-point probability distributions. Any two-point probability distribution can be equated to elliptic or spherical coordinates.
PLANCK measurements CMB	DM and DE are written into the PLANCK collaboration measurements of BAO in the CMB. PLANCK measurements rely on the establishment of Dipole moments. Dipole moments are spherical or elliptic probabilities *(This is covered in greater depth in Part I of this paper)*.

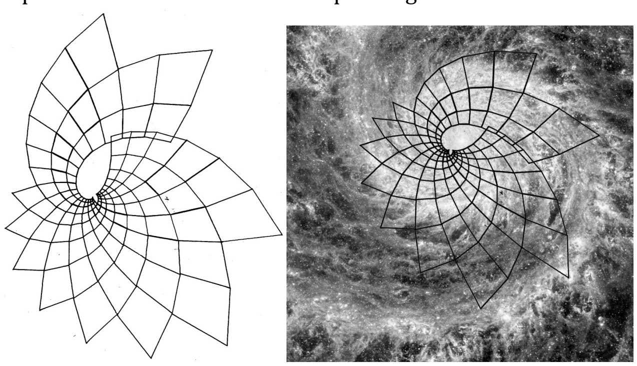
Any categorization of galaxies, including Edwin Hubble's first "fork" diagram of galactic evolution, are measures of completeness. Topological completeness and Lorentz invariance allow us to model galaxies as *complete* probabilities. Our topological definition of completeness allows us to jump between relative and quantum MFR, model actions at the horizon of a black hole, explain the nature of the Hubble constant and model the actions DM and DE without resorting to new particles or new physics. Gravity, in our model, is the Bayesian probability that exists for all, relative MFR, limited by the speed of light. All quantum frames of reference use Markovian time and Markov chain probabilities. This allows us to equate quantum and relative time scales utilizing Markovian and Bayesian statistical potentials. Statistical completeness can be equated to the measurements associated with Quantum Electrodynamics or Quantum Chromo-dynamics in High Energy Particle Physics (HEP). The difference we have to these models is that they are both based on the existence of zero-point vacua represented as "forces". Our model uses the completeness of probability measurements in real and complex space as the only invisible "force".

Thurston's definition of topological completeness, based on the minimum measure of curve and inverse curve, will serve as the conformal probability for all galaxies in our model. This eliminates the conflict between quantum and relative measurements. Unlike Quantum Field Theories (QFT, QCD), Gauge theory (GT) or Conformal Field Theories (CFT), our model is purely statistical and based on Measurement Theory (MT). By attaching a statistical MFR to the requirements of topology we can model actions at both the quantum and galactic time scales, giving us a complete measurement of Lorentz invariance. As we have demonstrated, statistical completeness allows us to model the universe as a complete probability limited only by the boundaries of topology and measurement. When the universe is viewed as a complete probability, all galaxies, galactic clusters, stars and black holes become probabilities that are linked together by Lorentz invariance. In the next section we discuss how completeness and conformal mapping can be used to describe the expansion of the universe and explain the nature of the Hubble constant.

2.2-Dark Energy and Hyperbolic conformal mapping in Euclidean space

When all probabilities and curves, both real and complex, exist simultaneously in hyperbolic space, they have a measureable, and perpendicular, equivalency in Euclidean space. Measurement in curved space-time is limited by the speed of light. Euclidean space is not. Of the three spaces we have been discussing, Euclidean space is the only coordinate space, which allows for the existence of parallel, relative and simultaneous, measurements and, therefore the only space that can accommodate the concept of quantum uncertainty. As we approach quantum scale measurements, we get closer and closer to, Euclidean, least time probabilities and must shift to Markovian time. Both Dark Energy and Dark Matter operate on least-time principles in Euclidean space. And work in Markovian time. This is one of the reasons DM and DE have been so difficult to measure for observers living with Bayesian probability and the limits of GR. In this model, gravity is equated to hyperbolic least-time potential and is not created by "mass". From the viewpoint of our statistical model, mass is simply a probability caused by the requirements of least time completeness at the quantum scale. Least-time completeness allows us to use topological boundaries to equate Bayesian and Markovian probability spaces.

Thurston uses the equivalencies between Euclidean and hyperbolic spaces to create conformal maps. Conformal mapping is dependent on the equivalency between topological spaces. Any co-evolving hyperbolic or spherical space has a Euclidean equivalent that is measurable in real and complex space. Conformal mapping of hyperbolic least time probabilities in a Euclidean background is how we will be structuring our model the expansion of the universe and explaining the nature of "Dark energy"*(Figure 10)*.



(Figure 10) *Topological, conformal mapping of an open torus on a recent JWST catalog of early release images.*

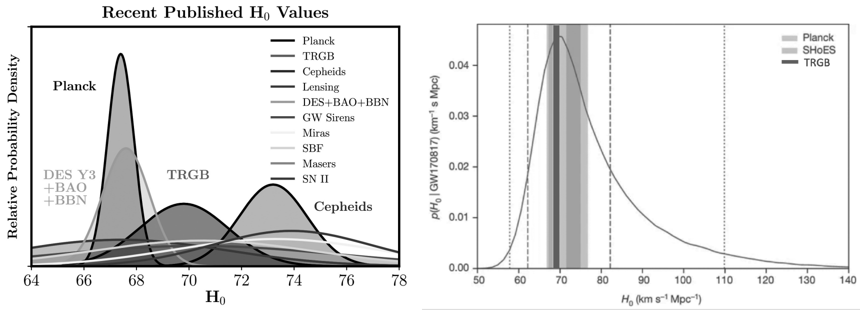
Lorentz invariance requires that all probabilities, relative and quantum, follow the curvature of space-time. Our use of topological completeness as a minimum measure for all MFR is also a measure of Lorentz invariance. Therefore, the closer we come to defining a least-time measure for all topological MFR, the closer we come to a complete, Lorentz-invariant, measurement of the completeness of the universe in real space. All observers in relative time are bounded by the speed of light and Bayesian probability. Measuring the Hubble constant requires the inclusion of all probabilities within a hyperbolic inverse curve limited by the speed of light. As we have demonstrated in our previous papers, *our value for the Hubble constant is, basically, a complete ring measurement of relative space limited by the speed of light:*

$$H_0 = \frac{2\pi i^2}{c^2} = .699i^2 = 69.9\frac{mpc}{s}$$

This same limit can be equated to the Schwarzschild radius of a black hole:

$$H_0 = \frac{2\pi i^2}{c^2} = \frac{GM}{c^2}$$

Our value for the Hubble constant is a topological requirement for all complete, hyperbolic, measurements of Lorentz invariance. Measuring each part of a Lorentz invariant distribution, separately, results in normal distributions with different means. Recent published values of the Hubble constant *(Freedman, Madore 2023),* based on the measurement of different standard candles, demonstrate this concept *(Figure 11).*



(Figure 11) *Measuring each part of a Lorentz invariant distribution, separately, results in normal distributions with different means. On the left are some of the recent measurements of the Hubble constant. On the right, we show the marginalized posterior density distribution for the Hubble constant derived from the strong gravitational lensing detection of GW170817. (Images: courtesy of Barry Madore,"Progress in direct measurements of the Hubble constant." Freedman, Madore, 2023)*

Each separate probability distribution yields a different value for the Hubble constant. It is only together that they fulfill the definition of measured completeness required by Lorentz invariance. Measurements of type 1a-Super Nova, (SN-1a) render a higher value of the Hubble constant because the limited sample size and, basis in the Levitt Law, create a normal distribution for any probability density measurement. If type 1A Super Nova represent measurements using the largest *scale* "ladder" in the universe and the Planck measurement of BAO represent the smallest ladder, then the three standard candles measured, recently by *(Freedman and Madore, 2023, see Appendix A- Observations)* represent the mean of a Lorentz invariant distribution. Our value of the Hubble constant, based only on the requirements of topology, is an exact match to this Lorentz invariant mean.

3.0-Galaxies and Clusters as knots

Thurston proved that, *every complete conformal knot contains an unknot.* Because all galaxies contain a black hole at their center, *galaxies, galaxy clusters and even the galactic web can be modeled as topological knots (Figure 12).*

(Figure 12) *Every knot contains an unknot and every galaxy contains a Black Hole in it's center (Left from Thurston). Ring theory allows for galactic clusters and the galactic web to equate to same complete knot. (Knot image, Bill Thurston) (Star cluster image: European Space Organization- ESO1825)*

When we model galaxies as knots, it gives us the ability to combine all elliptical orbits into a single, relative, path. The Black Hole at the center acts as an unknot, but we can "tighten" the knot by using the inverse of our value for the Hubble constant in place of the metric tensor:

$$T_{\mu v} = R_{\mu v} - \frac{1}{2} R g_{\mu v} + \Lambda g_{\mu v}$$

$$g_{\mu v} = \frac{c^2}{2\pi i^2}$$

$$T_{\mu v} = R_{\mu v} - \frac{R}{2}\left(\frac{c^2}{2\pi i^2}\right) + \Lambda\left(\frac{c^2}{2\pi i^2}\right)$$

$$(T_{\mu v} - R_{\mu v}) = \Lambda\left(\frac{1}{H_0}\right) - \frac{R}{2}\left(\frac{1}{H_0}\right)$$

$$2H_0(T_{\mu v} - R_{\mu v}) = 2\Lambda - R$$

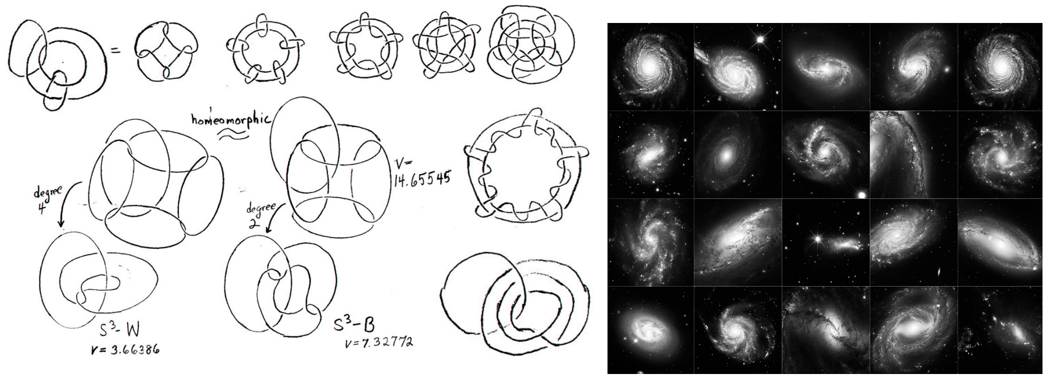
$$\frac{(T_{\mu v} - R_{\mu v})}{(2\Lambda - R)} = \frac{1}{2H_0} = \frac{1}{2}g_{\mu v} = \frac{c^2}{4\pi i^2} = D^n$$

Where D^n represents the imaginary hyperbolic disk limited by the speed of light, which lies perpendicular to the Euclidean tensors (on the other side of the equation). Equating the boundaries of a black hole to the topological boundaries of the galaxy unknot gives us the ability to equate to measurements of galactic DM halos using conformal mapping. Each galaxy or galaxy cluster DM halo becomes a Lorentz complete measurement of the sum of bound knots. *(Table 2)* demonstrates how galactic knots can define many of the characteristics associated with current galactic categories:

(Table 2)-Galaxy categories and topological equivalents

Galaxy category	Topological Equivalent
Elliptical	Elliptical galaxies can be thought of as the "loosest" knots, compared to Ultra-Compact Dwarfs (UCD). The center, unknot, acts as a center of gravity for any bound orbits sharing the same unknot.
Spiral	Spiral galaxies are the clearest example of the conformal geometry in the hyperbolic plane. Our model of a BH, as the center unknot, equates to ideal models of GR at the horizon of a BH.
Barred Spiral	Barred spiral galaxies are spirals with the addition of polarity. Unlike the hyperbolic probabilities of a simple spiral galaxy, Barred spirals have built in polarity of measurement. Both the hyperbolic and elliptical plane lie perpendicular to the central Euclidean unknot.
Irregular	Irregular galaxies are associated with the collision between different galaxies. IR galaxies have their own catalog of subtypes. Like merging black holes, colliding galaxies are in the process of forming a shared unknot.
Dwarf and Ultra-compact Dwarf	Dwarf galaxies have the largest ratio of measurements of the mass/luminosity relationship, (an indicator of the largest influence of DM on the measurement of mass). In our model of galaxies as knots, this translates to the tightest knot around the central unknot. Ultra-compact dwarfs have been tidally stripped of lowest mass stars, resulting tighter knot. This exaggerates the disparity between mass and luminosity measurements.

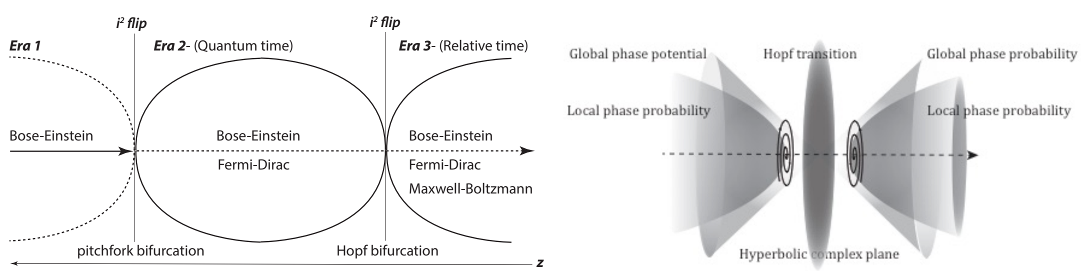
There are as many homeographic knot equivalencies as there are galaxies in the universe (Figure 13). Knots and rings have the ability to combine into larger structures while maintaining Lorentz invariance. Knots can also link together to form the largest structures in the universe, represented by measurements of the galactic web.



(Figure 13) *There are as many homeographic knot equivalents as there are galaxies in the universe. Knots can combine to form larger structures like galactic clusters and the galactic web while still maintaining hyperbolic completeness. We show just a few examples of Thurston's drawings of knot equivalents next to a compilation of Hubble galaxy images.*

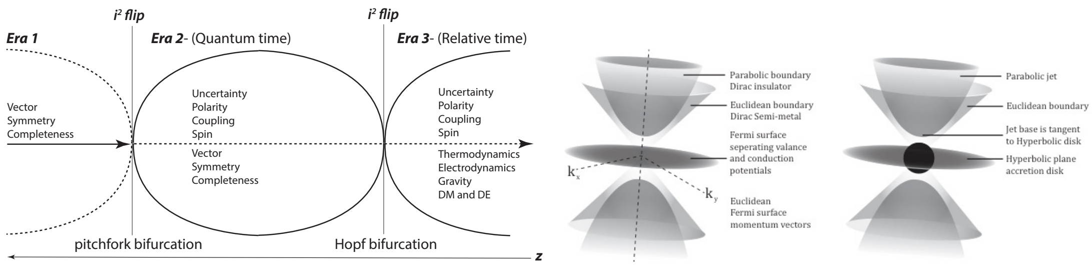
An example of observational evidence tying knot theory to galactic compactness can be found in Dwarf Galaxies (DG) and their evolution into Ultra-Compact Dwarfs (UCD). Ultra-compact dwarfs galaxies have the highest DM to luminosity ratio of all galaxies. Like DG, UCD galaxies have a massive black hole in their center. When a Dwarf galaxy compacts into a UCD, the super massive black hole remains the same size. This can be equated to the tightening of a Euclidean tensor on conformal Hyperbolic space acting like the tightening of a knot around the central unknot. We cover this, and the modeling of other astronomical and experimental observations, in *(Appendix A- Observations).*

In Part I of this discussion, we demonstrated how phase space transitions allow for simultaneous local and global probabilities. We discussed the boundary conditions of phase space bifurcation and the application of statistical principles to the modeling of the early universe. A statistical model, driven by the equivalencies found in topology, uses the complex hyperbolic plane as a Hopf transition between Markovian quantum potential and it's measurement as hyperbolic, spherical and elliptical probabilities in real space.



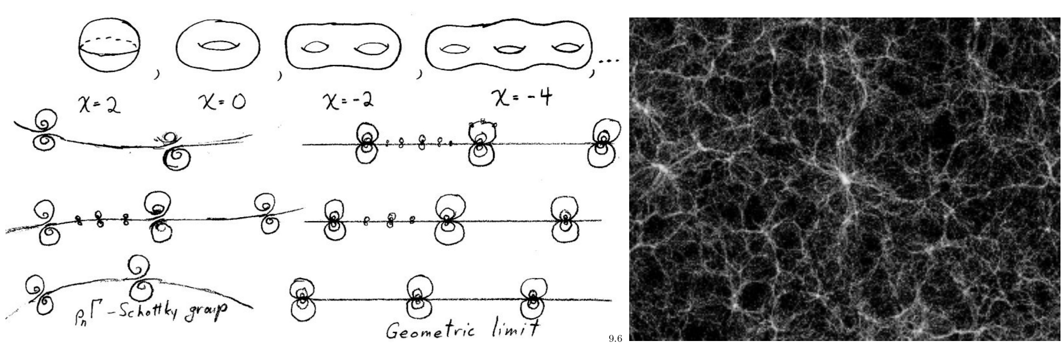
(Figure 14) *On the left, we show an illustration from Part I, showing the evolution of a statistical blackbody. Each phase space is Lorentz invariant and follows the limits of geometric completeness. This results in the globalization of local bifurcation nodes. Our local universe driven is by Bayesian cause and effect statistics and limited by the speed of light. On the right we show that Hopf phase transitions can be tied directly to Thurston's use of the imaginary hyperbolic plane.*

(Figure 14) Demonstrates how we can model the beginning of the universe using phase space bifurcation and Hopf transitions. In a topologically complete universe, Lorentz invariant probabilities are equivalent to measurements of topological completeness. Hopf transitions allow us use the complex plane as Hyperbolic potential for Markovian and Bayesian probabilities. *Galaxies are complete only when they include the Lorentz invariant probabilities at all scales.* As we have demonstrated, in all the previous sections, topology is not limited by scale. We can model any level of statistical completeness, both quantum and relative, using the boundaries of topology and geometry *(Figure 15).*



(Figure 15) *Topological equivalencies are not scale dependent. On the left we show how additive probability affects topological boundaries. On the right we use Thurston's equivalency to a hyperbole to model a Fermi surface and the dynamics of jet launching at the scale of a black hole.*

Observations of the early universe are indicating the early appearance of both massive galaxies and Black Holes. As we show here, those probabilities can be explained using the equivalencies found in topology. Thurston tells us any knot can be equated to a single unknot through Dehn surgery. This is not a scale dependent quality of knots. If we assume that the galactic web is just a large single knot, we can equate the galactic web to a single relative probability because topological equivalency is not limited by scale or the speed of light *(Figure 16).*



(Figure 16) *Thurston shows us that no matter how complicated and messy a knot is, it can still be equated to single torus and limited to the measurement of a simple closed curve. (Cosmic web image: V.Springel, Max-Planck Institut für Astrophysik, Garching bei München)*

4.0-Conclusion

On December 3, 2013, I received the inspiration that was to guide my investigations for the next ten years. As I write this conclusion, it is December 3, 2023. Since those early days, I have learned to depend on the robustness of this model. It has never let me down. I actually didn't run into Bill Thurston's work until 2022, after having already developed my model of a Black Hole as an ideal spin capacitor and my Theorem of geometric completeness. The resonance between Bill's work and my own writing, in measurement theory, was immediately apparent and I have now become, completely, "Thurstonized". In the coming months, I will be completing the Appendix for this paper, which will cover observational evidence from most of the major Astronomical surveys, focusing on JWST results from the early universe. Until then, enjoy the breadth and depth of this model.

Acknowledgements:

I am extremely grateful to Julian Thurston for the use of Bill's drawings. Hopefully, I was able to reflect just some of the clarity and beauty of his original work. Permission to use William Thurston's original illustrations was granted for distribution to the scientific community only. Any other reproduction or publication is strictly forbidden and covered by copyright. I would also like to thank Barry Madore for allowing me the use of his graphs regarding measurements of the Hubble constant.

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Conformal Mapping image: JWST early relaease images
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