Three paths for Fermat.

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Abstract

In this extension of our previous discussions, on the limits of measurement, we present three inductive proofs of Fermat's final theorem – using the geometric limits of Thales, Euclid, Apollonius of Perga, and Euler. We finish with a brief discussion of how these geometric limits compare to Andrew Wiles proof of the Taniyama-Shimura conjecture and Fermat's final theorem.

1.0 Introduction - A "point" in the history of analytic geometry.

Fermat straddled a formative time in the history of natural sciences and mathematics. By the 17th century, geometric proofs were shown to be limited in describing certain mathematical constructs. The squaring of a circle and the conics of Apollonius of Perga could not be proven using only a compass and straightedge. A quote from Descartes , *(in a letter to Beekman 1619 [5],* displays the unease he and many of his contemporaries felt regarding the transition away from geometric proofs and towards analytic and inductive mathematical proofs:

..."[In this new science] each problem will be solved according to its own nature as for example, in arithmetic some questions are resolved by rational numbers, others only by surd [irrational] numbers, and others finally can be imagined but not solved. So also I hope to show for continuous quantities that some problems can be solved by straight lines and circles alone; others only by other curved lines, which, however, result from a single motion and can therefore be drawn with new types of compasses, which are no less exact and geometrical, I think, than the common ones used to draw circles; and finally others that can be solved by curved lines generated by diverse motions not subordinated to one another, which curves are certainly only imaginary such as the rather well-known quadratrix. I cannot imagine anything that could not be solved by such lines at least, though I hope to show which questions can be solved in this or that way and not any other, so that almost nothing will remain to be found in geometry...." Rene Descartes[5].

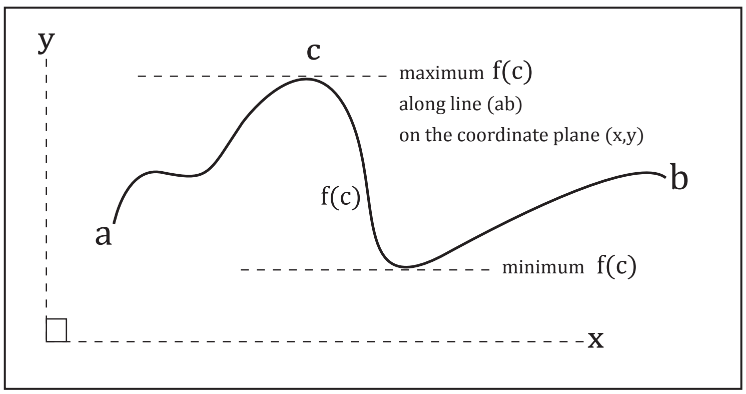
Descartes would have considered a geometric length to be the minimum measure for any algebraic sentence. For him, algebraic notation would have been a translation of geometric lengths, lines and planes. Descartes invented the use of the "ordinate" and "coordinate" to set a line. Points along the line or a plane were called "loci". Loci often indicated points of intersection of lines with other lines, or planes. Loci, unlike points in space, cannot exist without the finite boundaries established by the line or plane. Our modern concept of point functions and functions in Hilbert space, would seem quite alien to any mathematician of this era. We must always think of Fermat as a student of Euclid and Apollonius as well as a being one of the founders of modern analytic geometry probability and number theory. By the year 1637, when he wrote his taunting note from the margins of Diophantus, Fermat had already completed a number of treatises on plane geometry and the conic sections of Apollonius[1] For Fermat, Euclidian theory and geometry would still have been the most predominant form of accepted mathematical proof. Even while he was developing the basis for analytic geometry, he would still have been still deeply rooted in classical geometric principles. Therefore, in this investigation, the variables x^n, y^n *and* z^n should be thought of as geometric lengths and not exponential variations or algebraic functions. On our journey, back into the 17th century, we will attempt to walk three possible paths that Fermat may have taken towards a proof of his final theorem – using only the geometric limits of line and plane.

1.1 Fermat's concept of maxima and minima

One of the reasons I chose to write this paper is the affinity I have for Fermat's concepts of minima and maxima. In all of my papers[10-17] you will find boundaries for measurement that reflect Fermat's least time principles. The concepts of minimum and maximum values of a function on a curve were the precursor to Newton and Leibnitz's development of integral calculus. Establishing mathematical maxima and minima would set Fermat on the path towards the development of probability theory. However, Fermat would be the first to tell you that his concepts of minima and maxima of any function are rooted in geometry and the establishment of a line on a coordinate plane[2,3,4] In this, purely geometric, context, maxima and minima establish points of change on the function $f(c)$ which can be interpreted as 0 points but only as they relate to f(c) as a boundary to that function. When we convert Fermat's maxima and minima into our modern mathematical view of fuctions in Hilbert space, we allow for the creation of zero points – which were not part of Fermat's geometric toolkit. Maxima and minima are always established by two other points which form an interval or line. *In Fermat's theorem, (c) exists only because of the limits provided by the line (ab) and the plane (x,y) (Figure1).* It is our belief that Fermat's great insight may have come from an innate understanding of this basic principle of maxima and minima on a coordinate plane.

(Figure 1) *Fermat's theorem establishing maxima and minima of functions:*

$Suppose\ that\ a < c < b. If\ a\ function\ f is\ defined\ on\ the\ interval\ (a,b), and$
$it\ has\ a\ maximum\ or\ minimum\ at\ c, then\ either\ f'\ doesn't\ exist\ at\ c, or\ f(c) = 0$



1.2 - Fermat's insight limiting maxima and minima on the coordinate plane

Fermat spent much of his time devoted to re-creating lost ancient Greek texts. While he may be famous for his ability to "think outside of the box", his roots were always inside the box. He wrote a number of texts on plane geometry and the conics of Apollonius of Perga [2,3,4]. *When Fermat thought of infinity it would have been on the infinite 2-dimensional plane.* Fermat considered the technique of proof through infinite regression to be one of his greatest discoveries and he relied on it for most of his arguments. His solution to his final theorem, where n=4, was based on creating limitations to proof using infinite regression to a square. Fermat used a geometric argument to prove that roots have a limit. This technique allows the establishing of common roots of functions in Galois set theory. Andrew Wiles and Ken Ribet used many of the same techniques to align zero-points using the zeta function for modular forms and ellptic curves. Although modular forms and elliptic curves didn't exist as mathematical tools in the 17th century, the idea of geometric symmetry had been well established since the early beginnings of math and number theory in India. The question of how to divide something into equal, (or unequal), shares has been around human beings as long as fire. In fact, the need to find equal shares was the basis for Fermat and Blasé Pascal's later development of probability theory. Fermat would have been an expert in spotting symmetries based on his years working with geometry and limits to algebraic notation. When he first wrote down his note from the margins of Diophantus, Fermat may have understood that the basic geometric symmetry of this equation could equate to limits on an infinite coordinate plane:

$$x^n + y^n = z^n$$

The basic symmetry of this equation may have triggered an instant recognition of a connection to the symmetries and geometric proofs of Euclid and Apollonius. *Fermat may have realized that what seems like an* *infinite equation is actually a finite equation and that both minima and maxima are built into this same infinite equation. He has seen that he can count primes on a line and limit them to a plane – giving him a geometric coordinate system, based on right angles and minima, that he can use to prove his final theorem!*

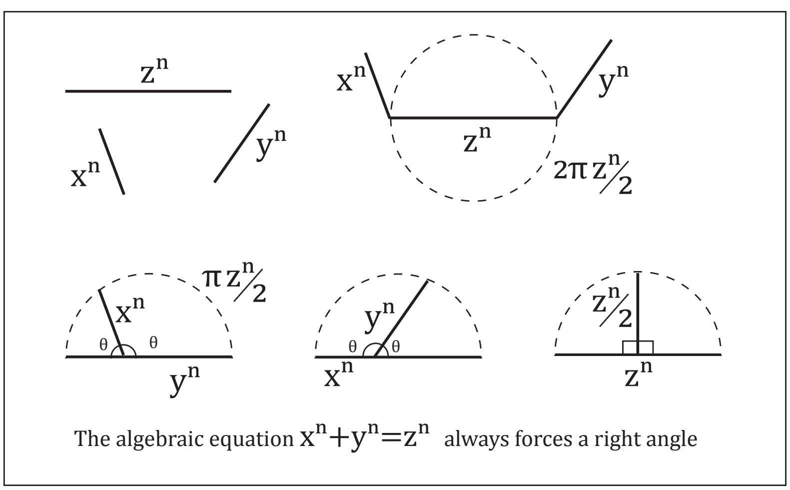
The only question that remained, for Fermat, was "which path to take?".

1.3 - Fermat takes the path through ancient Greece

Every mathematician of this era would have had Euclid's thirteen books on the *Elements* of Geometry drilled into them since childhood. Euclid's *Elements* was, for centuries, the second best selling book behind the Bible. It is likely that Fermat would have known all thirteen books by heart. He would have remembered that Euclid's Elements, Book 1, proposition 13; proves that connecting any two lines can force a symmetry of right angles on a semi-circle(Figure 2). One of Fermat's greatest gifts was the ability to "see" maxima and minima of functions. I believe he may have glimpsed this minimum measurement of the interaction of any two lines as a way to limit functions to the coordinate plane. This would have almost immediately brought to mind Thales theorem[1], *(also from Euclid: Book 3 proposition 31)[1]* which places all right angles on the circumference of a circle. It has been reported through the, somewhat suspect, annals of Greek mathematical legend, that Thales sacrificed an ox after he discovered the forcing of right angles on curve. The beauty of Thales theorem is it allows us to "count" right angles along a line that serves as the diameter of the circle. As you can see from our version of Euclid's proof (Figure 3) , *all right triangles can be counted along the line z^2 because all right angles are held to the circumference of a semicircle or circle with a diameter defined by z^2*. Remember, in our 17th century context, z^2 is still a measurement of the length of a line. This line, held to a plane, can become a square, a circle or any number of 2-dimensioal geometric shapes – including a right triangle. The forced right angle, in Thales theorem, is always limited to the semicircle (πz^n / 2). It allows us to set intervals on the line z^2. *In effect, this allows us to create defined intervals for a minima of "right triangle" along a line of any integer length which fulfills the Pythagorean theorem.* By counting Pythagorean triples on a line, we have created a prime number system based on geometric limits to minima on the coordinate plane. As we have shown, Fermat had to travel no further than his copy of Euclid's *Elements* [1] to see this path towards a final proof of his theorem.

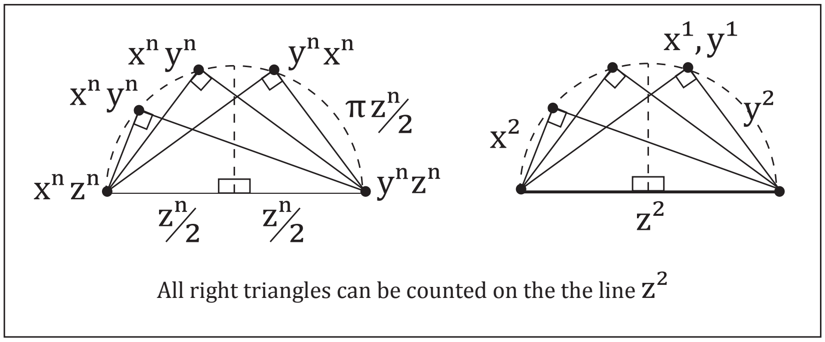
(Figure 2) *Euclid -Book 1 Proposition 13:*

$$If\ a\ straight\ line\ stands\ on\ a\ straight\ line, then\ it\ makes\ either\ two\ right\ angles\ or\ angles\ whose\ sum\ equals\ two\ right\ angles.$$



(Figure 3) *Euclid -Book 3 Proposition 31:*

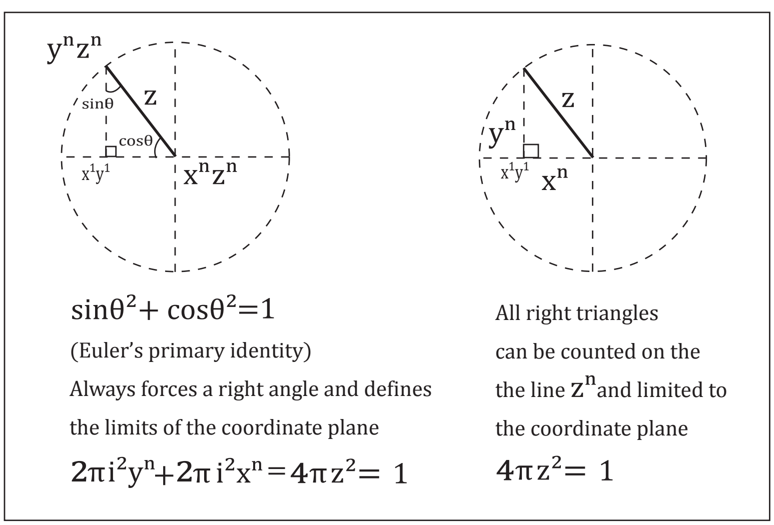
In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle; further the angle of the greater segment is greater than a right angle, and the angle of the less segment is less than a right angle.



1.4- Fermat discovers Euler's primary identity

Although Leonhard Euler wouldn't be born for another 100 years, Fermat was, clearly, clever enough to anticipate the development of Euler's primary identity. I believe that he may have seen the ability to count Pythagorean triples along the line z^n as a way to limit the coordinate plane. Using z^nas the radius of a circle allows Fermat to limit the coordinate plane to the suface area of a circle. Our, 17th century, version of Euler's identity (Figure 4), is what might have been conceived by Fermat without the use of the imaginary plane. Fermat could have easily seen that the basic relation between sine and cosine angles force a minimum of a right triangle on the coordinate plane (x^n, y^n) defined by the counting of an infinite number of Pythagorean triples along (z). This allows him to also confine any solutions to his final theorem to the coordinate plane represented by the limits placed on the intersection of the forced ordinate of the right triangle (x^1, y^1). I have included a reference to the imaginary plane, but it is not an equation that would have been used by Fermat, at this time. His restriction to the coordinate plane, in this example, comes from the counting of Pythagorean triples along the line (z^n) and not the creation of Euler's limit to the complex plane.

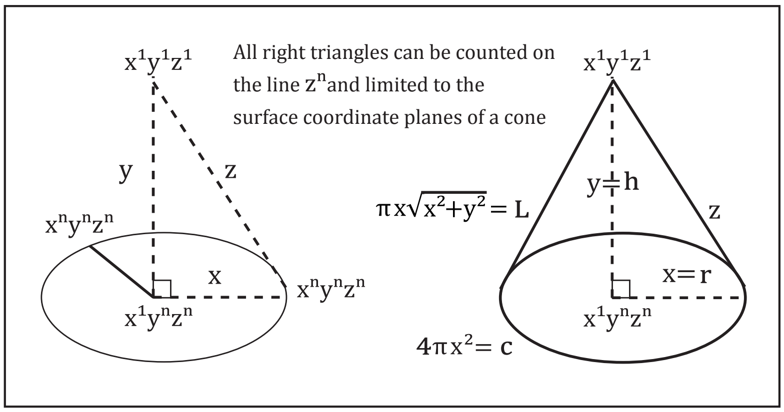
(Figure 4) *Fermat's version of Euler's primary identity*



1.5 - Fermat travels along the surface of a cone

The first place Fermat would go to limit a plane would be it's intersection with a cone. As was mentioned earlier, Fermat spent a great deal of effort to rework and rediscover the lost works of Apollonius of Perga. Apollonius was the first Greek mathematician to work with conics and , literally, wrote the book on it. The problem with his work is that the notation of multiple lengths and functions was cumbersome and difficult to follow even for 17th century mathematicians. Fermat's work on conics[4] is based on converting Apollonius's notation and also rediscovering some of the proofs that had been lost through time. One of the clear impressions I came away with while reading through Fermat's papers is his desire to reach for all-encompassing solutions. His writing on conics is no different and gives a clue to how he might find a path towards a solution to his final theorem using the works of Apollonius of Perga[1] as his guide . The coordinate system of a cone allows for the counting of Pythagorean triples along the slope of z while limiting all right triangles to the surface plane (Figure 5).

(Figure 5) *The conics of Apollonius and Fermat provide a coordinate plane based on the limits of the surface of a cone.*



1.6 Discussion

This is not a mathematics paper. It is intended for a broader audience and works with the geometry and symmetries of Pythagoras, Thales, Euler and Fermat. Connecting to the Wiles proof can be accomplished in a number of ways. The key difference between what we are presenting here, and the Wiles proof, is that, while Sir Andrew battles imaginary dragons in the complex upper half plane; we "exist", humbly, and with great respect, far below. This paper is a continuance of our discussion regarding the limits to measurement in the real and complex planes. You will find, upon closer examination, that we are in total agreement with Dr. Wiles when we separate real from complex potential using the maxima and minima. As we have stated, in all of our papers to date, *all zero point functions exist in the complex plane.* This would include all modular forms and elliptic curves, basically aligning the Wiles proof to our theories in number and set theory. Here are a few more direct connections to the Wiles proof:

- All the symmetries we presented here rely on creating geometric boundaries for points in real space and in the complex plane. The Euler primary identity projected on the surface of a cone has the advantage of an imaginary vertex. The vertex of a cone translates directly to elliptic curves and modular forms through Abel's theorem, regarding the establishment of real and imaginary points.

- Andrew Wiles and Ken Ribet's use of the zeta function to establish contradiction can be tied directly to Fermat's development of proof by infinite regression. This can be directly tied to the definition of minima and maxima of a geometric function on a coordinate plane.

-And finally, the use of Galois roots, in set theory, can be tied, directly, to the description of relative sets that we have employed in all our previous papers.

Hopefully, this has proven to be another fruitful discussion, regarding the establishment of boundaries for measurement based on Fermat's concepts of maxima and minima in probability and number theory. In our next paper we will discuss how the real measurement of zero-point limits to mathematical potential lead to a geometric proof of the discreteness of the Navier-Stokes equations.

1.7 References

Note to readers: *(Euclid's thirteen books of Elementa contain many of the ideas of other noted Greek mathematicians like Thales and Pythagorous. As primary references the works of Apollonius of Perga and Euler will also be assigned the same reference number [1] Hopefully, these gentlemen can be trusted without references. However, I would keep an(i) on Euler, if I were you. I have included references from the works of Fermat, my own work, historical publications and biographical references like Simon Singh's wonderful book and, of course, Dr. Wiles papers and lectures regarding his proof of Fermat's last theorem.)*

[1] ***Euclidis*** *Elementa, edidit et Latine interpretatus est I.L. Heiberg, in aedibus B.G. Teubneri, 1883–1885*
*[2]****Pierre de Fermat*** *-Methodus ad disquirendam maximam et minimam et de tangentibus linearum curvarum (1636)*
[3] ***Pierre de Fermat-*** *Varia opera mathematica*, as *Ad Locos Planos et Solidos Isagoge Pierre de Fermat (published posthumously 1679)*
*[4****] Catalan, E.C.****: Théorèmes et problèmes de Géométrie Elementaire (6e ed.). Dunod, Paris (1879)*
*[5]****Domski, M*** *"Descartes mathematics" Stanford Encyclopedia of Philosophy (Spring, 2017)*
[6] ***Wiles A.*** *On ordinary λ-adic representations associated to modular forms Inventiones Mathematicae. 94: 529-573. DOI: 10.1007/BF01394275*
[7] ***Wiles A.*** *Modular curves and the class group of Q(ςp) Inventiones Mathematicae. 58: 1-35. DOI: 10.1007/BF01402272*
[8] ***Wiles, A*** *"The Abel lectures." (Oslo, 2016)*
[9] ***Ribet, K*** *. "A modular construction of unramified p-extensions of (μ_p)". Inv. Math.* ***34*** *(3): 151–162. doi:10.1007/bf01403065.(1976)*
[9] ***Simon Singh*** *(1997). Fermat's Last Theorem. ISBN 1-85702-521-0*
[10] ***Blackwood, CJ*** *"Viewing paradox through the lens of general relativity" mtdi.org (Sept. 2019)*
[11] ***Blackwood, CJ*** *"Implementing rules for the measurement of Markovian and Bayesian time operations, establishes the measurement of charged and uncharged boundary conditions – without the need for observers." mtdi.org (May, 2018)*
*[12]****Blackwood, CJ*** *"Time symmetry and measurement at the event horizon of a black hole." mtdi.org (Dec., 2018)*
[13] ***Blackwood, CJ*** *"Establishing a minimum measurement boundary for the coupling of charged potentials eliminates the need for gluon fields." mtdi.org (Feb., 2019)*
[14] ***Blackwood, CJ*** *"Resolving the apparent paradox between Hubble and Planck measurements of H_0" mtdi.org (Oct, 2019)*
*[15]****Blackwood, CJ*** *"A black hole as an ideal spin capacitor" mtdi.org (Nov, 2019)*
*[16]****Blackwood, CJ*** *"Establishing geometric boundaries for the fundamental constants and dark energy." (March, 2020)*