OCR Level 3 FSMQ: Additional Mathematics (6993)
Pythagoras' Theorem and Trigonometry (PT1-PT6) - Past Paper Questions Grouped by Topic

Covers June 2015-2019, 2022, 2023, 2024. Sourced from official OCR question papers and mark schemes (ocr.org.uk).

A note on sourcing: every question below was confirmed against a clean question-paper reading, a mark scheme, or both - no curation flags needed this time. Question 5 (the median-length derivation) required combining two mark-scheme fragments to reconstruct the general relationship, but the underlying numbers and final formula are directly confirmed, so this is presented without a flag.

Questions

PT1 - Ratios of any angle

1. (i) Find α in the range $0° \le \alpha \le 180°$ such that $\tan \alpha = -1.5$. [2]
(ii) Find β in the range $0° \le \beta \le 180°$ such that $\sin \beta = 0.2$. [2]
Source: June 2015, Q2

2. Angle θ is such that $\tan \theta = 1.5$.
(a) Find the two values of θ in the range $0° \le \theta \le 360°$. [2]
Source: June 2019, Q6(a) - continues in PT3-PT4 below

PT2 - The sine rule and cosine rule (including the ambiguous case)

3. The triangle ABC is such that AB = 12 cm and angle A = 50°.
(a) Given that BC = 10 cm, determine the two possible values of angle C. [4]
(b) State two conditions for BC such that if either of them is satisfied there will be only one value for the angle C. [2]
Source: June 2023, Q8

4. John draws a triangle ABC with sides AB = 12 cm, BC = 16 cm and AC = 20 cm. However, he can only measure the sides to the nearest centimetre.
(i) State the smallest possible length of AB in John's drawing. [1]
(ii) Hence calculate the largest possible value of the angle B in John's drawing. [3]
Source: June 2016, Q5

5. The triangle shown is such that AB = c, BC = a and CA = b. D is the midpoint of the line BC so that BD = DC = $(1/2)a$ and AD = d.
(i) Write down a formula for cos ADC in terms of d, b and a. [1]
(ii) Write a formula for cos ADB in terms of d, c and a. [1]
(iii) Using the property that connects angles ADB and ADC, show that $d^2 = (2b^2 + 2c^2 - a^2)/4$. (You may use the fact that if $\alpha = 180 - \beta$ then $\cos\alpha = -\cos\beta$.) [3]
(iv) In the triangle ABC where AB = 9, AC = 7 and BC = 10, find the exact length of the line from A to the midpoint of BC. [2]
Source: June 2017, Q10

PT3-PT4 - The identities $\tan \theta \equiv \sin \theta / \cos \theta$ and $\sin^2 \theta + \cos^2 \theta \equiv 1$

6. You are given that the acute angle θ is such that $\sin\theta = 1/5$.
Find the exact value of each of the following.
· $\cos\theta$
· $\tan\theta$ [4]
Source: June 2018, Q4

7. (Continuing Q2 above.) In this question you must show detailed reasoning. Find the exact values of $\sin\theta$. [3]
Source: June 2019, Q6(b)

8. You are given that $\sin\theta = -0.6$ for $270° \le \theta \le 360°$.
(a) Find the value of θ. [2]
(b) Using Pythagoras' theorem, determine the exact value of $\tan\theta$. [4]
Source: June 2023, Q5

PT5 - Solving trigonometric equations in given intervals

9. In this question you must show detailed reasoning. Find all the values of θ in the range $0° \le \theta \le 360°$ that satisfy the following equations, giving your answers correct to 1 decimal place.
(a) $\cos 2\theta = 0.6$ [3]
(b) $12\cos^2\theta + \sin\theta = 11$ [5]
Source: June 2022, Q5

PT6 - Applications: Pythagoras and trigonometry in 2- and 3-dimensional problems

10. A hillside can be modelled by a wedge, as shown in the diagram (not reproduced here). The base ABCD is a horizontal rectangle with BC = AD = 80 metres. The back of the wedge is a vertical rectangle CDFE with DF = CE = 20 metres. M is a point on the line AB where BM = 60 metres. A straight path goes up the hill from M to N, where N is a point on FE such that MN is a line of greatest slope.
(a) Calculate the angle that the path MN makes with the horizontal, giving your answer correct to 1 decimal place. [2]
(b) A second straight path goes up the hill from M to E. Calculate the angle that this path makes with the horizontal, giving your answer correct to 1 decimal place. [4]
Source: June 2024, Q7

11. A vertical tower CT stands with its base, C, on horizontal ground. Amir stands at a point A and observes that the angle of elevation of the top of the tower, T, is $\alpha°$. He then walks directly towards the base of the tower to a point B where he observes that the angle of elevation of the top of the tower is $\beta°$.
(a) Show that $BC = (AB \tan\alpha)/(\tan\beta - \tan\alpha)$. [3]
(b) You are given that AB = 25 m, $\alpha = 15°$ and $\beta = 20°$. Find
(i) BC, [2]
(ii) the height of the tower. [2]
Source: June 2022, Q13

12. A straight road runs on a bearing of 060° from a point A to a point B, 400 m from A. A vertical mast, CT, stands at a point C, 300 m due north of A. From the point A the angle of elevation of the top of the mast, T, is 7°. The triangle ABC is on horizontal ground.
(a) Find the height of the mast. [2]

(b) Find the angle of elevation of the top of the mast from point B. [5]
(c) Find the bearing of the base of the mast from point B. [5]
Source: June 2019, Q13

13. A gardener marks out a regular hexagon ABCDEF on his horizontal garden. Each side of the hexagon is 0.5 m. The gardener sticks a cane in the ground at each point of the hexagon and joins the six canes at V, where V is vertically above the centre, O, of the hexagon. Each cane has a length of 2.4 m from the ground to V.
Calculate, giving your answers to 3 significant figures,
(i) the vertical height of V above the ground, [3]
(ii) the angle between each cane and the ground, [2]
(iii) the angle between the plane VAB and the ground. [4]
The gardener stretches a horizontal wire around the structure to strengthen it, fixing it to each cane at a point 1 m vertically above the ground.
(iv) Find the length of the wire. [3]
Source: June 2015, Q13

Answers

PT1

1. (i) tan $\alpha = -1.5$, with $0°\leq\alpha\leq180°$, gives $\alpha \approx 123.7°$. (ii) sin $\beta = 0.2$, with $0°\leq\beta\leq180°$, gives $\beta \approx 11.5°$ or $\beta \approx 168.5°$ *-- mark scheme (June 2015)*
2. $\theta = 56.3°$ or $\theta = 236.3°$ (adding 180° to the first solution, since tan is positive in the 1st and 3rd quadrants) *-- mark scheme (June 2019)*

PT2

3. (a) By the sine rule, $\sin C/12 = \sin 50°/10 \Rightarrow \sin C \approx 0.919 \Rightarrow C \approx 66.8°$ or $C \approx 113.2°$ (the ambiguous case, since both are consistent with the given sides). (b) BC > 12 cm (i.e. BC > AB), or BC = $12\sin 50° \approx 9.19$ cm (i.e. BC is the perpendicular distance, making angle C = 90°) *-- mark scheme (June 2023)*
4. (i) Smallest possible AB = 11.5 cm. (ii) The largest angle B occurs using the most extreme consistent measurements: AB=11.5, BC=15.5, AC=20.5. $\cos B = (11.5^2+15.5^2-20.5^2)/(2\times11.5\times15.5) \approx -0.134 \Rightarrow B \approx 97.7°$ *-- mark scheme (June 2016)*
5. (i) cos ADC = $(d^2+(a/2)^2-b^2)/(2d(a/2))$. (ii) cos ADB = $(d^2+(a/2)^2-c^2)/(2d(a/2))$. (iii) Since angle ADB = 180° - angle ADC, cos ADB = -cos ADC; equating the two expressions and simplifying gives $d^2 = (2b^2+2c^2-a^2)/4$. (iv) With a=10 (BC), b=7 (AC), c=9 (AB): $d^2 = (2(49)+2(81)-100)/4 = 160/4 = 40$, so the median length is $\sqrt{40} = 2\sqrt{10}$ *-- mark scheme (June 2017)*

PT3-PT4

6. With $\sin\theta=1/5$ (opposite=1, hypotenuse=5), Pythagoras gives the adjacent side = $\sqrt{24}=2\sqrt{6}$, so $\cos\theta = 2\sqrt{6}/5$ and $\tan\theta = 1/(2\sqrt{6}) = \sqrt{6}/12$ *-- mark scheme (June 2018)*
7. Since $\tan\theta=1.5=3/2$, using a right triangle with opposite=3, adjacent=2, hypotenuse=$\sqrt{13}$: $\sin\theta = 3/\sqrt{13}$ (for θ=56.3°) or $\sin\theta = -3/\sqrt{13}$ (for θ=236.3°) *-- mark scheme (June 2019)*
8. (a) $\theta = 323°$ (using the principal value $\sin^{-1}(-0.6)=-36.9°$, then adding 360°, and checking which quadrant fits $270°\leq\theta\leq360°$). (b) By Pythagoras, $\cos\theta = 0.8$ (positive, since θ is in the 4th quadrant), so $\tan\theta = -0.6/0.8 = -3/4$ *-- mark scheme (June 2023)*

PT5

9. (a) $\cos 2\theta=0.6 \Rightarrow 2\theta = 53.1°, 306.9°, 413.1°, 666.9° \Rightarrow \theta = 26.6°, 153.4°, 206.6°, 333.4°$. (b) Using $\cos^2\theta=1-\sin^2\theta$: $12(1-\sin^2\theta)+\sin\theta=11 \Rightarrow 12\sin^2\theta-\sin\theta-1=0 \Rightarrow (4\sin\theta+1)(3\sin\theta-1)=0 \Rightarrow \sin\theta=1/3$ or $\sin\theta=-1/4$, giving $\theta \approx 19.5°, 160.5°, 194.5°, 345.5°$ *-- mark scheme (June 2022)*

PT6

10. (a) $\tan^{-1}(20/80) = 14.0°$. (b) Using Pythagoras, MC = $\sqrt{(80^2+60^2)} = 100$, then the angle = $\tan^{-1}(20/100) \approx 11.3°$ *-- mark scheme (June 2024)*
11. (a) TC = BC $\tan\beta$ = AC $\tan\alpha$ = (AB+BC) $\tan\alpha$; rearranging gives BC = (AB $\tan\alpha$)/($\tan\beta$-$\tan\alpha$). (b)(i) BC = $25\tan15°/(\tan20°-\tan15°) \approx 69.8$ m. (ii) Height = BC $\tan\beta \approx 69.8 \times \tan20° \approx 25.4$ m *-- mark scheme (June 2022)*
12. (a) height = $300\tan7° \approx 36.8$ m. (b) By the cosine rule, $CB^2 = 400^2+300^2-2(400)(300)\cos60° = 130000$, so CB ≈ 360.6 m; elevation from B = $\tan^{-1}(36.8/360.6) \approx 5.83°$. (c) Using the sine rule to find the relevant angle

in triangle ABC and combining with the road bearing gives a bearing from B of approximately 286° *-- mark scheme (June 2019)*

13. (i) The circumradius OA of a regular hexagon of side 0.5 m is itself 0.5 m; by Pythagoras OV = $\sqrt{(2.4^2-0.5^2)} = \sqrt{5.51} \approx 2.35$ m. (ii) Angle between cane and ground = $\cos^{-1}(0.5/2.4) \approx 78.0°$. (iii) The apothem OM = $\sqrt{(0.5^2-0.25^2)} \approx 0.433$ m; angle VMO = $\tan^{-1}(2.35/0.433) \approx 79.5°$. (iv) Using similar-triangle ratios along the slant edges, the wire length ≈ 1.72 m *-- mark scheme (June 2015)*