OCR Level 3 FSMQ: Additional Mathematics (6993)
Calculus (CA1-CA15) - Past Paper Questions Grouped by Topic

Covers June 2015-2019, 2022, 2023, 2024, and the 2020 Specimen paper. Sourced from official OCR question papers and mark schemes (ocr.org.uk).

Questions

CA1-CA7 - Differentiation: gradient, tangents, normals, stationary points

1. Find the equation of the tangent to the curve $y = x^2 - 3x$ at the point (4, 4). [4]
Source: June 2017, Q3

2. Find the equation of the tangent to the curve $y = x^3 + 3x - 5$ at the point (2, 9). [5]
Source: June 2015, Q3

3. Find the equation of the normal to the curve $y = x^3 - 2x^2 + 2x + 4$ at the point (2, 8). [6]
Source: June 2019, Q3

4. A curve has equation $y = x^3 - 3x^2 - 3x + 4$. Points P and Q lie on the curve. The coordinates of P are (3, -5).
(i) Find the equation of the tangent to the curve at P. [4]
The tangent to the curve at Q is parallel to the tangent to the curve at P.
(ii) Find the coordinates of Q. [3]
Source: June 2016, Q9

5. Determine the coordinates of the stationary points on the curve $y = x^3 - 6x^2 + 9x + 4$. [5]
Source: June 2024, Q10

6. In this question you must show detailed reasoning. You are given that the curve $y = 2x^3 + 3x^2 - 12x + 8$ has two stationary points.
(a)(i) Show that one of the stationary points has coordinates (1, 1). [4]
(ii) Determine the nature of this stationary point. [2]
(b) Find the coordinates of the other stationary point. [2]
Source: June 2023, Q15

7. You are given that $y = x^3 + 2x - 7$.
(a) Find dy/dx. [2]
(b) Use your result to part (a) to show that the graph of $y = x^3 + 2x - 7$ has no turning points. [2]
Source: 2020 Specimen, Q3

8. In the diagram, the curve with equation $y = (1/2)x^2 - 2x$ crosses the x-axis at the origin, O, and the point A. The tangent to this curve at O and the normal to this curve at A intersect at the point B.
(a) Determine the equation of the line OB. [3]
(b) Determine the equation of the line AB. [6]
(c) Hence determine the coordinates of the point B. [3]
Source: June 2022, Q12

CA8-CA13 - Integration: reversing differentiation, definite integrals, area

9. The gradient function of a curve is given by $dy/dx = 3x^2 - 4x + 2$, and the curve passes through the point (1, 3).
Find the equation of the curve. [4]
Source: June 2016, Q2

10. The gradient function of a curve is given by $dy/dx = 2 + 4x - x^2$, and the curve passes through the point (1, 2).
Determine the equation of the curve. [4]
Source: June 2022, Q8

11. (i) Find $\int_2^5 (x^2 - 7)\,dx$. [4]
(ii) Explain, by means of a sketch, why the area between the curve $y = x^2 - 7$ and the lines $x = 2$ and $x = 5$ is NOT the same as your answer to part (i). [2]
Source: June 2017, Q7

12. Fig. 5 shows part of the graph of the curve with equation $y = 6x^2 - 2x^3$.
Find the area of the shaded region enclosed by the curve and the x-axis. [4]
Source: June 2018, Q5

13. The curve C1 has equation $y = -x^2 + 10x + k$ and passes through the point (5, 10).
(a) Show that $k = -15$. [2]
(b) Show that there is a maximum value at the point (5, 10). [3]
The curve C2 with equation $y = (x - 3)^2$ has been plotted on the grid provided.
(c) On the same grid, sketch the curve C1. [2]
(d) Find the coordinates of the points of intersection of the curves C1 and C2. [2]
(e) Find the area between the curves C1 and C2. [5]
Source: June 2019, Q12

CA14-CA15 - Application to kinematics

14. An object is falling through a liquid. The distance fallen is modelled by the formula $s = 48t - t^3$ until it comes to rest, where s is the distance fallen in centimetres and t is the time in seconds measured from the point when the object entered the liquid.
(a) Find
(i) the acceleration when $t = 1$, [5]
(ii) the time when the object comes to rest, [2]
(iii) the distance fallen when the object comes to rest. [2]
(b) Sketch the velocity/time graph for the period of time until the object comes to rest. [1]
Source: 2020 Specimen, Q14

15. A car moves from rest away from traffic lights such that after t seconds its velocity, v ms^{-1}, is given by $v = (7/128)t^2(12 - t)$.
(i) Show that the acceleration is 0 when $t = 0$ and $t = 8$. [3]
(ii) Find the distance travelled in the first 8 seconds. [4]
Source: June 2017, Q9

Answers

CA1-CA7

1. $dy/dx = 2x-3$, which at $x=4$ gives gradient 5. Tangent: $y-4=5(x-4) \Rightarrow y = 5x-16$ -- *mark scheme (June 2017)*

2. $dy/dx = 3x^2+3$, which at $x=2$ gives gradient 15. Tangent: $y-9=15(x-2) \Rightarrow y = 15x-21$ -- *mark scheme (June 2015)*

3. $dy/dx = 3x^2-4x+2$, which at $x=2$ gives gradient 6, so the normal gradient is -1/6. Normal: $y-8=-1/6(x-2) \Rightarrow x+6y=50$ -- *mark scheme (June 2019); see flag above*

4. (i) $dy/dx=3x^2-6x-3$, which at $x=3$ (point P) gives gradient 6. Tangent: $y+5=6(x-3) \Rightarrow y=6x-23$. (ii) Setting $3x^2-6x-3=6$ gives $x^2-2x-3=0 \Rightarrow (x-3)(x+1)=0$, so $x=-1$ (the other solution besides P itself); at $x=-1$, $y=3$, so Q=(-1,3) -- *mark scheme (June 2016)*

5. $dy/dx=3x^2-12x+9=0 \Rightarrow x^2-4x+3=0 \Rightarrow (x-1)(x-3)=0 \Rightarrow x=1$ or $x=3$. The stationary points are (1, 8) and (3, 4) -- *mark scheme (June 2024)*

6. (a)(i) $dy/dx=6x^2+6x-12=0 \Rightarrow x^2+x-2=0 \Rightarrow (x-1)(x+2)=0 \Rightarrow x=1$ or $x=-2$; at $x=1$, $y=2+3-12+8=1$, confirming (1,1). (ii) The second derivative is $12x+6$, which at $x=1$ gives 18>0, so (1,1) is a minimum. (b) At $x=-2$, $y=-16+12+24+8=28$, giving the other stationary point (-2, 28) -- *mark scheme (June 2023)*

7. (a) $dy/dx = 3x^2+2$. (b) Since $x^2 \geq 0$ for all real x, $3x^2+2 \geq 2 > 0$ always, so dy/dx is never zero and the curve has no turning points -- *official specimen mark scheme*

8. Curve crosses the x-axis at O(0,0) and A(4,0). Gradient at O = -2, so line OB: $y=-2x$. Gradient at A = 2, so normal gradient = -1/2; line AB: $y=-x/2+2$. Solving simultaneously: $-2x=-x/2+2 \Rightarrow x=-4/3$, $y=8/3$, so B=(-4/3, 8/3) -- *mark scheme (June 2022)*

CA8-CA13

9. Integrating: $y=x^3-2x^2+2x+c$. Substituting (1,3): $3=1-2+2+c \Rightarrow c=2$, so $y=x^3-2x^2+2x+2$ -- *mark scheme (June 2016)*

10. Integrating: $y=2x+2x^2-x^3/3+c$. Substituting (1,2): $2=2+2-1/3+c \Rightarrow c=-5/3$, so $y=2x+2x^2-x^3/3-5/3$ -- *own working, consistent with mark scheme method (June 2022)*

11. (i) $\int (x^2-7)dx = [x^3/3-7x]$; evaluating between 2 and 5 gives 20/3 - (-34/3) = 18. (ii) The curve $y=x^2-7$ crosses the x-axis at $x=\sqrt{7}\approx 2.65$, which lies between $x=2$ and $x=5$. Part of the region (from $x=2$ to $\sqrt{7}$) lies below the x-axis, so its contribution to the definite integral is negative and partly cancels the positive area above the axis - the true enclosed area (all measured as positive) is therefore larger than the value of the plain integral -- *own working, consistent with mark-scheme reasoning (June 2017)*

12. $\int_0^3 (6x^2-2x^3)dx = [2x^3-x^4/2]$ from 0 to 3 = 54 - 40.5 = 13.5 -- *mark scheme (June 2018)*

13. (a) Substituting (5,10): $-25+50+k=10 \Rightarrow k=-15$. (b) $dy/dx=-2x+10=0$ at $x=5$, and the second derivative $-2<0$, confirming a maximum. (d) Solving $-x^2+10x-15=(x-3)^2$ gives $x^2-8x+12=0 \Rightarrow (x-2)(x-6)=0$, so the curves meet at (2,1) and (6,9). (e) Area = $\int_2^6$ [C1-C2] $dx = \int_2^6 (-2x^2+16x-24)dx = 64/3$ -- *mark scheme (June 2019)*

CA14-CA15

14. $v=ds/dt=48-3t^2$, $a=dv/dt=-6t$. (a)(i) At $t=1$, $a=-6$ cm s^{-2}. (ii) The object comes to rest when $v=0$: $48-3t^2=0 \Rightarrow t=4$ s. (iii) At $t=4$, $s=48(4)-4^3=192-64=128$ cm. (b) The velocity/time graph is a downward-curving parabola starting at (0,48) and meeting the t-axis at (4,0) -- *official specimen mark scheme*

15. $v=(7/128)(12t^2-t^3)$, so $a=dv/dt=(7/128)(24t-3t^2)=(21/128)t(8-t)$, which is zero at $t=0$ and $t=8$. (ii)

Distance = $\int_0^8 v\,dt = (7/128)[4t^3 - t^4/4]$ from 0 to 8 = (7/128)(2048-1024) = 56 m -- *mark scheme (June 2017)*