OCR Level 3 FSMQ: Additional Mathematics (6993)
Exponentials and Logarithms (EL1-EL7) - Past Paper Questions Grouped by Topic

Covers June 2019, 2022, 2023, and the 2020 Specimen paper. Sourced from official OCR question papers and mark schemes (ocr.org.uk).

Questions

EL1 and EL6 - The exponential function and its graph; solving equations of the form $a^x = b$

1. (a) On the grid provided, sketch the curve $y = 5^{1-2x}$. [2]
(b) Solve algebraically the equation $5^{1-2x} = 3$, giving your answer correct to 3 significant figures. [3]
Source: June 2019, Q4

EL2-EL3 - The logarithmic function and the laws of logarithms

2. Write $\log_{10}(2a) + 2\log_{10} b$ as a single logarithm. [2]
Source: June 2024, Q2

3. In this question you must show detailed reasoning.
(a) You are given that $y = 3\log_4 x$. Rewrite this equation with x as the subject. [2]
(b) Write $2\log_{10} 5 + (1/2)\log_{10} 16$ as a single number. [3]
(c) The equation $a^x = 17$ has the solution $x = 2.58$, correct to 3 significant figures. Given that a is an integer, determine the value of a. [2]
Source: June 2022, Q9

4. In this question you must show detailed reasoning.
Solve the equation $\log_{10} x + \log_{10}(x+2) = 3\log_{10} 2$. [5]
Source: June 2019, Q5

EL4-EL5, EL7 - Reducing to linear form; estimating constants from a graph; growth and decay

5. Sarah brings a saucepan of water to the boil. She leaves the water to cool, measuring its temperature every 10 minutes for 30 minutes. The results are shown in the table below.
Time (t minutes): 0, 10, 20, 30 Temperature (T °C): 100, 60, 40, 30
Sarah believes that the temperature of the water as it cools can be modelled by the equation $T - 20 = A \times 2^{-t/b}$ where A and b are constants.
(a)(i) Explain the significance of the number 20 in this equation. [1]
(ii) Use the fact that the initial temperature of the water is 100°C to determine the value of A. [2]
(b) By taking logs of both sides of Sarah's equation, show that plotting $\log_{10}(T-20)$ against t will give a straight line. [3]
(c) Complete the table below. [2]
Time (t minutes): 0, 10, 20, 30 T-20: (to be completed) $\log_{10}(T-20)$: (to be completed)
(d) Plot the values of $\log_{10}(T-20)$ against t on the grid provided. [1]
(e) Hence estimate the value of b. [2]

Source: June 2023, Q14

6. John bought a car in January 2015 for £28 000. He investigated how the value of his car might depreciate over the years and found the following data.
Number of years after buying car (t years): 0, 2, 4, 6, 8   Value (£v): 28000, 18000, 11500, 7350, 4700
He believes that the relationship between the age of the car and its value can be modelled by the equation $v = ka^t$ where v is the value in pounds and t is the age in years.

(a) Write down the value of k. [1]

(b) Show that the equation can be rewritten in the form $\log_{10} v = \log_{10} k + t \log_{10} a$. [2]

John plotted $\log_{10} v$ against t and obtained a straight-line graph.

(c) Use the graph to estimate a value for a. [3]

(d) Use this model to estimate the age of John's car when its value drops below £3000. [3]

Source: 2020 Specimen, Q15

Answers

EL1, EL6

1. (a) A decreasing exponential curve, approaching the x-axis (asymptotic) as $x \to \infty$, crossing the y-axis at (0,5). (b) Taking logs: $(1-2x)\log 5 = \log 3 \Rightarrow x = [1 - (\log 3/\log 5)]/2 \approx 0.159$ *-- mark scheme (June 2019)*

EL2-EL3

2. $\log_{10}(2a) + 2\log_{10} b = \log_{10}(2a) + \log_{10}(b^2) = \log_{10}(2ab^2)$ *-- mark scheme (June 2024)*

3. (a) $y=3\log_4 x \Rightarrow y/3=\log_4 x \Rightarrow x=4^{y/3}$. (b) $2\log_{10}5+(1/2)\log_{10}16 = \log_{10}25+\log_{10}4 = \log_{10}100 = 2$. (c) $x=\log_a 17=2.58 \Rightarrow a^{2.58}=17$; testing $a=3$ gives $3^{2.58}\approx 17$, so $a=3$ *-- mark scheme (June 2022)*

4. $\log_{10}x+\log_{10}(x+2)=3\log_{10}2 \Rightarrow \log_{10}[x(x+2)]=\log_{10}8 \Rightarrow x^2+2x-8=0 \Rightarrow (x+4)(x-2)=0 \Rightarrow x=2$ (rejecting $x=-4$, since logarithms of negative numbers are undefined) *-- mark scheme (June 2019)*

EL4-EL5, EL7

5. (a)(i) 20°C is the long-term (room/ambient) temperature that the water settles towards as t increases, since $2^{-t/b} \to 0$. (ii) At $t=0$: $100-20=A\times 2^0 \Rightarrow A=80$. (b) Taking logs: $\log_{10}(T-20)=\log_{10}A-(t/b)\log_{10}2$, which is of the form $Y=c+mt$ (a straight line). (c) $T-20$: 80,40,20,10; $\log_{10}(T-20)$: 1.90,1.60,1.30,1.00. (e) Gradient of the graph ≈ -0.03, so $\log_{10}2/b=0.03 \Rightarrow b \approx 10$ *-- mark scheme (June 2023)*

6. (a) $k=28000$ (the value when $t=0$). (b) Taking logs of $v=ka^t$ gives $\log_{10}v=\log_{10}k+t\log_{10}a$. (c) The gradient of the $\log_{10}v$ vs t graph gives $\log_{10}a$ in the range [-0.10,-0.09], so $a \approx 0.8$. (d) Using $v=28000(0.8)^t$, setting $v=3000$ gives $0.8^t\approx 0.107 \Rightarrow t=\log(0.107)/\log(0.8) \approx 10$ years *-- official specimen mark scheme*