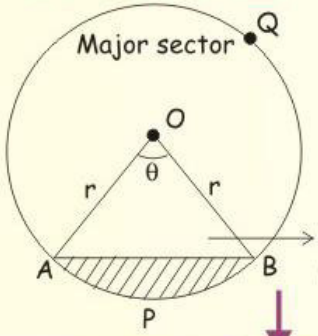
AREAS RELATED TO CIRCLES

Sector of a circle



Generally sector implies minor sector

Length of arc

$\overset{\frown}{AB} = \frac{\theta}{360^\circ} \times 2\pi r$

e.g. $\theta = 60^\circ$, $r = 3$ m

$\overset{\frown}{AB} = \frac{60^\circ}{360^\circ} \times 2\pi \times 3$

$= \pi$ m

Area of sector

$AOBP = \frac{\theta}{360^\circ} \times \pi r^2$

e.g. $\theta = 60^\circ$, $r = 3$ m

$\text{Area} = \frac{60^\circ}{360^\circ} \times \pi \times 3^2$

$= \frac{3\pi}{2}$ m

Area of segment (APB) =

Area of sector (AOB) – Area of ΔAOB

Note:

1. If two circles touch each other externally, then the distance between their centres is equal to sum of their radii.
2. If two circles touch each other internally, then the distance between their centres is equal to difference of their radii.
3. The distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel.

Circumference = $2\pi R$

Area = πR^2

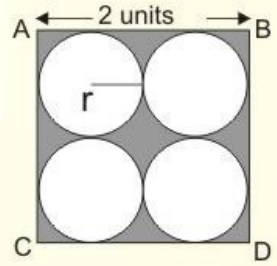
Some Important Formulas :

1. Heron's formula : Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ Where s = Semi-perimeter and a, b, c are the sides of the triangle.
2. Area of a triangle = $\frac{1}{2}$ × base × altitude
3. Area of an equilateral triangle = $\frac{\sqrt{3}}{4}$ a^2.
4. Area of a rectangle = Length × breadth
6. Area of a square of side 'a' = a^2.
7. Length of diagonal of a square of a side 'a' = $\sqrt{2}a$.
8. Area of a parallelogram = Base × Height
9. Area of a rhombus = $\frac{1}{2}d_1d_2$.

Where d_1 and d_2 are the lengths of its diagonals.

Area of combination of plane figures

e.g.



Sol.

$r + r + r + r$ = length of square

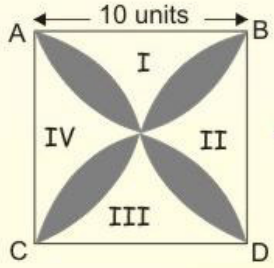
$\Rightarrow 4r = 2$; $r = 1/2$

Area of shaded reg. = Area of square – Area of 4 circles

$= (2\times2) - 4 \times \frac{\pi(1)^2}{4}$

$= 4 - \pi$

e.g.



Sol.

Area of square ABCD = 10 × 10 = 100 cm^2

Ar. (R_1) + Ar. (R_3)

= Ar. square – Ar. two semicircle of radius 5cm

$= 100 - 2\left(\frac{1}{2}\pi(5)^2\right) = 100 - 25\pi$

similarly Ar. (R_2) + Ar. (R_4) = $100 - 25\pi$

So, Area of the shaded region

= Ar. square – (Ar. R_1 + Ar. R_2 + Ar. R_3 + Ar. R_4)

= 100 – ($100 - 25\pi + 100 - 25\pi$)

= $(50\pi - 100)$ sq. units