Three Dimension Geometry

Type 1:	**Distance Formula and Section Formula based question:**
	1) Show that the triangle whose vertices are (2,5,3),(5,3,2),(3,2,5) is an equilateral triangle. 2) By distance formula show that the points A(1,2,3), B(4,0,4) and C (-2,4,2) are collinear. 3) Show that the circum-center of the triangle with vertices A (1,1,0) , B(1,2,1), C(-2,2,-1) in ΔABC is D (- 1/2 , 2,0). 4) In which ratio does the plane XY divide the line joining the points (-3,4,-8)and (5,-6,4)? 5) A (3,2,0), B(5,3,2) and C(-9,6,-3) are three vertices of ΔABC. Bisector of angle A meets BC in D. Find the coordinate of the point D. 6) The mid-point of the sides of sides of a triangle are (1,5,-1), (0,4,-2) and (2,3,4). Find its vertices. 7) The coordinates of centroid of a triangle are (4/3, 4/3, 1/3). The coordinate of two vertices of a triangle are (1,3,4) and (5,-2,1). Find third
Type 2:	**Direction Ratios and Direction cosines based questions :**
	8) Find the direction ratios and direction cosines of the line joining the points (1,2,3) and (4,5,-6). 9) Prove by use of direction ratios that the points (1,-2,3), (2,3,-4) and (0,-7,10) are collinear. 10) The projection of a line segment on the axes are 2,3,6. Find the length and dc's of the line. 11) If the coordinate of points P,Q,R,S respectively are (3,4,5), (4,6,3),(-1,2,4) and (1,0,5) then find the projection of RS on PQ. 12) Find the Direction cosines of the line which is perpendicular to the lines whose direction cosines proportional to **1,-2,-2** and **0,2,1.** 13) If two lines joining the pair of points **(4,1,2),(p,3,0)**and **(2,1,-1), (4,3,2)** are mutually perpendicular to each other then find the value of **p**.

Plane

- General Equation of Plane : **ax + by+ cz + d =0**
- Equation of Plane Passing through a given point (x_1 , y_1, z_1): $\mathbf{a(x - x_1) + b(y - y_1) + c(z - z_1) = 0}$
  (a , b , c are the direction ratios to the normal of the plane)
- Equation of plane passing through origin : **ax + by + cz = 0**
- Normal Form of the equation of plane : **lx + my +nz = p**
  (l ,m, n are direction cosines and p is the perpendicular distance of the plane from origin)
- Equation to the plane in Intercept form: $\frac{x}{a} + \frac{y}{b} = 1$
- Equation of the co-ordinate planes :- YZ plane: x=0 , XZ plane : y =0 , XY plane : z =0.
- Distance of a point (x', y', z')from the give plane : ax + by + cz +d =0.

$$P = \frac{ax' + by' + cz' + d}{\sqrt{a^2+b^2+c^2}}$$

- The distance between two parallel planes $\mathbf{ax + by + cz + d_1 = 0}$ and $\mathbf{ax + by + cz + d_2 = 0}$.

$$P_2 - P_1 = \frac{d_2 - d_1}{\sqrt{a^2+b^2+c^2}}$$

- Angle between two planes $\mathbf{a_1x + b_1y + c_1z + d_1 = 0}$ and $\mathbf{a_2x + b_2y + c_2z + d_2 = 0}$

$$Cos\ \theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{({a_1}^2 + {b_1}^2 + {c_1}^2)\,({a_2}^2 + {b_2}^2 + {c_2}^2)}}$$

- Equation of plane passing through the intersection of two plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$.
  $\mathbf{(a_1x + b_1y + c_1z + d_1) + \lambda\, (a_2x + b_2y + c_2z + d_2) = 0}$
- Equation of the plane Bisecting the angles between two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$

$$\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{({a_1}^2 + {b_1}^2 + {c_1}^2)}} = \pm \frac{a_2x + b_2y + c_2z}{\sqrt{({a_2}^2 + {b_2}^2 + {c_2}^2)}}$$

	Types of Questions
	Plane
Type 1 :	**Find the Equation of Planes : General form, Normal Form , Intercept Form**
	1) Find the equation of the plane on which the normal drawn from the origin has length equal to 4 and its direction cosines are proportional to the (2, -3, 6) 2) Find the equation of plane which makes equal intercept on the axes and passes through the point (1, 2, 3). 3) A plane cuts the co-ordinate axes at A, B, C respectively. The Centroid of ΔABC is(3, -6, 9). Find the equation of the plane. 4) A variable plane is at a constant distance p from the origin and meets the axes in A,B, and C. Show that the locus of the centroid of the tetrahedron OABC is $\mathbf{x^{-2}+y^{-2}+z^{-2} = 16p^{-2}}$. 5) A variable plane is at a constant distance p from the origin and meets the axes at A,B,C. through A,B,C the planes are drawn parallel to the co-ordinate planes. Show that the locus of their points of intersection is given by $\mathbf{x^{-2}+y^{-2}+z^{-2} = p^{-2}}$
Type 2 :	**Find the Equation of plane passes through *a point and perpendicular to the given line*.**
	6) Find the equation to the plane that passes through (2,- 3, 1) and is perpendicular to the line joining the points (3 , 4, -1) and (2, -1 , 5). 7) Find the equation of plane which passes through the points (2, 6, 3) and is perpendicular to that line whose direction ratios are 2, 7, and -7. 8) A line joining the points (4,-1,3) and (3, -2, -3) meets a plane at point (-6, 5, 4) normally. Find the equation of the plane.
Type 3 :	**Find the Equation of plane passes through *a points and perpendicular to the given planes*.**
	9) Find the equation to the plane passing through the point (-1,-1,2) and perpendicular to the planes $3x+ 2y - 3z =1$ and $5x - 4y + z=5$. 10) Find the equation of plane passing through the points (2,1,-1) and (-1,3,4) and perpendicular to the plane $x - 2y + 4z =10$.
Type 4:	**Find the Equation of Plane passes *through 3 points* :**
	11) Find the equation of planes passes through the the points (4,5,1) , (0, -1,-1) and (-4,4,4). 12) Prove that the points (1,2,3) , (3,0,3) , (-2,-3,-3) and (3,4,6) are coplanar.
Type 5:	**Find the Angle between two planes:**
	13) Find the angle between two planes **x+2y+z+7=0** and **2x+y-z+13**=0. 14) Find the value of K if planes $3x - 6y - 2z = 7$ and $2x + y - kz =5$ are perpendicular to each other.
Type 6:	**Distance of the plane from a point and Distance between two planes.**
	15) Find the distance between two planes $2x-2y+z+3 =0$ and $4x - 4y + 2z + 5 = 0$. 16) Find the equation of planes which are parallel to the plane $x - 2y + 2z =3$ and perpendicular distance from the point (1,2,3) is 1. 17) Find the foot of the perpendicular and image of the point (2,5,7) on the plane $6x+6y+3z=12$.
Type 7 :	**Equation of planes passing through the intersection of two planes:** $\mathbf{(a_1x + b_1y + c_1z +d_1) + \lambda (a_2x + b_2y + c_2z +d_2)=0}$
	18) Find the equation of plane which passes through the intersection of two planes $x+2y+3z-4=0$ and $2x+y-z+5=0$ and is perpendicular to the plane $5x+2y+6z-8=0$ 19) Find the equation of plane which passing through the point (0,1,1) and which is passes through the point of intersection of the planes $x+2y+3z+5=0$ and $2x-4y+z-3=0$.
Type 8:	**Equation of the Plane bisecting the angles between the given plane (angle bisector plane) :**
	20) Find the Equation of the Bisector planes of the angles between the planes $3x-2y+6z+8=0$ and $2x-y+2z+3=0$. 21) Find the equation of the plane that bisects the angle containing the origin between the planes $3x-6y+2z+5=0$ and $4x - 12y +3z -3 =0$.

Straight Line

Type 1 :	**Equation of Line passes through a point or two points.**
	1) Find the equation of line passing through a point (2,1,3) and having the direction ratios (1,3,2). 2) Find the equation of Line joining the points (0,2,5) and (1,3,-3).
Type 2 :	**Angle between two lines :**
	3) Find the angle between lines : $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{-3}$ and $\frac{x+3}{-1}=\frac{y-5}{8}=\frac{z-1}{4}$ 4) Prove that the lines x = ay+b, z=cy+d and x=a'y +b' , z=c'y+d' are perpendicular if aa'+cc'+1=0
Type 3:	**Equation of Line passing through a point and parallel to the given line.**
	5) Find the equation of line passing through the point (1,2,3) and parallel to the line $$\frac{x-6}{12}=\frac{y-2}{4}=\frac{z+5}{5}$$ 6) Find the equation of line through the point (0,-3,2) and perpendicular to the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{-3}$ and $\frac{x-3}{2}=\frac{y-2}{-2}=\frac{z-1}{1}$
Type 4:	**Perpendicular Distance of the line from given point.**
	7) Find the perpendicular distance of the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$ from a point (1,2,3).
Type 5:	**Find the intersection point of a line and plane:**
	8) Find the point where the line $\frac{x-1}{2}=\frac{y-2}{-3}=\frac{z+3}{4}$ meets the plane 2x+4y-z+1=0.
Type 6:	**Foot of the perpendicular from the point to the line.**
	9) Find the foot of the perpendicular from the point (2,3,4) to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$. Also find the perpendicular distance from the given point to the line and image of the point. 10) Find the equation of the perpendicular from the point (3,-1,11) to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$. Also find the foot of the perpendicular and length of the perpendicular.
Type 7:	**Conversion in Symmetric form:**
	11) Find the Equation of lines x+y+z+1=0 and 4x+y-2z+2=0 in symmetric form. 12) Show that the lines 3x+2y+z-5=0=x+y-2z-3 and 8x-4y-4z=0=7x+10y-8z are mutually perpendicular.
Type 8:	**Plane containing a line:**
	13) Find the equation to the plane containing the line $\frac{x+2}{2}=\frac{y+3}{3}=\frac{z-4}{-2}$ and passing through the point (0,6,0) 14) Find the equation to the plane containing the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{4}$ and is perpendicular to the plane x+2y+z=12. 15) Show that the equation to the plane through the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ and perpendicular to the plane containing the lines $\frac{x}{m}=\frac{y}{n}=\frac{z}{l}$ and $\frac{x}{n}=\frac{y}{l}=\frac{z}{m}$ is **(m-n)x + (n-l)y + (l-m)z =0**
Type 9:	**Condition for two coplanar lines or intersecting lines :**
	16) Show that the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{3}=\frac{z-6}{5}$ intersect. Find also their point of intersection. 17) Show that the lines $\frac{x+1}{1}=\frac{y+1}{2}=\frac{z+1}{3}$ and x+2y+3z-8=0=2x+3y+4z-11 are coplanar and find the point of intersection.

Vectors

Type 1: Vector algebra :

1) ABCD is a parallelogram ,AC and BD are its diagonals , show that $\overrightarrow{AC} + \overrightarrow{BD} = 2\overrightarrow{BC}$ and $\overrightarrow{AC} - \overrightarrow{BD} = 2\overrightarrow{AB}$.
2) Show that the points A,B,C with the position vectors$\vec{a} - 2\vec{b} + 3\vec{c}$, $\overrightarrow{2a} - 3\vec{b} + 4\vec{c}$and $-7\vec{b} + 10\vec{c}$ are collinear.
3) D,E,F are mid - points of sides BC , CA and AB of a ΔABC . Prove that $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF}$=0
4) If G is Centroid of a ΔABC . Prove that $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC}$= 0
5) If the position vector of the vertices A,B,C of a ΔABC are respectively $\vec{a}, \vec{b}, \vec{c}$.So find the Psition vector of centroid. (using section formula).
6) If $\vec{a}$ and $\vec{b}$ are position vector of A and B respectively, Find the position vector of C produced in AB such that AC = 3AB and D in BA produced such that BD = 2BA.
7) If C is a point on a line segment AB such that λ.AC = μ.CB, prove that λ.$\overrightarrow{OA}$ + μ.$\overrightarrow{OB}$= (λ+μ) $\overrightarrow{OC}$
8) From vector method prove that the medians of triangle bisect each other in ratio 2:1.
9) Prove by vector method that medians of triangle are concurrent.
10) Show by vector method that the line joining the mid points of two sides of a triangle is parallel to and half of the third side.
11) ABCD is a Parallelogram. E and F are the mid points of its arms AB and BC. Prove that DE and DF bisect the diagonal of AC in three equal parts.

Type 2 : Vector Components (î, ĵ, k)

12) Show that the points A (2,3,6), B(-1,-1,2) and (5,7,10) are collinear.
13) Find the unit vector in the direction of the $\vec{a} - \vec{b}$ where $\vec{a}$ = î +2ĵ-k and $\vec{b}$ = 3î +ĵ-5k
14) If position vector of P,Q,R and S are 2î +4k , 5î +3$\sqrt{3}$ĵ+4k , -2$\sqrt{3}$ĵ+k and 2î + k respectively. Prove that $\overrightarrow{RS}$ is parallel to $\overrightarrow{PQ}$ and $\overrightarrow{RS} = \frac{2}{3}\overrightarrow{PQ}$. Find the direction cosines of $\overrightarrow{PQ}$ and $\overrightarrow{RS}$.
15) If three points whose position vector are 60î +3ĵ, 40î - 8ĵ and λî - 52ĵ respectively are collinear , then find the value of λ.

Type 3: Scalar Product or DOT product based question :

16) Find the angle between the vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}$ = î +2ĵ-k and $\vec{b}$ = 3î +ĵ-5k.
17) If $\vec{a} + \vec{b} + \vec{c}$ = 0 and $|\vec{a}|$ = 3, $|\vec{b}|$ = 4, and $|\vec{c}|$ = 4, then find the angle between them.
18) Find the scalar projection and vector projection of vector 7î +ĵ+4k , on vector 7î +ĵ-3k.
19) Prove that $(\vec{a}.\vec{b})^2 \leq |a|^2 . |b|^2$ and $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |b|^2$
20) A particle acted upon by constant forces 2î +4k , 5î +3 ĵ+4k , -2 ĵ+k is displaced from î +2ĵ+3k to the point 5î +4ĵ+k . Find the work done by the forces.

Type 4: Vector Product or CROSS product based question :

21) If $|\vec{a}|$ = 2, $|\vec{b}|$ = 5 and $|\vec{a} \times \vec{b}|$ = 8 , find $\vec{a}.\vec{b}$
22) Find the area of the parallelogram determined by the vectors î +2ĵ+3k and 3î - 2ĵ+k.
23) Find the area of triangle whose vertices are A (1,2,3) , B(0,-1,-2) and C (2,3,5) by vector method.
24) If $\vec{a}, \vec{b}$ and $\vec{c}$ determine the vertices of a triangle , show that $\frac{1}{2}|\vec{b} \times\times \vec{c} + \vec{c} \times\times \vec{a} + \vec{a} \times \overrightarrow{\times b}|$ gives the vector area of the triangle. Hence deduce condition that $\vec{a}, \vec{b}, \vec{c}$ are collinear
25) Prove that $(\vec{a} \times \vec{b})^2 = \overrightarrow{|a|^2|b|}^2 - |\vec{a}.\vec{b}|^2$
26) Show thata unit vector perpendicular to the plane of vector 3î + ĵ+2k and 2î - 2ĵ + 4k is $\frac{\hat{i} - \hat{j} - k}{\sqrt{3}}$ and the sine of the angle between them is $\frac{2}{\sqrt{7}}$.
27) Show that the area of the triangle whose two adjacent sides are determined by the vectors 3î +4 ĵ and -5î +7ĵ is 20 ½ sq unit.
28) Show that the area of parallelogram having diagonals 3î +ĵ-2k and î - 3ĵ+4k is 5$\sqrt{3}$.
29) If $\vec{a} + \vec{b} + \vec{c}$ = 0 show that $\vec{b} \times\times \vec{c} = \vec{c} \times\times \vec{a} = \vec{a} \times \overrightarrow{\times b}$

Type 5 :	**Triple product :**
	30) Prove that the four points (4,5,1), (0,-1,-1), (3,9,4) and (-4,4,4) are coplanar. (Vector Method) 31) If $\vec{a}$ = 2î – ĵ + k and $\vec{b}$ = î + 2ĵ-k and $\vec{c}$ = 2î + 3ĵ, then find the values of $(\vec{a} \times \vec{b}) \times \vec{c}$ and $\vec{a} \times (\vec{b} \times \vec{c})$ 32) If $\vec{a}$ = î - 2ĵ + 3k ,$\vec{b}$ = 2î +ĵ – k and $\vec{c}$ = ĵ + k, prove that $\vec{a} \times (\vec{b} \times\times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$. 33) The volume of the parallelopiped whose edges are -12î + λk , 3ĵ - k and 2î + ĵ – 15k is 546, find the value of λ.
Type 6:	**Three Dimensional Geometry in Vector form :**
	34) Find the direction cosine of the vector 6î+2ĵ – 3k. 35) By vector method find the foot of the perpendicular drawn from the point A (1,2,2) to the line joining the points B (1,2,5) and C (5,4,6). 36) The Modulus of a vector $\vec{r}$ is 21 and its direction ratios are 2, -3, 6. Find the components of $\vec{r}$ along the axes and the direction cosines of $\vec{r}$.
Type 7:	**<u>Plane(Vector form)</u>** • General Equation of plane : $\vec{r}.\vec{n} - d = 0$ • Normal form of plane : $\vec{r}.\hat{n} = p$ • Equation of plane passing through a given point $\vec{a}$: $(\vec{r} - \vec{a}) . \vec{n} = 0$ • Equation of plane passing through two points $\vec{a}$, $\vec{b}$ and parallel to a vector $\vec{t}$: $(\vec{r} - \vec{a}) .[(\vec{b} - \vec{a}) \times \vec{t}] = 0$ • Eq. of Plane passing through three given points $\vec{a}$, $\vec{b}$, $\vec{c}$:$(\vec{r} - \vec{a}) .[(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})]$ • Angle between two planes $\vec{r}.\overrightarrow{n1} - d = 0$ and $\vec{r}.\vec{n}2 - d = 0$: $\cos\theta = \frac{\overrightarrow{n1}.\vec{n}2}{\lvert\vec{n}1\rvert\lvert\vec{n}2\rvert}$ • Distance of a point $\vec{a}$ from a plane $\vec{r}.\vec{n} - d = 0$: $P = \frac{\lvert\vec{a}.\vec{n} - d\rvert}{\lvert\vec{n}\rvert}$ • Eq. of Plane passing through the intersection of two planes $\vec{r}.\overrightarrow{n1} - d = 0$ and $\vec{r}.\vec{n}2 - d = 0$. $(\overrightarrow{r}.\overrightarrow{n1} - d) + \lambda (\vec{r}.\vec{n}2 - d) = 0$
	37) Find the vector equation of plane 5x -9y +2z = 3. 38) Find the Vector equation of plane passing through (2,2,-1), (3,4,2) and (7,0,6). And show that the point (3,3,0) lie on same plane. 39) The length of perpendicular from the origin to a plane is 11 units and direction ratios of a normal to the plane are -3,4,12. Find the vector equation of plane. 40) Find the equation of the plane passing through a point A and normal to the line joining the points A (î - 2ĵ - 4k) and B (3î + ĵ + 2k). 41) Find the angle between the planes :$\vec{r}$. (2î+3ĵ – 6k) = 3 and $\vec{r}$. (î- 2ĵ + 2k) = 7. 42) Find the distance of the point 2î- ĵ – 4k from the plane :$\vec{r}$. (4î- 12ĵ – 3k) = 6. 43) Find the equation of plane through the point (î+ 2ĵ – k) which is perpendicular to the line of intersection of the planes $\vec{r}$. (3î- ĵ + k) = 1 and $\vec{r}$. (î+ 4ĵ - 2k) = 2. 44) Find the equation of plane which passes through the intersection of planes $\vec{r}$. (2î- 3ĵ + k) = 5 and $\vec{r}$. (î+ 5ĵ - 2k) = 1 and through the point (3, -1,1).

Indefinite Integration

Type 1:	**Basic integration based questions :**
	1) $\int \frac{\sec x}{\sec x+\tan x}dx$, $\int \frac{1}{1+\cos 2x}dx$, $\int \frac{\cos 2x-\cos 2\theta}{\cos x-\cos\theta}dx$, $\int \frac{\cos 3x-\cos 2x}{\cos x}dx$, **2)** $\int \sin 2x \,.\cos 3x\, dx$
Type 2:	**Integration by substitution:**
	3) $\int \frac{\cos(\log x)}{x}\,dx$, $\int \frac{e^{\log x}}{x}dx$, $\int x\sqrt{1-x^2}\,dx$, $\int \sec^2 x \,.\, \tan^2 x\, dx$, $\int \sin^5 x\, dx$, **4)** $\int \frac{(\sin^{-1}x)^3}{\sqrt{1-x^2}}dx$, $\int \frac{1}{(\tan^{-1}x)^2(1+x^2)}dx$, $\int \tan^4 x\, dx$, $\int \frac{2x(\sin^{-1}x^2)}{\sqrt{1-x^4}}dx$, **5)** $\int \frac{dx}{(2\sin x+\cos x)^2}$,
Type 3:	**Standard Formulas:** $\int \frac{dx}{x^2+a^2}$, $\int \frac{dx}{x^2-a^2}$, $\int \frac{dx}{a^2-x^2}$, $\int \frac{dx}{\sqrt{a^2-x^2}}$, $\int \frac{dx}{\sqrt{a^2+x^2}}$, $\int \frac{dx}{\sqrt{x^2-a^2}}$, $\int \sqrt{(a^2-x^2)}\,dx$, $\int \sqrt{(a^2+x^2)}\,dx$, $\int \sqrt{(x^2-a^2)}\,dx$,
	6) $\int \frac{x^2dx}{x^2-4}$, $\int \frac{dx}{x^2+9}$, $\int \frac{dx}{25-x^2}$, $\int \frac{2e^x dx}{\sqrt{4-e^{2x}}}$, $\int \frac{dx}{\sqrt{25+36x^2}}$, $\int \frac{dx}{x\sqrt{x^2-4}}$ **7)** $\int \sqrt{(1-4x^2)}\,dx$, $\int \sqrt{(36+x^2)}\,dx$, $\int \sqrt{(x^2-25)}\,dx$,
Type 4:	**Integration by parts : (product formula) : ILATE** $\int (U.V)dx = U\int V\,dx - \int[\frac{dU}{dx}.\int V\,dx]$
	8) $\int x^3 .\tan^{-1}x\,dx$, $\int (\cos^{-1}x)^2 dx$, $\int (\tan^{-1}\sqrt{x})\,dx$.
Type 5:	$\int e^{ax}\sin bx\, dx$, $\int e^{ax}\cos bx\, dx$ **form.**
	9) $\int e^{x}\sin(x)\cos(x)\,dx$, $\int e^{3x}.\cos^2 x\,dx$, $\int e^{x}.\sec x .[1+\tan x]\,dx$
Type 6:	**Trigonometric Substitution :** *For* (a^2+x^2) **substitute x = a tan θ** *For* (a^2-x^2) **substitute x = a sin θ** *For* (x^2-a^2) **substitute x = a sec θ**
	10) $\int \frac{x(\tan^{-1}x)^2}{(1+x^2)^{3/2}}dx$, $\int \frac{\log(\sec^{-1}x)}{x\sqrt{x^2-1}}dx$, $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$, $\int \cos^{-1}(\frac{1-x^2}{1+x^2})\,dx$

Harder Integration

Type 1 :	**Integration through Partial Fraction.**
	11) $\int \frac{dx}{x^2-7x+12}$, $\int \frac{x^3}{x^2-3x+2}\,dx$, $\int \frac{dx}{x\,(1+\log x)\,(3+\log x)}$
Type 2:	**Evaluate :** $\int \sqrt{ax^2+bx+c}\;dx$ **by using standard formulas** $\int \sqrt{(a^2-x^2)}\,dx$, $\int \sqrt{(a^2+x^2)}\,dx$, $\int \sqrt{(x^2-a^2)}\,dx$,
	12) $\int \sqrt{x^2+4x+6}\;dx$, $\int \sqrt{-4x+5-x^2}\,dx$
Type 3:	**Evaluate :** $(px+q)\int \sqrt{ax^2+bx+c}\;dx$ & $\int \frac{(px+q)}{\sqrt{ax^2+bx+c}}$ dx **: Put** $(px+q) = \lambda \frac{d}{dx}(ax^2+bx+c) + \mu$
	13) $\int (2x-5)/\sqrt{4+5x-x^2}\;dx$, $\int (4x+1)\sqrt{x^2-x-2}\;dx$, $\sqrt{2ax-x^2}\,dx$
Type 4:	$\int \frac{dx}{a+b\sin x}$, $\int \frac{dx}{a+b\cos x}$, $\int \frac{dx}{a+b\cos x+c\sin x}$ **Substitute:** t = tan $\frac{x}{2}$, then sin x = $\frac{2t}{1+t^2}$, cos x = $\frac{1-t^2}{1+t^2}$, $dx = \frac{2}{1+t^2}dt$
	14) $\int \frac{dx}{5-4\sin x}$, $\int \frac{dx}{4\cos x-1}$, $\int \frac{dx}{5+3\cos x-4\sin x}$, $\int \frac{dx}{\cos x-\sin x}$
Type 5 :	$\int \frac{a\cos x+b\sin x}{c\cos x+d\sin x}$ dx : **Numerator = λ $\frac{d}{dx}$ (Denominator) + μ (Denominator)** $\int \frac{a\cos x+b\sin x+m}{c\cos x+d\sin x+n}$ dx **: Numerator = λ $\frac{d}{dx}$ (Denominator) + μ (Denominator) + K**
	15) $\int \frac{2\cos x-3\sin x}{6\cos x+4\sin x}$ dx , $\int \frac{4\sin x+5\cos x+3}{5\sin x+4\cos x-5}$ dx
Type 6:	$\int \frac{dx}{a+b\sin^2 x}$, $\int \frac{dx}{a+b\cos^2 x}$, $\int \frac{dx}{a\cos^2 x+b\sin^2 x}$ **[Divide the Numerator and Denominator by cos²x and, then substitute tan x = t and sec²x dx = dt]**
	16) $\int \frac{dx}{4\cos^2 x+\sin^2 x}$, $\int \frac{dx}{\cos^2 x-3\sin^2 x}$
Type 7:	$\int \cos^m x\,.\,\sin^n x\;\;dx$
	17) $\int \cos^5 x\,.\,\sin^3 x\;\;dx$, $\int \frac{1}{\sin^2 x\,.\cos^3 x}\;dx$, $\int \cos^7 x\;\;dx$
Type 8:	$\int \frac{x^2+1}{x^4+\lambda x^2+1}\,dx$, $\int \frac{x^2-1}{x^4+\lambda x^2+1}\,dx$, $\int \frac{1}{x^4+\lambda x^2+1}\,dx$ **where λ is constant.** **Divide the numerator and denominator by x^2.**
	18) $\int \frac{x^2+1}{x^4+x^2+1}\,dx$, $\int \frac{x^2+1}{x^4-2x^2+1}\,dx$, $\int \frac{1}{x^4+1}$ **19)** $\int \sqrt{\tan x}$ dx , $\int \sqrt{\cot x}\;dx$, $\int (\sqrt{\cot x}+\sqrt{\tan x})$ dx

Differential Equation

	• Order of Differential Equation: - The order of the differential equation is the order of the highest derivatives occurring in the differential equation. • Degree of Differential Equation :- The degree of differential equation is the exponent of the derivatives of the highest order after the equation has been freed from the radicals and fractions in the dependent variable and its derivatives. • Linear Equation : A differential equation is said to be linear if the dependent variable and all of its derivatives occur only in first degree.
Type 1:	**Formation of Differential Equation :**
	1) Find the Differential equation of the following curves and eliminate constants : $X^2+y^2=a^2$, $y = mx + \frac{a}{m}$, $(x - h)^2 = 4a(y - k)$, $x^2+y^2+2gx =0$, $Y = Ae^{2x} + B e^{-3x}$, $y = e^x (A \cos x + B \sin x)$
Type 2:	**Solution of a Differential Equation :**
	2) Prove that $y = e^{-3x}$, is the solution of differential equation $\frac{d^2y}{dx^2} - 9y = 0$. 3) Prove that $y = ae^x + be^{2x}$ is the solution of the differential equation $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+ 2y = 0$ 4) Verify that the solution of $\frac{d^2y}{dx^2}+ 9y = 0$ is $y = 4 \sin 3x$ 5) If $y = x^3 + ax^2 + bx +c$ then prove that $\frac{d^3y}{dx^3} = 0$. 6) Verify that the solution of the equation $x\left[y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] - y \frac{dy}{dx} = 0$ is $Ax^2 + By^2 = 1$.
Type 3:	**Solution of type $\frac{dy}{dx} = f(x)$;** **Change in form of $\int dy = \int f(x)\, dx$ and solve.**
	7) (i) $(e^x + e^{-x}) \frac{dy}{dx} = (e^x - e^{-x})$, (ii) $(1 + \cos x) dy = (1- \cos x) dx$, (iii) $\frac{dy}{dx} = \frac{1}{\sin^4 x + \cos^4 x}$
Type 4 :	**Equation with variable separated : $f(y)\, dy = f(x)\, dx$**
	8) (i) $(1-x^2)dy + xy\, dx = xy^2\, dx$ (ii) $\cos x . \cos y\, dy = - \sin x . \sin y\, dx$ (iii) $x (1 + y^2) dx - y (1 + x^2) dy = 0$ (iv) $\sec^2x \tan y + \sec^2y \tan x\, dy =0$ (v) $3 e^x \tan y\, dx + (1- e^x) \sec^2y\, dy = 0$
Type 5:	**Homogeneous Differential Equations : a function f (x , y) in x and y is homogeneous function of degree n, if the degree of each term is n.** **Working rule : Substitute y = vx and $\frac{dy}{dx} = v + x \frac{dv}{dx}$**
	9) $(3xy + y^2) dx + (x^2 + xy) dy = 0$ 10) $x^2\frac{dy}{dx} = x^2 + xy +y^2$ 11) $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$ 12) $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$ if x = 1, y=1 13) $(1 + e^{\frac{x}{y}}) dx + e^{\frac{x}{y}} (1 - \frac{x}{y}) dy = 0$
Type 6 :	**Linear Equation : $\frac{dy}{dx} + py = Q$, where P and Q are constant of f(x).** **Solution : $x\, e^{\int p\, dx} = \int Q .\ e^{\int p\, dx}\, dx$ where $e^{\int p\, dx} = Integrating\ Factor$**
	14) $(1 + x^2)\frac{dy}{dx} + 2xy - 4 x^2 = 0$ 15) $(\cos^2x) \frac{dy}{dx} + y = \tan x$ 16) $\sin x \frac{dy}{dx} + y \cos x = 2 \sin^2x \cos x$ 17) $\cos^3 x \frac{dy}{dx} + y \cos x = \sin x$ 18) $(x + y + 1) \frac{dy}{dx} = 1$

Application of Derivatives

Type 1 : **Motion in a straight line :** **Velocity (v) = $\frac{ds}{dx}$, Accelaration (a) = $\frac{dv}{dt} = \frac{d^2s}{dt^2}$**

1) The distance travelled by a particle in ***t*** seconds is given by **s = 7t^2 – 4t + 1**. Find the velocity and acceleration at ***t*** **=** $\frac{3}{2}$ seconds.
2) The distance ***s*** meters covered by a particle in t seconds is given by **s= 2t^3 – 5t^2 + 4t -3** find the time when acceleration is 14. Also find the velocity and displacement at that time.
3) A point moves along x – axis such that its position at time ***t*** is given by **x = 3t + cos 3t.**show that its velocity and acceleration both vanish together.
4) The distance ***s*** in meters described by a particle in ***t*** seconds is given by s = $ae^t + \frac{b}{e^t}$ show that the acceleration of the particle at time ***t*** is equal to the distance travelled by it upto time ***t***.

Type 2: **Motion under Gravity :**

5) A ball thrown vertically upwards, moves according to the formula s = 13.8t – 4.9t^2, where s is in metres and t is in seconds, Find :
   i) Its velocity at t = 1 second.
   ii) Its acceleration at t = 1 second.
   iii) The maximum height reached by the ball.
6) The motion of a particle projected vertically upwards satisfies s = at^2 + bt, where **s** and **t** are measured in metres and seconds respectively. If acceleration is – 9.8 m/sec^2 and maximum height is 44 metres, Find the velocity after 1 second.

Type 3: **Mensuration Based Problems. (Rate of change of Quantities)**

7) The radius of circular plate is increasing at the rate of 0.2 cm per second. At what rate is the area increasing when the radius of the plate 10 cm.
8) A ladder 5 cm long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of 3 cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4 cm away from the wall.
9) The volume of a cube is increasing at the rate of 0.003 m^3/sec. At the instant when the edge is 20 cm, Find the rate at which the edge is changing.

Type 4: **Increasing and Decreasing Functions :**

10) Prove that the function x3 – 3x2 + 3x – 100 is increasing function on R.
11) Find the interval in which the function x3 – 9x2 + 15x + 11 are increasing or decreasing.

Type 5: **Maxima and Minima :**

Working Rule :

- **Find the points at f'(x) = 0i.ex$_1$, x$_2$,….**
- **At each of the points , find the sign of f''(x)**
- **If f''(x)>0 (+ve value) at x$_1$ so x$_1$ is point of minima and f(x$_1$) Minima value.**
- **If f''(x)<0 (-ve value) at x$_1$ so x$_1$ is point of maxima and f(x$_1$) Maxima value.**

12) For what value of x is y = x(5-x) maximum or minimum.
13) Determine the maximum and minimum values of f(x) = cos x + sin x in $0 \leq x \leq \pi$
14) Find both Maximum and minimum value of f(x) = 3x^4 – 8x^3 + 12x^2-24 x + 25 in [0,3].
15) Determine two positive numbers whose sum is 15 and the sum of squares is minimum.
16) Show that the area of a rectangle of a given perimeter is maximum, when the rectangle is a square.
17) Prove that the perimeter of a right angled triangle of a given hypotenuses is maximum when the triangle is isosceles.

	18) Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base. 19) The combined resistance R of two resistors R1 and R2 is given by $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$. If $R_1 + R_2 = c$ (a constant), show that the maximum resistance R is obtained by choosing $R_1 = R_2$. 20) Prove that the height of the right circular cylinder of maximum volume that can be inscribed in a space of radius a is $\frac{2a}{\sqrt{3}}$. 21) Find the volume of the largest cone that can be inscribed in a sphere of radius **r** cm. 22) Find the volume of the largest Cylinder that can be inscribed in a sphere of radius **r** cm. 23) A square of piece of tin of side 18cm is to be made in to a box without top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum? Also find this maximum volume. 24) A wire of length 25 m is to be cut into two pieces. One of the pieces is to be made into a square and the either in to a circle. What should be the lengths of the two pieces so that the combined area of the square and the circle is maximum. 25) Find the radius of right circular cylinder of 100 cm3 with maximum surface area.
Type 6:	**Rolle's Theorem** **I) f(x) is continuous on the closed interval [a,b].** **II) f(x) is differentiable on the open interval (a,b).** **III) f(a) = f(b)** **Then there exists at least one value 'c' of x in the open interval (a,b) such that f′(c) = 0** Verify **Rolle's Theorem** for the following functions : 26) $f(x) = x^3 - 6x^2 + 11x - 6$ in the interval [1,3] 27) $f(x) = \sqrt{4 - x^2}$. In [-2,2] 28) $f(x) = \log(x^2 + 2) - \log 3$ in [-1 , 1] 29) If Rolle's theorem holds for the function $f(x) = x^3 - 6x^2 + ax + b$ in [1,3] with $c = \left[2 + \frac{1}{\sqrt{3}}\right]$, Find the value of **a** and **b**.
Type 7 :	**Lagrange's Mean Value theorem :** **I) f(x) is continuous on the closed interval [a,b].** **II) f(x) is differentiable on the open interval (a,b).** **then there exists at least one c ε (a,b) such that** $f'(c) = \frac{f(b) - f(a)}{b-a}$ Verify **Lagrange's Mean Value theorem**for the following functions : 30) $f(x) = (x-1)(x-2)(x-3)$ 31) $f(x) = f(x) = x - 2\sin x$ 32) Find c of **Lagrange's mean value theorem** , for the function f(x) = x2 – 3x – 2, x ε [-1,2] 33) Using **Lagrange's mean value theorem**, find a point P on the curve $y = x^3 - 3x$ defined in the interval [1,2], where the tangent is parallel to the chord joining the end points on the curve.