Time Allowed : 3 Hours **MATHEMATICS-10+2** **Max. Marks :80**

Instructions:

1. All the questions are compulsory.
2. The question paper consists of 16 questions divided into 4 sections A,B,C and D.
3. Section A comprises of 3 questions :
   (i) Q.No.1 consists of 16 Multiple Choice Questions carrying 1 mark each.
   (ii) Q.No.2 consists of 8 Fill in the Blank type questions carrying 1 mark each.
   (iii) Q.No.3 consists of 8 True/False type questions carrying 1 mark each.
4. Section B comprises of 5 questions of 2 marks each.
5. Section C comprises of 5 questions of 4 marks each.
6. Section D comprises of 3 questions of 6 marks each.
7. Internal choice has been provided in three questions of 2 marks, three questions of 4 marks and three questions of 6 marks. You have to attempt only one of the alternatives in all such questions.
8. Use of calculator is not permitted.

Section – A

Q1 Choose the correct options in the following questions :

(i) Function $f: R \to R,\ f(x) = 3x - 5$ is : **1**
(a)one-one only (b)onto only (c)one-one and onto (d)none of these

(ii) Relation given by $R = \{(1,1),(2,2),(1,2),(2,1)\}$ is **1**
(a)reflexive only (b)symmetric only (c)transitive only (d) equivalence relation

(iii) $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)$ is equal to : **1**
(a)$\frac{\pi}{5}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

(iv) If $\begin{bmatrix} 1 & -x \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 4 & -3 \end{bmatrix}$ then value of x is: **1**
(a)8 (b)-4 (c)3 (d)-8

(v) If order of matrix A is 2×3 and order of matrix B is 3×5 then order of matrix BA is : **1**
(a)5×2 (b) 2×5 (c) 5×3 (d) 3×2

(vi) If$f(x) = \begin{cases} kx + 1, & x \le 5 \\ 3x - 5, & x > 5 \end{cases}$ is continuous then value of k is : **1**
(a)$\frac{9}{5}$ (b) $\frac{5}{9}$ (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

(vii) $\frac{d}{dx}\{\tan^{-1}(e^x)\}$is equal to : **1**
(a)$e^x \tan^{-1} e^x$ (b) $\frac{e^x}{1+e^{2x}}$ (c) 0 (d)$e^x \sec^{-1} x$

(viii) Critical point of the function $f(x) = x^2 - 10x + 2$ is : **1**
(a)$x = 4$ (b)$x = 6$ (c)$x = 5$ (d)$x = 2$

(ix) $\int 3x^2 dx$ is equal to : **1**
(a)$x + c$ (b)$x^2 + c$ (c)$x^3 + c$ (d)$x^4 + c$

(x) $\int_0^{\pi/2} \frac{\sin^{1/2} x}{\sin^{1/2} x + \cos^{1/2} x} dx$is equal to : **1**
(a)0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

(xi) Degree of differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = 0$ is: **1**
(a)3 (b) 2 (c)1 (d)0

(xii) If $\vec{a}.\vec{b} = |\vec{a} \times \vec{b}|$ then angle between vector $\vec{a}$ and vector $\vec{b}$ is : **1**
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

(xiii) If $\vec{a}.\vec{b} = 0$ then angle between vectors $\vec{a}$ and $\vec{b}$ is : **1**
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

(xiv) Direction ratios of line given by $\frac{x-1}{3} = \frac{2y+6}{12} = \frac{1-z}{-7}$ are : **1**
(a) $< 3,12,-7 >$ (b) $< 3,-6,7 >$ (c) $< 3,6,7 >$ (d)$< 3,6,-7 >$

(xv) Common area for each constraint is called : **1**
(a)infeasible region (b)feasible region (c)useless region (d)main region

(xvi) If $P(A)=\frac{1}{2}$, $P(B)=\frac{3}{8}$ and $P(A\cap B)=\frac{1}{5}$ then $P(A/B)$ is equal to : 1

(a) $\frac{2}{5}$ (b) $\frac{8}{15}$ (c) $\frac{2}{3}$ (d) $\frac{5}{8}$

Q2 Fill in the blanks from the given options

(i) Value of $\sin^{-1}(1)$ is ____________ 1

(ii) If $A=[a_{ij}]_{2\times 3}$ such that $a_{ij}=i+j$ then $a_{11}=$__________ 1

(iii) If $\begin{vmatrix} x & 0 \\ 7 & 1 \end{vmatrix}=\begin{vmatrix} 3 & 0 \\ 7 & 2 \end{vmatrix}$ then $x=$ _______ 1

(iv) If $y=\cos x$ then at $x=0$, $\frac{dy}{dx}=$ ______________ 1

(v) $\int_0^5 dx=$__________ 1

(vi) Order of the differential equation $\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^3+y=0$ is _________ 1

(vii) Direction ratios of x −axis are _______________ 1

(viii) Probability of occurrence of impossible event = _______________ 1

Q3 State true or false for the following statements :

(i) If A is a square matrix then $(A+A')$ is a skew-symmetric matrix. 1

(ii) If $y=10x$ then $\frac{dy}{dx}=0$. 1

(iii) If $y=\tan x$ then $\frac{dy}{dx}=\sec^2 x$ 1

(iv) $\int dx=x^2+c$ 1

(v) $xdy-ydx=0$ is a variable separable type of differential equation. 1

(vi) Scalar product of two perpendicular vectors is zero. 1

(vii) The point $(3,-4,2)$ lies on the $y-$ axis. 1

(viii) If $P(E)=0.4$ then $P(not\ E)=0.6$ 1

Section – B

Q4 Given a matrix $A=\begin{bmatrix} 3 & 5 \\ 2 & 7 \end{bmatrix}$,show that matrix $P=A+A'$ is a symmetric matrix. 2

Q5 Find the interval in which function $f(x)=x^2+2x-7$ is increasing. 2

OR

If $y=\log x$, then find $\frac{d^2y}{dx^2}$ 2

Q6 Evaluate $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$. 2

OR

Evaluate $\int\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$ 2

Q7 Using integration find the area bounded by the parabola $y^2=4x$ straight lines $x=1, x=4$ in the first quadrant. 2

Q8 Find the value of m if the vectors $\vec{a}=2\hat{\imath}-\hat{\jmath}-m\hat{k}$ and $\vec{b}=5\hat{\imath}+2\hat{\jmath}-\hat{k}$ are perpendicular to each other. 2

OR

Find the angle between the lines : $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-7}{-5}$ and $\frac{x+5}{3}=\frac{y-2}{2}=\frac{z-6}{4}$ 2

Section – C

Q9 Show that function : $\mathbb{R}\to\mathbb{R}$, $f(x)=\frac{4-3x}{2}$ is one-one and onto. 4

Q10 If $y=x^{\sin x}+(\sin x)^x$ then find $\frac{dy}{dx}$. 4

OR

Find the general solution of the differential equation $(\tan^{-1}y-x)dy=(1+y^2)dx$. 4

Q11 Evaluate $\int_0^{\pi/2} \log \sin x \, dx$. **4**

OR

Evaluate $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] \, dx$ **4**

Q12 Solve the following linear programming problem graphically:
Maximize and minimize $Z = 4x + 3y$ subject to the constraints

4

$$x + y \leq 8, \quad 4x + y \geq 8, \quad x - y \geq 0, \quad x \geq 0 \quad , y \geq 0$$

Q13 Probability of solving a specific problem independently by A and B are 1/2 and 1/3 respectively. If both try to solve the problem independently, find the probability that : **4**
(i)the problem is solved (ii)exactly one of them solves the problem

OR

In an examination , 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If coin falls heads, he answers true and if it falls tails, he answers false. Find the probability that he answers at least 12 questions correctly. **4**

Section – D

Q14 (a) Express the matrix $A = \begin{bmatrix} 2 & 2 & 5 \\ 3 & 9 & 5 \\ 8 & 7 & 1 \end{bmatrix}$ as a sum of a symmetric matrix and a skew-symmetric matrix. **4**

(b) If $A = \begin{bmatrix} 5 & -2 \\ 4 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 0 \\ 3 & 2 \end{bmatrix}$ then show that $(AB)' = B'A'$ **2**

OR

Solve the following system of linear equations by matrix method : **6**

$$2x - 4y + 5z = 3 \quad 3x + y - 4z = 0 \quad x + y - z = 1$$

Q15 Show that height of the cylinder of maximum volume that can be inscribed in a sphere of 30 cm is $\frac{60}{\sqrt{3}}$ cm. **6**

OR

Solve $\int \frac{1}{x^4+1} dx$ **6**

Q16(a) Find the projection of the vector $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{\jmath}$ on the vector $\vec{b} = 6\hat{\imath} + \hat{\jmath} - 2\hat{k}$ **2**

(b) Find any daigonal of the parallelogram whose adjacent sides are given by the vectors $\vec{a} = 5\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 9\hat{\jmath} + 2\hat{k}$ .Also find the area of the parallelogram. **4**

OR **6**

A line makes angles α, β, γ and δ with the diagonals of a cube, prove that **6**

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$$