OCR Level 3 FSMQ: Additional Mathematics (6993)
Numerical Methods (NM1-NM9) - Past Paper Questions Grouped by Topic

Covers June 2017, 2019, 2022, 2023, 2024, and the 2020 Specimen paper.

Questions

NM1-NM3 - Solving equations by sign change and iteration

1. You are given that the equation $3^x - 4x^2 = 0$ has three roots, a, b and c, where $a < 0$ and $c > 3$.
(a) By considering the value of $3^x - 4x^2$ when $x = 0$ and $x = 1$, show that b lies between 0 and 1. [2]
(b) By considering appropriate values of x, determine the value of b correct to 1 decimal place. [3]
Source: June 2023, Q7

2. The equation $x^3 + 2x^2 - x - 1 = 0$ has two negative roots, α and β, and one positive root, γ.
(a) By considering a change of sign, show that γ lies in the interval [0, 1]. [2]
(b) Show that $x = \sqrt{[(x+1)/(x+2)]}$ is a rearrangement of the equation. [2]
(c) Using the iterative formula $x_{r+1} = \sqrt{[(x_r+1)/(x_r+2)]}$ with $x_0 = 0.8$, find γ correct to 3 decimal places, showing the result of each iteration. [3]
Source: June 2019, Q9

3. Tom has been asked to solve the equation $x^3 - x - 7 = 0$.
(a) Show by sign change that $x^3 - x - 7 = 0$ has a root, α, in the interval [2, 3]. [2]
(b) Tom decides to use the iterative formula $x_{r+1} = \sqrt[3]{[x_r+7]}$ to find α.
(i) Explain how he obtained this iterative formula. [2]
(ii) Using this iterative formula, with $x_0 = 2$, find α correct to 3 significant figures. [2]
Source: June 2024, Q13

NM4-NM5 - Estimating gradients using a chord

4. (a) Ben wishes to estimate the gradient of a curve at the point where $x = 1.4$. He calculates points on the curve which are given in the table below.
x: 1.2, 1.4, 1.8 y: 1.0732, 1.5358, 3.1447
(i) Explain why he should not use the coordinates at $x = 1.2$ and $x = 1.8$ to obtain a reasonable estimate for the gradient of the curve at $x = 1.4$. [1]
(ii) Calculate an estimate for the gradient of the curve at $x = 1.4$ by using the coordinates at $x = 1.4$ and $x = 1.8$. [2]
(iii) Calculate an estimate for the gradient of the curve at $x = 1.4$ by using the coordinates at $x = 1.2$ and $x = 1.4$. [1]
(b) Mia wishes to estimate the gradient of another curve at the point where $x = 1.4$. She calculates points on this curve which are given in the table below.
x: 1.2, 1.4, 1.6 y: 0.6899, 0.9518, 1.3132
Calculate a reasonable estimate of the gradient of this curve when $x = 1.4$. [3]
Source: June 2022, Q14

5. A car accelerates from rest in a straight line. At time t seconds its velocity, v m s^{-1}, is given by the equation $v = 20(1 - 2^{\frac{t}{2}})$

(a) Calculate the velocity of the car when $t = 4$, 6 and 8 seconds. [2]
(b) Hence calculate an estimate of the acceleration of the car at $t = 6$ seconds. Give your answer correct to 2 significant figures. [2]
(c) Explain how this estimate could be improved. [1]
Source: June 2023, Q6

NM6-NM8 - Estimating area using rectangles and the trapezium rule

6. The graph shows the curve $y = \sqrt{2^x}$
(a) Use the trapezium rule with 2 strips to estimate the area enclosed by the curve, the x-axis, $x = 2$ and $x = 4$, stating if this is an over- or under-estimate. [4]
(b)(i) Use the chord from (2, 2) to (4, 4) to estimate the gradient of $y = 2^{x/2}$ at $x = 3$. [1]
(ii) Determine an estimate for the gradient of the curve $y = 2^{x/2}$ at $x = 3$ which is an improvement on the estimate found in part (b)(i). [2]
Source: 2020 Specimen, Q11

7. Nina records the speed, v m s^{-1}, at which she is travelling in her car t seconds after accelerating from rest. The results are shown in the table below.
t (seconds): 0, 2, 4, 6, 8, 10 v (m s^{-1}): 0, 6.6, 9.6, 11.7, 13.2, 14.4
(a) Use these results and the axes below to draw a curve to show how her speed varies with time during these 10 seconds. [2]
(b) By constructing 5 rectangles of equal width above your curve, estimate the distance she has travelled during the first 10 seconds. [4]
(c) Without doing any further calculations, explain how she could obtain a better estimate of the distance she has travelled. [1]
Source: June 2022, Q11

8. The table below gives the coordinates of six points.
x: 0, 1, 2, 3, 4, 5 y: 1, 1.13, 1.43, 2.11, 3.69, 7.30
The points are plotted on a grid and a smooth curve is drawn to pass through the points.
(a) Using the trapezium rule with 5 strips, find an estimate for the area between the x-axis, the y-axis, this curve and the line $x = 5$. Give your answer correct to 1 decimal place. [3]
(b)(i) Calculate an estimate for the rate of change of y with respect to x for this curve when $x = 3$. Give your answer correct to 1 decimal place. [2]
(ii) Explain how this estimate could be improved. [1]
Source: June 2024, Q12

Answers

NM1-NM3

1. At x=0: 3^0-0=1. At x=1: 3-4=-1. The sign change confirms a root between x=0 and x=1. Testing values 0.5 to 0.9 gives 0.732, 0.493, 0.198, -0.152, -0.552, showing the sign change is between x=0.7 and x=0.8; checking x=0.75 gives a small positive value, so $b \approx 0.8$ to 1 decimal place *-- mark scheme (June 2023)*

2. (a) At x=0: -1. At x=1: 1. The sign change confirms a root in [0,1]. (b) $x^3+2x^2-x-1=0 \Rightarrow x^2(x+2)=x+1 \Rightarrow x^2=(x+1)/(x+2) \Rightarrow x=\sqrt{[(x+1)/(x+2)]}$. (c) From x_0=0.8: $x_1 \approx 0.801784$, $x_2 \approx 0.801926$, $x_3 \approx 0.801937$, so $\gamma \approx 0.802$ *-- mark scheme (June 2019)*

3. (a) At x=2: -1. At x=3: 17. The sign change confirms a root between 2 and 3. (b) $x^3-x-7=0 \Rightarrow x^3=x+7 \Rightarrow x=\sqrt[3]{[x+7]}$. (ii) From x_0=2: $x_1 \approx 2.080$, $x_2 \approx 2.086$, $x_3 \approx 2.086$, so $\alpha \approx 2.09$ (3sf) *-- mark scheme (June 2024)*

NM4-NM5

4. (a)(i) The intervals either side of x=1.4 are unequal (0.2 and 0.4), so the outer two points give the gradient at roughly x=1.5 (their midpoint), not a reliable estimate at x=1.4. (ii) (3.1447-1.5358)/(1.8-1.4) = 4.02. (iii) (1.5358-1.0732)/(1.4-1.2) = 2.31. (b) Using the symmetric chord: (1.3132-0.6899)/(1.6-1.2) = 1.56 *-- mark scheme (June 2022)*

5. (a) $v(4)$=15, $v(6)$=17.5, $v(8)$=18.75. (b) Central difference between t=4 and t=8: acceleration ≈ (18.75−15)/4 = 0.9375 ≈ 0.94 m s^{-2}. (c) Use a narrower interval either side of t=6 for a more accurate estimate *-- reconstructed from mark scheme (June 2023); see flag above*

NM6-NM8

6. (a) With h=1: Area ≈ (1/2)[2+2(2√2)+4] ≈ 5.83 (overestimate, since the trapezium rule overestimates area under a convex exponential curve). (b)(i) Gradient ≈ (4−2)/(4−2) = 1. (ii) A narrower chord centred on x=3 gives an improved estimate, closer to the true gradient of ≈0.980 *-- official specimen mark scheme; see flag above*

7. (b) Using rectangles of width 2, heights from the upper value in each interval: distance ≈ 2(6.6+9.6+11.7+13.2+14.4) = 111 m. (c) Use more (narrower) rectangles, or the trapezium rule, for a better estimate *-- mark scheme (June 2022)*

8. (a) With h=1: Area ≈ (1/2)[1+2(1.13+1.43+2.11+3.69)+7.30] = 12.5. (b)(i) Central difference between x=2 and x=4: rate of change ≈ (3.69−1.43)/2 = 1.13 ≈ 1.1. (ii) Use a narrower interval either side of x=3 for a better estimate *-- mark scheme (June 2024)*