NCERT – Relation & function

Excercise 1.1

Question 11 :

Show that the relation R in the set A of points in a plane given by R = {(P, Q): Distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all point related to a point P ≠ (0, 0) is the circle passing through P with origin as centre.

Answer 11:

R = {(P, Q): Distance of point P from the origin is the same as the distance of point Q from the origin}

Clearly, (P, P) ∈ R since the distance of point P from the origin is always the same as the distance of the same point P from the origin.

∴ **R is reflexive.**

Now, Let (P, Q) ∈ R. The distance of point P from the origin is the same as the distance of point Q

from the origin.The distance of point Q from the origin is the same as the distance of point P from the origin.

⇒ (Q, P) ∈ R ∴ **R is symmetric.**

Now, Let (P, Q), (Q, S) ∈ R.

The distance of points P and Q from the origin is the same and also, the distance of points Q and S from the origin is the same. The distance of points P and S from the origin is the same.

⇒ (P, S) ∈ R ∴ **R is transitive.**

Therefore, R is an equivalence relation.

The set of all points related to P ≠ (0, 0) will be those points whose distance from the origin is the same as the distance of point P from the origin. In other words, if O (0, 0) is the origin and OP = k, then the set of all points related to P is at a distance of k from the origin. Hence, this set of points forms a circle with the centre as the origin and this circle passes through point P.

Question 12:

Show that the relation R defined in the set A of all triangles as R = {(T1, T2): T1 is similar to T2}, is equivalence relation. Consider three right angle triangles T1 with sides 3, 4, 5, T2 with sides 5, 12, 13 and T3 with sides 6, 8, 10. Which triangles among T1, T2 and T3 are related?

Answer 12:

R = {(T1, T2): T1 is similar to T2} , R is reflexive since every triangle is similar to itself.

∴ **R is reflexive.**

Further,

If (T1, T2) ∈ R, then T1 is similar to T2.

T2 is similar to T1, (T2, T1) ∈ R

∴ **R is symmetric.**

Now,

Let (T1, T2), (T2, T3) ∈ R. , T1 is similar to T2 and T2 is similar to T3. T1 is similar to T3.

∴ **R is transitive.**

Thus, R is an equivalence relation. Now,

We can observe that (3 /6) = (4 / 8) = (5 / 10) = (1 / 2)

∴The corresponding sides of triangles T1 and T3 are in the same ratio.

Then, triangle T1 is similar to triangle T3.

Hence, T1 is related to T3.

Question 13:

Show that the relation R defined in the set A of all polygons as R = {(P1, P2): P1 and P2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

Answer 13:

R = {(P1, P2): P1 and P2 have same the number of sides}

R is reflexive,

Since (P1, P1) ∈ R, as the same polygon has the same number of sides with itself.

Let (P1, P2) ∈ R.

⇒ P1 and P2 have the same number of sides.

⇒ P2 and P1 have the same number of sides.

⇒ (P2, P1) ∈ R

∴ R is symmetric.

Now,

Let (P1, P2), (P2, P3) ∈ R.

⇒ P1 and P2 have the same number of sides.

Also, P2 and P3 have the same number of sides.

⇒ P1 and P3 have the same number of sides.

⇒ (P1, P3) ∈ R

∴ R is transitive.

Hence, R is an equivalence relation.

The elements in A related to the right-angled triangle (T) with sides 3, 4, and 5 are those polygons which have 3 sides (Since T is a polygon with 3 sides).

Hence, the set of all elements in A related to triangle T is the set of all triangles.

Question 14:

Let L be the set of all lines in XY plane and R be the relation in L defined as R = {(L1, L2): L1 is parallel to L2}. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x + 4.

Answer 14:

R = {(L1, L2): L1 is parallel to L2}

R is reflexive as any line L1 is parallel to itself i.e., (L1, L1) ∈ R.

Now, let (L1, L2) ∈ R.

⇒ L1 is parallel to L2 ⇒ L2 is parallel to L1.

⇒ (L2, L1) ∈ R

∴ R is symmetric.

Now, let (L1, L2), (L2, L3) ∈R.

⇒ L1 is parallel to L2. Also, L2 is parallel to L3.

⇒ L1 is parallel to L3.

∴ R is transitive.

Hence, R is an equivalence relation.

The set of all lines related to the line y = 2x + 4 is the set of all lines that are parallel to the line y = 2x + 4.

Slope of line y = 2x + 4 is m = 2

It is known that parallel lines have the same slopes.

The line parallel to the given line is of the form y = 2x + c, where c ∈ R.

Hence, the set of all lines related to the given line is given by y = 2x + c, where c ∈ R.

Exercise : 1.3

Question 10:

Let f: X → Y be an invertible function. Show that f has unique inverse. (Hint: suppose $g1$ and $g2$ are two inverses of f. Then for all y ∈ Y, $fog1(y)$ = $IY(y)$ = $fog2(y)$. Use one – one ness of f).

Answer 10:

Let f: X → Y be an invertible function. Also, suppose f has two inverses (sayg1 and g2) +

Then, for all y ∈Y, we have fog1(y) = IY(y) = fog2(y) ⇒ f(g1 (y)) = f(g2 (y)) ⇒ g1 (y) = g2 (y)

[as f is invertible ⇒ f is one – one] ⇒ $g1$ = $g2$ [as g is one – one] Hence, f has a unique inverse.

Question 11:

Consider f: {1, 2, 3} → {a, b, c} given by f(1) = a, f(2) = b and f(3) = c. Find f^{-1} and show that $(f^{-1})^{-1}$ = f.

Answer 11:

Function f: {1, 2, 3} → {a, b, c} is given by f(1) = a, f(2) = b, and f(3) = c

If we define g: {a, b, c} → {1, 2, 3} as g(a) = 1, g(b) = 2, g(c) = 3.

We have

(fog)(a) = f(g(a)) = f(1) = a

(fog)(b) = f(g(b)) = f(2) = b

(fog)(c) = f(g(c)) = f(3) = c

and

(gof)(1) = g(f(1)) = f(a) = 1

(gof)(2) = g(f(2)) = f(b) = 2

(gof)(3) = g(f(3)) = f(c) = 3

∴ gof = I_X and fog = I_Y, where X = {1, 2, 3} and Y= {a, b, c}.

Thus, the inverse of f exists and f^{-1} = g. ∴ f^{-1} :{a, b, c} → {1, 2, 3} is given by

$f^{-1}(a) = 1$, $f^{-1}(b) = 2$, $f^{-1}(c) = 3$

Question 12:

Let f: X → Y be an invertible function. Show that the inverse of f −1 is f, i.e., (f −1) −1 = f.

Answer 12:

Let f: X → Y be an invertible function. Then, there exists a function g: Y → X such that

gof = I_X and fog = I_Y. Here, f^{-1} = g. Now, gof = I_X and fog = I_Y ⇒ $f^{-1}$of = I_X and fof $^{-1}$ = I_Y Hence,

f^{-1} : Y → X is invertible and f is the inverse of f −1 i.e., $(f^{-1})^{-1}$ = f