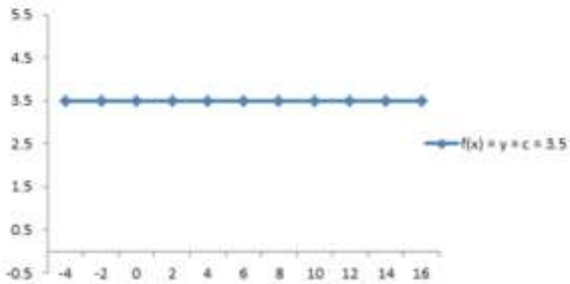
Functions

We can define a function as a special relation which maps each element of set A with one and only one element of set B. Both the sets A and B must be non-empty. A function defines a particular output for a particular input. Hence, f: A → B is a function such that for a ∈ A there is a unique element b ∈ B such that (a, b) ∈ f

Types of Functions

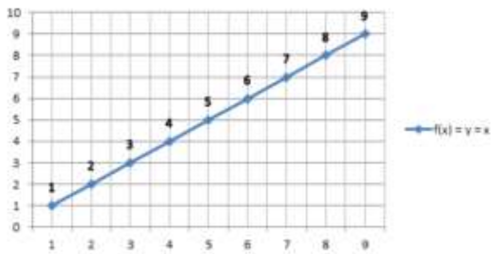
Constant Function

If the function f: **R**→**R** is defined as f(x) = y = c, for x ∈ R and c is a constant in **R**, then such function is known as Constant function.



Identity Function

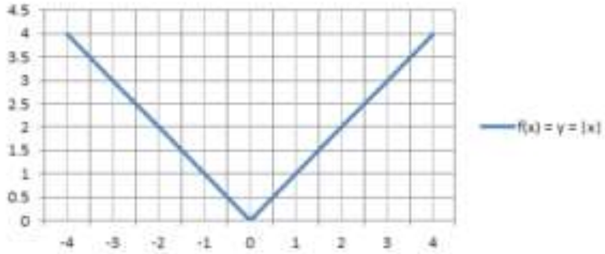
Let **R** be the set of real numbers. If the function f: **R**→**R** is defined as f(x) = y = x, for x ∈ R, then the function is known as Identity function. The domain and the range being **R.** The graph is always a straight line and passes through the origin.



Modulus Function

The absolute value of any number, x is represented in the form of |x|. If any function f: **R**→ **R** is defined by f(x) = |x|, it is known as Modulus Function. For each non-negative value of x, f(x) = x and for each negative value of x, f(x) = -x, i.e.,

$$f\ (x) = \begin{cases} x & if\ x \geq 0 \\ -x & if\ x < 0 \end{cases}$$

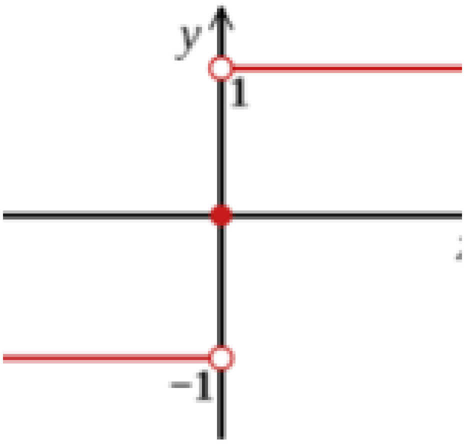


Signum Function

A function f: **R**→ **R** defined by
f(x) = { 1, if x > 0; 0, if x = 0; -1, if x < 0

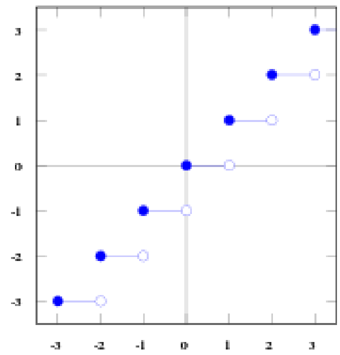
$$f(x) = \begin{cases} 1 & if\ x > 0 \\ 0 & if\ x = 0 \\ -1 & if\ x < 0 \end{cases}$$

Signum or the sign function extracts the sign of the real number and is also known as step function.



Greatest Integer Function

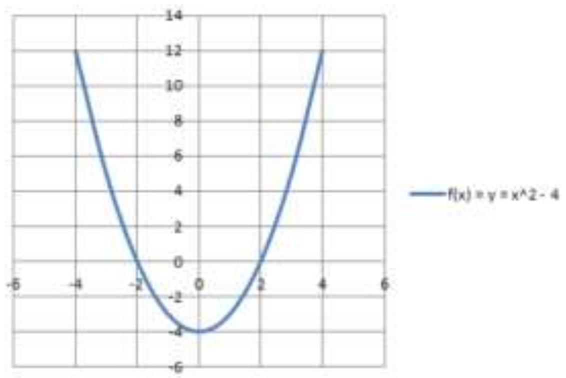
If a function f: **R**$\rightarrow$ **I** is defined by f(x) = [x], $x \in X$. It round-off to the real number to the integer less than the number. For example: [- 2.1] = - 3, [5.12] = 5.



Polynomial Function

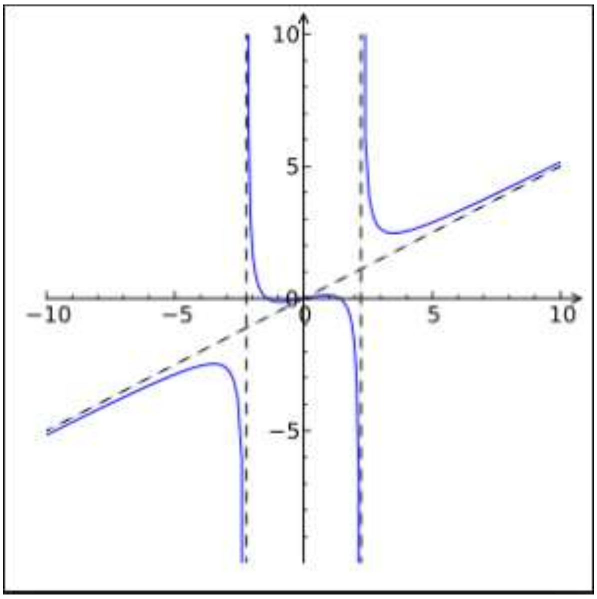
A polynomial function is defined by $y = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$, where n is a non-negative integer and $a_0, a_1, a_2, \ldots, {}_n \in \mathbf{R}$. The highest power in the expression is the degree of the polynomial function. Polynomial functions are further classified based on their degrees:

Quadratic Function: If the degree of the polynomial function is two, then it is a quadratic function. It is expressed as $f(x) = ax^2 + bx + c$, where $a \neq 0$ and a, b, c are constant & x is a variable. The domain and the range are **R.** The graphical representation of a quadratic function say, $f(x) = x^2 - 4$ is



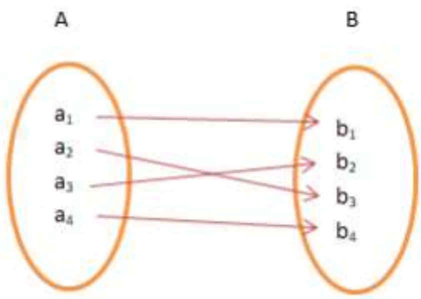
Rational Function

A rational function is any function which can be represented by a rational fraction say, f(x)/g(x) in which numerator, f(x) and denominator, g(x) are polynomial functions of x, where g(x) ≠ 0. Let a function f: **R** → **R** is defined say, f(x) = 1/(x + 2.5). The domain and the range are **R**. The Graphical representation shows asymptotes, the curves which seem to touch the axes-lines.



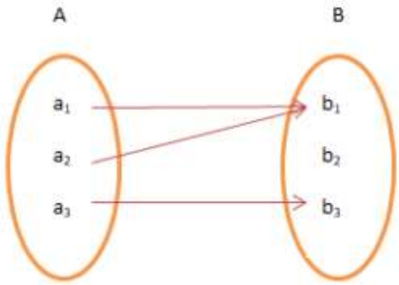
One to One Function

A function f: A → B is One to One if for each element of A there is a distinct element of B. It is also known as Injective.



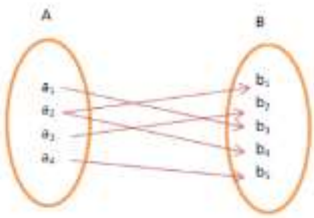
Many to One Function

It is a function which maps two or more elements of A to the same element of set B. Two or more elements of A have the same image in B.



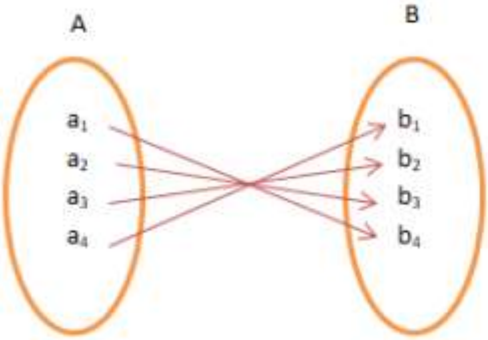
Onto Function

If there exists a function for which every element of set B there is (are) pre-image(s) in set A, it is Onto Function. Onto is also referred as Surjective Function.



One – One and Onto Function

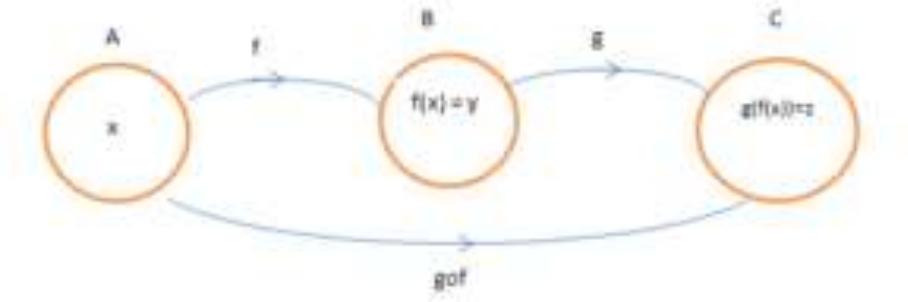
A function, f is One – One and Onto or Bijective if the function f is both One to One and Onto function. In other words, the function f associates each element of A with a distinct element of B and every element of B has a pre-image in A.



Composite Functions

When two functions combine in a way that the output of one function becomes the input of other, the function is a composite function.

A composite function is denoted by **fog(x) = f (g(x))** and **gof (x) = g (f(x)).**

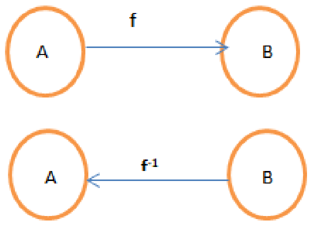


The composition of functions is associative in nature i.e., g o f = f o g. It is necessary that the functions are one-one and onto for a composition of functions.

Invertible Function

A function is invertible if on reversing the order of mapping we get the input as the new output. In other words, if a function, f whose domain is in set A and image in set B is invertible if f^{-1} has its domain in B and image in A.

$f(x) = y \Leftrightarrow f^{-1}(y) = x.$



It is necessary that the function is one-one and onto to be invertible, and vice-versa. It is interesting to know the composition of a function and its inverse returns the element of the domain.

$f^{-1} \text{ o } f = f^{-1}(f(x)) = x$