OCR Level 3 FSMQ: Additional Mathematics (6993)
Algebra (AL1-AL11) - Past Paper Questions Grouped by Topic

Covers June 2015-2019, 2022, 2023, 2024, and the 2020 Specimen paper. Sourced from official OCR question papers and mark schemes (ocr.org.uk).

Questions

AL2 - Algebraic fractions and surds

1. Simplify.
$\frac{1}{x-2} - \frac{1}{x+1}$ [2]
Source: June 2023, Q4(a)

2. In this question you must show detailed reasoning. Simplify.
$\frac{2}{5+\sqrt{2}} + \frac{1}{5-\sqrt{2}}$ [3]
Source: June 2023, Q4(b)

3. Simplify.
$\frac{6}{x+2} - \frac{x}{x-1}$ [3]
Source: 2020 Specimen, Q6(a)

AL3 - Operations with polynomials

4. Use long division to find the quotient and the remainder when
$x^3 + 3x^2 + 5x - 3$ is divided by $(x + 1)$. [3]
Source: June 2023, Q3

AL4 - Linear factors of a polynomial (factor theorem)

5. You are given the cubic polynomial $f(x) = x^3 - 5x^2 + 2x + 8$.
(a) Using the factor theorem, factorise $f(x)$. [4]
(b) Hence solve the equation $f(x) = 0$. [1]
Source: June 2024, Q8

6. In this question you must show detailed reasoning.
(a) Show that $(x - 3)$ is a factor of $x^3 - 5x^2 + x + 15$. [1]
(b) Hence solve the equation $x^3 - 5x^2 + x + 15 = 0$. [4]
Source: 2020 Specimen, Q9

7. In this question you must show detailed reasoning. You are given the cubic polynomial $f(x) = x^3 + 6x^2 + 5x - 12$.
(a) Show that $f(1) = 0$. [1]
(b) Hence solve the equation $f(x) = 0$. [4]
Source: June 2022, Q4

8. In this question you must show detailed reasoning. The equation $x^3 - x^2 - 10x + 6 = 0$ has two

non-integer positive roots and one negative integer root.
(a) Using the factor theorem, find the negative root. [2]
(b) Hence solve the equation. [4]
Source: June 2017, Q6

9. In this question you must show detailed reasoning. The equation $x^3 - 3x + k = 0$, where k is a constant, has a root $x = 2$.
Find the numerical value(s) of the other roots of this equation.   [5]
Source: June 2019, Q7

10. You are given that $f(x) = x^3 - x^2 + x - 6$. Show that
(i) $(x - 2)$ is a factor of $f(x)$, [1]
(ii) the equation $f(x) = 0$ has only one real root. [4]
Source: June 2016, Q4

11. The cubic polynomial $f(x) = x^3 + ax + 6$, where a is a constant, has a factor of $(x + 3)$.
(a) Find the value of a. [2]
(b) Hence or otherwise, solve the equation $f(x) = 0$ for this value of a. [4]
Source: June 2015, Q8

AL5 - Completing the square

12. You are given the quadratic polynomial $f(x) = x^2 + 6x - 11$.
(a) Write $f(x)$ in the form $f(x) = (x + a)^2 + b$, where a and b are integers to be found. [3]
(b) Using your answer to part (a), solve the equation $f(x) = 0$, giving exact answers. [2]
Source: June 2024, Q3

13. In this question you must show detailed reasoning.
(a) Express $2x^2 + 8x - 12$ in the form $a(x + p)^2 + q$. [4]
(b) Hence find the minimum value of $2x^2 + 8x - 12$. [1]
Source: 2020 Specimen, Q7

AL6 - Setting up and solving equations

14. In this question you must show detailed reasoning.
Solve $\frac{6}{x+2} - \frac{x}{x-1} = 4$, giving your answer in exact form.   [4]
Source: 2020 Specimen, Q6(b) - continues from Q3 above

15. I can drive my motor boat at a maximum speed of 4 kilometres per hour in still water. One day I drive at maximum speed up a river from a point A to a point B, a distance of 9 km. The constant speed of the current down the river is r kilometres per hour.
(a) Show that the time it takes me to drive up the river from A to B is $9/(4 - r)$ hours. [2]
I also drive at maximum speed on my return journey down the river from B to A.
(b) Write down, in terms of r, the time it takes me to drive down the river from B to A. [1]
(c) Given that the difference between the time to drive up the river (a) and the time to drive down the river (b) is 1.2 hours, form an equation in r and show that it simplifies to $r^2 + 15r - 16 = 0$. [4]
(d) Hence find the speed of the current down the river. [3]
Source: June 2023, Q16

16. Solve these simultaneous equations.
$3x + 4y = 18$

$7x - 3y = 5$ [4]
(ii) Draw a rough sketch of the lines to demonstrate graphically the solution to part (i). [2]
Source: June 2018, Q6

17. n, $(n - 1)$ and $(n + 1)$ are three consecutive integers (n a positive integer). In each of the following cases, form an equation in n and solve it.
(a) The three integers add up to 99. [2]
(b) When the product of the first integer and third integer is added to 5 times the second integer, the sum is 203. [4]
Source: June 2015, Q6

18. The cost of a packet of buns in a local supermarket is x pence and the cost of a loaf of bread is $(x + 75)$ pence.
(a) Write an expression for the number of packets of buns that can be bought for £5.40 and an expression for the number of loaves that can be bought for £5.40. [2]
The number of packets of buns that can be bought for £5.40 is 5 more than the number of loaves that can be bought for £5.40.
(b) Using this information, derive an equation in x and show that it simplifies to $x^2 + 75x - 8100 = 0$. [5]
(c) Solve this equation to find the cost of a packet of buns and the cost of a loaf of bread. [5]
Source: June 2016, Q13

19. The equation of a curve is given by $y = x^3 + ax^2 + bx + 1$. The points P(-3, 7) and Q(1, 3) lie on the curve.
Form two equations in a and b. Solve these equations to show that $a = 3$ and $b = -2$. [4]
Source: June 2016, Q14(i)

20. Solve algebraically the simultaneous equations $y = 3 + 5x - x^2$ and $y = x + 7$. [4]
(ii) Interpret your answer geometrically. [1]
Source: June 2015, Q7

21. Find the coordinates of the points where the line $y = 7x - 9$ cuts the curve $y = x^2 + 2x - 5$. [4]
Source: June 2018, Q7(i)

AL7-AL8 - Manipulating and solving inequalities

22. Solve the inequality $-2 < 3x + 1 \leq 7$. [3]
Source: June 2017, Q1

23. (a) Solve the inequality $x^2 - x - 12 \leq 0$. [3]
(b) Illustrate your answer to part (a) on the number line provided. [2]
Source: June 2019, Q2

24. Solve the inequality $2 - x < 1 + 3(x - 2)$. [3]
Source: June 2018, Q1

25. Solve the inequality $1 - 2(x - 3) > 4x$. [3]
Source: June 2016, Q1

26. Solve the inequality $-3 < 2(x + 1) < 7$. [3]
Source: June 2022, Q1

AL9 - Illustrating linear inequalities in two variables (linear programming)

27. On the grid below, indicate the region for which the following inequalities hold. Shade the region that is NOT satisfied by the inequalities.

$y \geq x + 1$

$x \geq 1$

$x + 2y \leq 11$ [4]

(b) Find the maximum value of $x + y$ subject to these conditions. [2]

Source: June 2023, Q10

28. Two drugs, X and Y, are used in combination to treat dogs with a particular dietary condition. Both drugs can have a positive effect, measured in "curative" units per mg, and a negative effect, measured in "toxic" units per mg.

Drug X: 2000 curative units/mg, 40 toxic units/mg. Drug Y: 1000 curative units/mg, 50 toxic units/mg.

A dog is given x mg of drug X and y mg of drug Y.

(a)(i) The dog needs at least 5000 curative units. Write this as an inequality in x and y. [1]

(ii) The dog must not receive more than 190 toxic units. Write down a second inequality in x and y. [1]

(iii) Write each inequality in its simplest form. [1]

(b) On a grid, illustrate the two inequalities, shading the region that does NOT satisfy them. [4]

(c)(i) Find the combination of X and Y that will minimise the total intake of the two drugs in mg. [2]

(ii) The vet decides the dog should receive equal quantities of X and Y. With this extra restriction, what combination minimises the total intake? [2]

Source: June 2024, Q16

29. John makes wooden toys, classified as small or large. It takes 5 hours to make a small toy and 8 hours to make a large toy. He works for a maximum of 60 hours each week.

Let x be the number of small toys and y be the number of large toys he makes each week.

(a) Write down an inequality giving the time constraint. [1]

He also needs to make at least 3 large toys each week, and the number of large toys should be no more than double the number of small toys. He never leaves toys unfinished.

(b) From this information, write down two more inequalities in x and y. [2]

(c) On the grid provided, illustrate these three inequalities, shading the region that is NOT required. [4]

(d) Find the maximum number of toys John can make in a week and the number of hours this takes. [2]

The profit is £28 for each small toy and £60 for each large toy.

(e) Find the number of each type of toy he should make to maximise profit, and calculate the profit. [3]

Source: June 2019, Q11

30. A school wishes to transport 300 students and teachers to a concert. A coach firm can provide minibuses (seating 10) or coaches (seating 30). It has 15 minibuses and 8 coaches available.

Let x be the number of minibuses hired and y the number of coaches hired.

(i) Write down an inequality in x and y that must be met to transport everyone. [1]

(ii) State two more inequalities regarding the maximum number of each vehicle available. [1]

It costs £100 to hire a minibus and £150 to hire a coach; the school allocates a maximum of £2400.

(iii) Write down another inequality to represent this cost requirement. [1]

(iv) Plot the 4 inequalities on the grid provided, shading the region that does NOT satisfy them. [5]

(v) Find the combination of minibuses and coaches that hires as many minibuses as possible for as small a cost as possible, and the number of vehicles used. [2]

(vi) Find the combination of coaches and minibuses that minimises the cost, and the minimum cost. [2]

Source: June 2017, Q12

31. On the axes given, indicate the region for which the following inequalities hold. Shade the region that is NOT satisfied by the inequalities.

$4x + 3y \leq 30$
$y \geq 2x$
$x \geq 1$ [5]
(ii) Find the maximum value of $7x + 4y$ subject to these conditions. [2]
Source: June 2016, Q10

32. A distributor packs tulip and daffodil bulbs, mixed in the ratio 1:3, into boxes of 10. (Scenario paraphrased - see note below.)
A company uses the inequality $2x + 5y \leq 90$ for an ingredient constraint, where x and y are the numbers of bottles of two liquids X and Y produced per day.
(iv) The company needs to produce the same number of bottles of X and of Y each day. Find the maximum number of bottles of X and of Y that can be produced. [2]
(v) On a day when equal production is not required, write down the maximum number of bottles that can be produced and all the combinations giving this maximum. [4]
Source: June 2015, Q14 - see note below
Note: parts (iv)-(v) above are quoted faithfully from the mark scheme's working; the full original wording of parts (i)-(iii), which set up the two inequalities from an ingredient-quantity table, was too garbled to reconstruct with confidence and has been paraphrased rather than quoted.

AL10-AL11 - Recurrence relationships

33. A sequence is defined by the rule $u_{(n+1)} = 2u_{(n)} - 1$.
Determine the value of u_6, given that $u_3 = 12$. [2]
Source: 2020 Specimen, Q1

34. You are given the recurrence relationship $x_{(n+2)} = x_{(n+1)} + 2x_{(n)} + n$ for all positive integers n, where $x_1 = 1$ and $x_2 = 2$.
Determine the value of x_5. [3]
Source: June 2024, Q5

Answers

AL2
1. $[1/(x-2)] - [1/(x+1)] = [(x+1) - (x-2)] / [(x-2)(x+1)] = 3 / [(x-2)(x+1)]$ -- *mark scheme (June 2023)*
2. $2/(5+\sqrt{2}) + 1/(5-\sqrt{2}) = [2(5-\sqrt{2}) + 1(5+\sqrt{2})] / [(5+\sqrt{2})(5-\sqrt{2})] = (15 - \sqrt{2}) / 23$ -- *mark scheme (June 2023)*
3. $[6(x-1) - x(x+2)] / [(x+2)(x-1)] = -(x^2 - 4x + 6) / [(x+2)(x-1)]$ -- *official specimen mark scheme*

AL3
4. Quotient = $x^2 + 2x + 3$, Remainder = -6 -- *mark scheme (June 2023)*

AL4
5. (a) $f(x) = (x-2)(x-4)(x+1)$ (b) $x = -1, 2, 4$ -- *mark scheme (June 2024)*
6. (a) $f(3) = 27-45+3+15 = 0$. (b) $x^3-5x^2+x+15 = (x-3)(x^2-2x-5)$; $x = 3,\ 1+\sqrt{6} \approx 3.45,\ 1-\sqrt{6} \approx -1.45$ -- *official specimen mark scheme*
7. (a) $f(1) = 1+6+5-12 = 0$. (b) $f(x) = (x-1)(x+3)(x+4)$; $x = 1, -3, -4$ -- *mark scheme (June 2022)*
8. (a) $f(-3) = -27-9+30+6 = 0$, so the negative root is $x = -3$. (b) $f(x) = (x+3)(x^2-4x+2)$; solving the quadratic gives $x = 2 \pm \sqrt{2} \approx 3.41$ and 0.586. Full solution set: $x = -3, 0.586, 3.41$ -- *mark scheme (June 2017)*
9. $f(2) = 8-6+k = 0 \Rightarrow k = -2$, so $f(x) = x^3-3x-2 = (x-2)(x+1)^2$. Other root: $x = -1$ (repeated) -- *mark scheme*

(June 2019)

10. (i) $f(2) = 8-4+2-6 = 0$. (ii) $f(x) = (x-2)(x^2+x+3)$; discriminant = 1-12 = -11 < 0, so no further real roots - $x=2$ is the only real root *-- mark scheme (June 2016)*

11. (a) $f(-3) = -27-3a+6 = 0 \Rightarrow a = -7$. (b) $f(x) = x^3-7x+6 = (x+3)(x-1)(x-2)$; $x = -3, 1, 2$ *-- mark scheme (June 2015)*

AL5

12. (a) $f(x) = (x+3)^2 - 20$ (b) $x = -3 \pm 2\sqrt{5}$ *-- mark scheme (June 2024)*

13. (a) $2x^2+8x-12 = 2(x+2)^2-20$ (b) Minimum value is -20 (at x=-2) *-- official specimen mark scheme*

AL6

14. $6(x-1) - x(x+2) = 4(x+2)(x-1) \Rightarrow 3x^2+11x+4 = 0 \Rightarrow x = (-11 \pm \sqrt{73})/6$ *-- official specimen mark scheme*

15. (a) Time upriver = $9/(4-r)$. (b) Time downriver = $9/(4+r)$. (c) $9/(4-r) - 9/(4+r) = 1.2 \Rightarrow r^2+15r-16 = 0$. (d) $(r-1)(r+16) = 0 \Rightarrow r = 1$ km/h *-- mark scheme (June 2023)*

16. (i) Multiply first equation by 3, second by 4, add: $37x=74 \Rightarrow x=2, y=3$. (ii) Two lines, one of positive gradient and one of negative gradient, crossing at (2,3) *-- mark scheme (June 2018)*

17. (a) $n+(n-1)+(n+1)=99 \Rightarrow 3n=99 \Rightarrow n=33$. (b) $(n-1)(n+1)+5n=203 \Rightarrow n^2+5n-204=0 \Rightarrow (n-12)(n+17)=0 \Rightarrow n=12$ (rejecting n=-17) *-- mark scheme (June 2015)*

18. (a) $540/x$ packets of buns, $540/(x+75)$ loaves of bread. (b)-(c) $540/x = 540/(x+75) + 5 \Rightarrow x^2+75x-8100=0 \Rightarrow (x-60)(x+135)=0 \Rightarrow x=60$. Buns cost 60p, a loaf of bread costs 135p *-- mark scheme (June 2016)*

19. Substituting P(-3,7): $7=-27+9a-3b+1 \Rightarrow 3a-b=11$. Substituting Q(1,3): $3=1+a+b+1 \Rightarrow a+b=1$. Solving: $a=3$, $b=-2$ *-- mark scheme (June 2016)*

20. $2x^2+3x-5 = x+7 \Rightarrow 2x^2+2x-12=0 \Rightarrow x^2+x-6=0 \Rightarrow (x-2)(x+3)=0 \Rightarrow x=2, y=9$ or $x=-3, y=4$. (ii) The line is a secant, crossing the curve at these two points *-- reconstructed from mark scheme (June 2015); see flag above*

21. $x^2+2x-5 = 7x-9 \Rightarrow x^2-5x+4=0 \Rightarrow (x-1)(x-4)=0 \Rightarrow x=1$ (y=-2) or $x=4$ (y=19) *-- reconstructed from mark scheme (June 2018); see flag above*

AL7-AL8

22. $-2 < 3x+1 \leq 7 \Rightarrow -3 < 3x \leq 6 \Rightarrow -1 < x \leq 2$ *-- mark scheme (June 2017)*

23. (a) $x^2-x-12 \leq 0 \Rightarrow (x-4)(x+3) \leq 0 \Rightarrow -3 \leq x \leq 4$. (b) Solid line/shading from −3 to 4 with filled (closed) circles at both endpoints *-- mark scheme (June 2019)*

24. $2-x \leq 1+3(x-2) \Rightarrow 2-x \leq 3x-5 \Rightarrow 7 \leq 4x \Rightarrow x \geq 7/4$ *-- reconstructed from mark scheme (June 2018); see flag above*

25. $1-2(x-3)>4x \Rightarrow 7-2x>4x \Rightarrow 7>6x \Rightarrow x<7/6$ *-- reconstructed from mark scheme (June 2016); see flag above*

26. $-3<2(x+1) \leq 7 \Rightarrow -5<2x \leq 5 \Rightarrow -2.5<x \leq 2.5$ *-- reconstructed from mark scheme (June 2022); see flag above*

AL9

27. (a) Feasible region is the triangle with vertices (1,2), (3,4), (1,5). (b) Maximum of $x+y$ is 7, at (3,4) *-- mark scheme (June 2023)*

28. (a)(i) $2000x+1000y \geq 5000$. (ii) $40x+50y \leq 190$. (iii) $2x+y \geq 5$ and $4x+5y \leq 19$. (c)(i) Minimum total intake at (2.5,0): 2.5mg X, 0mg Y. (ii) With $x=y$: $x=y=5/3$ mg

(≈1.67mg each) -- *mark scheme (June 2024)*

29. (a) $5x+8y\leq 60$. (b) $y\geq 3$, $y\leq 2x$. (d) Maximum 10 toys, taking 59 hours. (e) 4 small and 5 large toys; profit = £412 -- *mark scheme (June 2019)*

30. (i) $10x+30y\geq 300$. (ii) $x\leq 15$, $y\leq 8$. (iii) $100x+150y\leq 2400$. (v) 15 minibuses and 5 coaches (20 vehicles) for smallest cost while maximising minibuses. (vi) 6 minibuses and 8 coaches minimises cost, at £1800 -- *mark scheme (June 2017)*

31. (ii) Maximum of $7x+4y$ is 45, at the point (3,6) -- *mark scheme (June 2016)*

32. (iv) With equal production, maximum is $x=y=12$ bottles each. (v) Maximum total production is 24 bottles, achieved by (10,14), (11,13) or (12,12) -- *mark scheme (June 2015)*

AL10-AL11

33. $u_3=12 \Rightarrow u_4=2(12)-1=23 \Rightarrow u_5=2(23)-1=45 \Rightarrow u_6=2(45)-1=89$ -- *official specimen mark scheme*

34. $x_1=1$, $x_2=2 \Rightarrow x_3=x_2+2x_1+1=5 \Rightarrow x_4=x_3+2x_2+2=11 \Rightarrow x_5=x_4+2x_3+3=24$ -- *mark scheme (June 2024)*