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3.3 worksheet projectile motion with answers

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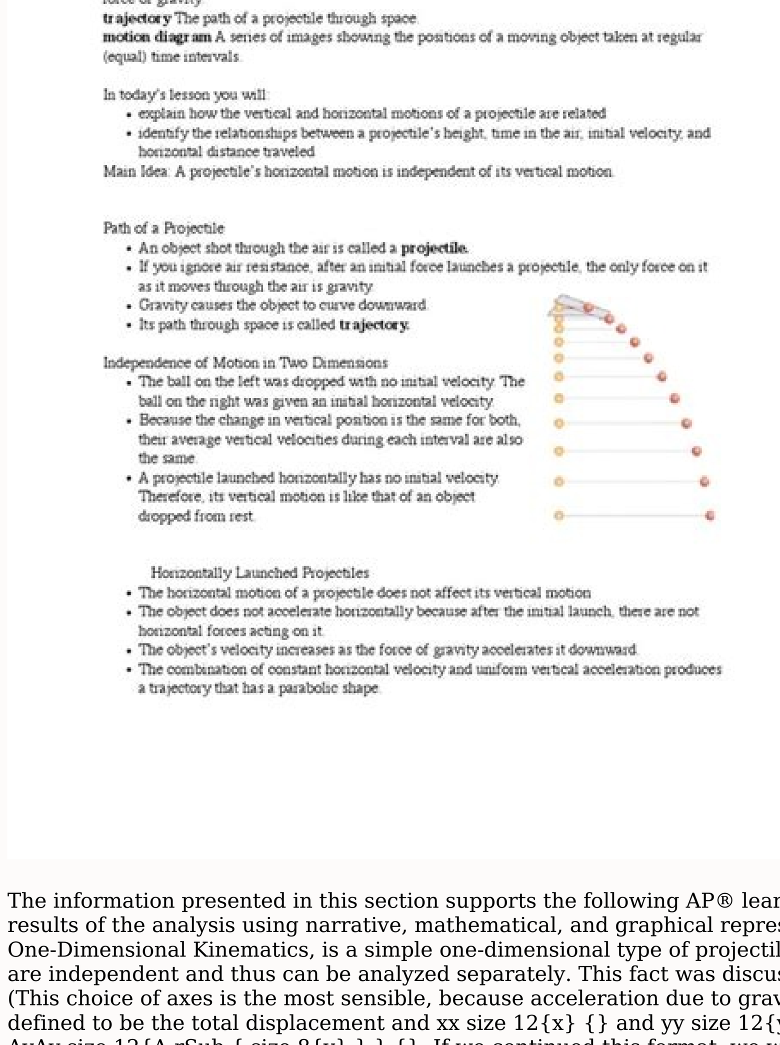
By the end of this section, you will be able to: Identify and explain the properties of a projectile, such as acceleration due to gravity, range, maximum height, and trajectory. Determine the location and velocity of a projectile at different points in its trajectory. Apply the principle of independence of motion to solve projectile motion problems.


Chapter 3 Motion in Two Dimensions

3.3: Projectile Motion

projectile An object shot through the air, such as a football, that has independent vertical and horizontal motions and, after receiving an initial thrust, travels through the air only under the force of gravity.
trajectory The path of a projectile through space.
motion diagram A series of images showing the positions of a moving object taken at regular (equal) time intervals.

In today's lesson you will:
- explain how the vertical and horizontal motions of a projectile are related
- identify the relationships between a projectile's height, time in the air, initial velocity, and horizontal distance traveled

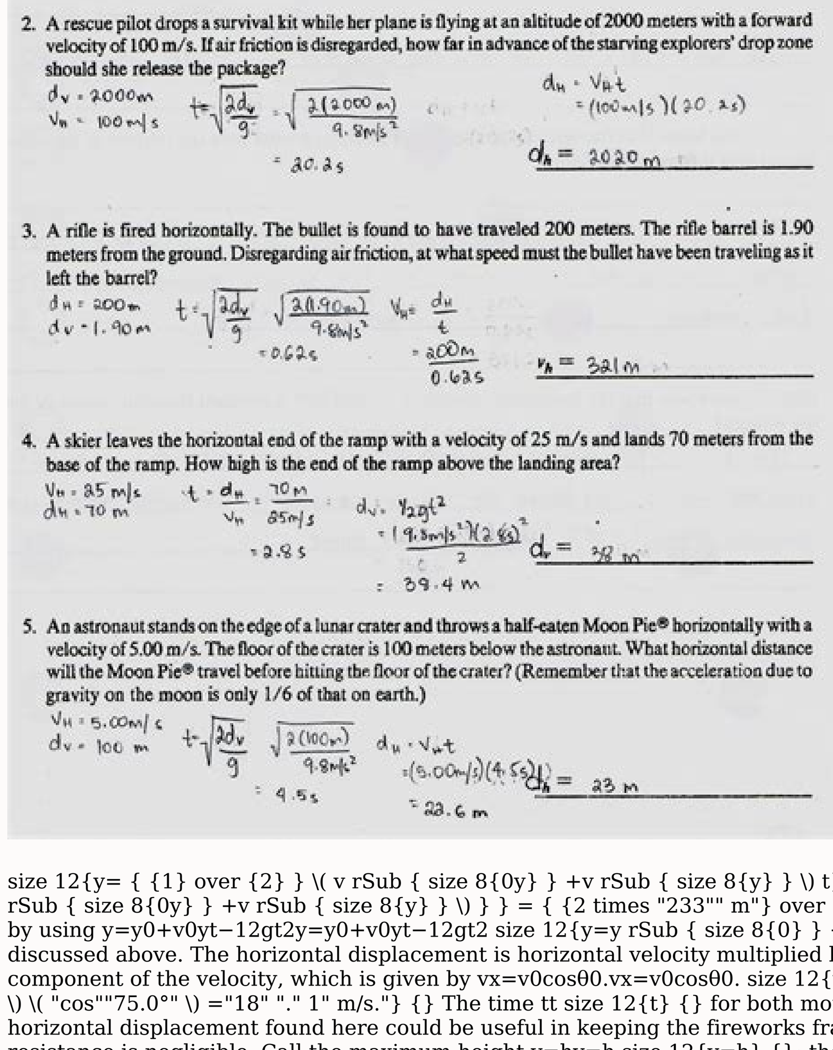
Main Idea: A projectile's horizontal motion is independent of its vertical motion.

Path of a Projectile
- An object shot through the air is called a **projectile.**
- If you ignore air resistance, after an initial force launches a projectile, the only force on it as it moves through the air is gravity.
- Gravity causes the object to curve downward.
- Its path through space is called **trajectory.**

Independence of Motion in Two Dimensions
- The ball on the left was dropped with no initial velocity. The ball on the right was given an initial horizontal velocity.
- Because the change in vertical position is the same for both, their average vertical velocities during each interval are also the same.
- A projectile launched horizontally has no initial velocity. Therefore, its vertical motion is like that of an object dropped from rest.

Horizontally Launched Projectiles
- The horizontal motion of a projectile does not affect its vertical motion.
- The object does not accelerate horizontally because after the initial launch, there are not horizontal forces acting on it.
- The object's velocity increases as the force of gravity accelerates it downward.
- The combination of constant horizontal velocity and uniform vertical acceleration produces a trajectory that has a parabolic shape.

The information presented in this section supports the following AP® learning objectives: 3.A.1.1 The student is able to express the motion of an object using narrative, mathematical, and graphical representations. (S.P. 1.5, 2.1, 2.2) 3.A.1.3 The student is able to analyze experimental data describing the motion of an object and is able to express the results of the analysis using narrative, mathematical, and graphical representations. (S.P. 5.1) Projectile motion is the motion of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a projectile, and its path is called its trajectory. The motion of falling objects, as covered in Problem-Solving Basics for One-Dimensional Kinematics, is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football or other object for which air resistance is negligible.The most important fact to remember here is that motions along perpendicular axes are independent and thus can be analyzed separately. This fact was discussed in Kinematics in Two Dimensions: An Introduction, where vertical and horizontal motions were seen to be independent. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. (This choice of axes is the most sensible, because acceleration due to gravity is vertical—thus, there will be no acceleration along the horizontal axis when air resistance is negligible.) As is customary, we call the horizontal axis the x-axis and the vertical axis the y-axis. Figure 3.36 illustrates the notation for displacement, where ss size 12{s} {} is defined to be the total displacement and xx size 12{x} {} and yy size 12{y} {} are its components along the horizontal and vertical axes, respectively. The magnitudes of these vectors are s, x, and y. (Note that in the last section we used the notation AA size 12{A} {} to represent a vector with components AxAx size 12{A rSub { size 8{x} } } {} and AyAy size 12{A rSub { size 8{y} } } {}. If we continued this format, we would call displacement ss size 12{s} {} with components sxsx size 12{s rSub { size 8{x} } } {} and sysy size 12{s rSub { size 8{y} } } {}. However, to simplify the notation, we will simply represent the component vectors as xx size 12{x} {} and yy size 12{y} {}.)Of course, to describe motion we must deal with velocity and acceleration, as well as with displacement. We must find their components along the x- and y-axes, too. We will assume all forces except gravity (such as air resistance and friction, for example) are negligible. The components of acceleration are then very simple: ay=–g=–9.80 m/s2ay=–g=–9.80 m/s2 size 12{a rSub { size 8{y} } ="-g"="-9.80" "m/s" rSup { size 8{2} } } {}. (Note that this definition assumes that the upwards direction is defined as the positive direction. If you arrange the coordinate system instead such that the downwards direction is positive, then acceleration due to gravity takes a positive value.) Because gravity is vertical, ax=0ax=0 size 12{a rSub { size 8{x} } } {}. Both accelerations are constant, so the kinematic equations can be used. x = x 0 + v - t x = x 0 + v - t size 12{x=`x rSub { size 8{0} } `+` { bar {v}}t} {} v - = v 0 + v 2 v - = v 0 + v 2 size 12{ { bar {v}}=` { {v rSub { size 8{0} } +v} over {2} } } {} v = v 0 + at v = v 0 + at size 12{v=v rSub { size 8{0} } + ital "at"} {} x = x 0 + v 0 t + 1 2 at 2 x = x 0 + v 0 t + 1 2 at 2 size 12{x=x rSub { size 8{0} } +v rSub { size 8{0} } t+ { {1} over {2} } ital "at" rSup { size 8{2} } } {} v 2 = v 0 2 + 2a (x − x 0) . v 2 = v 0 2 + 2a (x − x 0) . size 12{v rSup { size 8{2} } =v rSub { size 8{0} } rSup { size 8{2} } +2a \(x - x rSub { size 8{0} } \) } {} Figure 3.36 The total displacement ss size 12{s} {} of a soccer ball at a point along its path. The vector ss size 12{s} {} has components xx size 12{x} {} and yy size 12{y} {} along the horizontal and vertical axes.
Its magnitude is ss size 12{s} {}, and it makes an angle θθ size 12{θ} {} with the horizontal. Given these assumptions, the following steps are then used to analyze projectile motion: Step 1. Resolve or break the motion into horizontal and vertical components along the x- and y-axes.
These axes are perpendicular, so Ax=AcosθAx=Acosθ size 12{A rSub { size 8{x} } =A"cos"θ} {} and Ay=AsinθAy=Asinθ size 12{A rSub { size 8{y} } =A"sin"θ} {} are used. The magnitude of the components of displacement ss size 12{s} {} along these axes are xx size 12{x} {} and y.y. size 12{y} {} The magnitudes of the components of the velocity vv size 12{v} {} are vx=vcosθvx=vcosθ size 12{v rSub { size 8{x} } =v"cos"θ} {} and vy=vsinθ,vy=vsinθ, size 12{v rSub { size 8{y} } =v"sin"θ} {} where vv size 12{v} {} is the magnitude of the velocity and θθ size 12{θ} {} is its direction, as shown in Figure 3.37. Initial values are denoted with a subscript 0, as usual.Step 2. Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical. The kinematic equations for horizontal and vertical motion take the following forms: Horizontal Motion (a x = 0) Horizontal Motion (a x = 0) size 12{"Horizontal Motion " \(a rSub { size 8{x} } =0 \) } {} 3.33 x = x 0 + v x t x = x 0 + v x t size 12{x=x rSub { size 8{0} } +v rSub { size 8{x} } t} {} 3.34 vx=v0x=vx=velocity is a constant.vx=v0x=vx=velocity is a constant. size 12{v rSub { size 8{x} } =v rSub { size 8{0x} } =v rSub { size 8{x} } ="velocity is a constant."} {} 3.35 Vertical Motion (assuming positive is up a y = − g = − 9. 80 m/s 2) Vertical Motion (assuming positive is up a y = − g = − 9. 80 m/s 2) size 12{"Vertical Motion " \("assuming positive is up "a rSub { size 8{y} } = - g= - 9/"80"" m/s" rSup { size 8{2} } \) } {} 3.36 y = y 0 + 1 2 (v 0y + v y) t y = y 0 + 1 2 (v 0y + v y) t size 12{y=y rSub { size 8{0} } + { {1} over {2} } \(v rSub { size 8{0y} } +v rSub { size 8{y} } \) t} {} 3.37 v y = v 0y − gt v y = v 0y − gt size 12{v rSub { size 8{y} } =v rSub { size 8{0y} } - ital "gt"} {} 3.38 y = y 0 + v 0y t − 1 2 gt 2 y = y 0 + v 0y t − 1 2 gt 2 size 12{y=y rSub { size 8{0} } +v rSub { size 8{0y} } t - { {1} over {2} } ital "gt" rSup { size 8{2} } } {} 3.39 vy2=v 0y 2−2g(y−y0).vy2=v 0y 2−2g(y−y0). size 12{v rSub { size 8{y} } rSup { size 8{2} } =v rSub { size 8{0y} } rSup { size 8{2} } - 2g \(y - y rSub { size 8{0} } \) "."} {} 3.40 Step 3. Solve for the unknowns in the two separate motions—one horizontal and one vertical. Note that the only common variable between the motions is time tt size 12{t} {}. The problem solving procedures here are the same as for one-dimensional kinematics and are illustrated in the solved examples below.Step 4.
Recombine the two motions to find the total displacement ss size 12{s} {} and velocity vv size 12{v} {}. Because the x - and y -motions are perpendicular, we determine these vectors by using the techniques outlined in the Vector Addition and Subtraction: Analytical Methods and employing A = A x 2 + A y 2 A = A x 2 + A y 2 size 12{v= sqrt {v rSub { size 8{x} } rSup { size 8{2} } +v rSub { size 8{y} } rSup { size 8{2} } } } {} and θ=tan−1(Ay/Ax)θ=tan−1(Ay/Ax) size 12{θ="tan" rSup { size 8{ - 1} } \(A rSub { size 8{y} } /A rSub { size 8{x} } \) } {} in the following form, where θθ size 12{θ} {} is the direction of the displacement ss size 12{s} {} and θvθv size 12{θ rSub { size 8{v} } } {} is the direction of the velocity vv size 12{v} {}: Total displacement and velocity s = x 2 + y 2 s = x 2 + y 2 size 12{s= sqrt {x rSup { size 8{2} } +y rSup { size 8{2} } } } {} 3.41 θ = tan − 1 (y / x) θ = tan − 1 (y / x) size 12{θ="tan" rSup { size 8{ - 1} } \(y/x \) } {} 3.42 v = v x 2 + v y 2 v = v x 2 + v y 2 size 12{v= sqrt {v rSub { size 8{x} } rSup { size 8{2} } +v rSub { size 8{y} } rSup { size 8{2} } } } {} 3.43 θv=tan−1(vy/vx).θv=tan−1(vy/vx). size 12{θ rSub { size 8{v} } ="tan" rSup { size 8{ - 1} } \(v rSub { size 8{y} } /v rSub { size 8{x} } \) "."} {} 3.44 Figure 3.37 (a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because ax=0ax=0 size 12{a rSub { size 8{x} } =0} {} and vxvx size 12{v rSub { size 8{x} } } {} is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The x - and y -motions are recombined to give the total velocity at any given point on the trajectory. During a fireworks display, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of 75.0°75.0° above the horizontal, as illustrated in Figure 3.38. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passed between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes?StrategyBecause air resistance is negligible for the unexploded shell, the analysis method outlined above can be used. The motion can be broken into horizontal and vertical motions in which ax=0ax=0 size 12{ a rSub { size 8{x} } =0} {} and ay=–gay=–g size 12{ a rSub { size 8{y} } =-g} {}. We can then define x0x0 size 12{x rSub { size 8{0} } } {} and y0y0 size 12{y rSub { size 8{0} } } {} to be zero and solve for the desired quantities.Solution for (a)By “height” we mean the altitude or vertical position yy size 12{y} {} above the starting point. The highest point in any trajectory, called the apex, is reached when vy=0vy=0 size 12{ v rSub { size 8{y} } =0} {}. Since we know the initial and final velocities as well as the initial position, we use the following equation to find yy size 12{y} {}: vy2=v0y2−2g(y−y0).vy2=v0y2−2g(y−y0). size 12{v rSub { size 8{y} } rSup { size 8{2} } =v rSub { size 8{0y} } rSup { size 8{2} } - 2g \(y - y rSub { size 8{0} } \) "."} {} Figure 3.38 The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally. Because y0y0 size 12{y rSub { size 8{0} } } {} and vyvy size 12{v rSub { size 8{y} } } {} are both zero, the equation simplifies to 0=v 0y 2−2gy.0=v 0y 2−2gy. size 12{0=v rSub { size 8{0y} } rSup { size 8{2} } - 2 ital "gy."} {} Solving for yy size 12{y} {} gives y = v 0y 2 2g . y = v 0y 2 2g . size 12{y= { {v rSub { size 8{0y} } rSup { size 8{2} } } over {2g} } "." } {} Now we must find v0yv0y size 12{v rSub { size 8{0y} } } {}, the component of the initial velocity in the y-direction. It is given by v0y=v0sinθv0y=v0sinθ size 12{v rSub { size 8{0y rSup} =v rSub {0 rSup size 12{"sin"θ}} {}, where v0yv0y is the initial velocity of 70.0 m/s, and θ0=75.0°θ0=75.0° size 12{θ rSub { size 8{0} } } {} is the initial angle. Thus,v0y=v0sinθ0=(70.0 m/s)(sin 75°)=67.6 m/s.v0y=v0sinθ0=(70.0 m/s)(sin 75°)=67.6 m/s. size 12{v rSub { size 8{0y} } =v rSub { size 8{0} } "sin"θ rSub { size 8{0} } = \("70" "." 0" m/s" \) \("sin""75" { size 12{ circ } } \) ="67" "." 6" m/s."} {} and yy size 12{y} {} is y = (67 .6 m/s) 2 2 (9 . 80 m /s 2) , y = (67 .6 m/s) 2 2 (9 . 80 m /s 2) , size 12{y= { { \("67" "." 6" m/s" \) rSup { size 8{2} } } over {2 \(9 "." "80"" m/s" rSup { size 8{2} } \) } } } {} so that y=233 m.y=233 m. size 12{y="233"" m."} {} Discussion for (a)Note that because up is positive, the initial velocity is positive, as is the maximum height, but the acceleration due to gravity is negative. Note also that the maximum height depends only on the vertical component of the initial velocity, so that any projectile with a 67.6 m/s initial vertical component of velocity will reach a maximum height of 233 m (neglecting air resistance). The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding. In practice, air resistance is not completely negligible, and so the initial velocity would have to be somewhat larger than that given to reach the same height.Solution for (b)As in many physics problems, there is more than one way to solve for the time to the highest point. In this case, the easiest method is to use y=y0+12(v0y+vy)ty=y0+12(v0y+vy)t size 12{y=y rSub { size 8{0} } + { {1} over {2} } \(v rSub { size 8{0y} } +v rSub { size 8{y} } \) t} {}. Because y0y0 size 12{y rSub { size 8{0} } } {} is zero, this equation reduces to simply y = 1 2 (v 0y + v y) t . y = 1 2 (v 0y + v y) t .


1. A ball rolls with a speed of 2 m/s across a table top that is 1 meter above the floor. Upon reaching the edge of the table, it follows a parabolic path to its landing spot on the floor. How far along the floor is this spot from the table?

$V_H = 2$ m/s, $d_V = 1.0$ m; $t = \sqrt{\frac{2d_V}{g}} = \sqrt{\frac{2(1.0\,m)}{9.8\,m/s^2}} = 0.45$ s; $d_H = V_H t = (2\,m/s)(0.45\,s) = 0.9$ m; $d_H = 0.9$ m

2. A rescue pilot drops a survival kit while her plane is flying at an altitude of 2000 meters with a forward velocity of 100 m/s. If air friction is disregarded, how far in advance of the starving explorers' drop zone should she release the package?

$d_V = 2000$ m, $V_H = 100$ m/s; $t = \sqrt{\frac{2d_V}{g}} = \sqrt{\frac{2(2000\,m)}{9.8\,m/s^2}} = 20.2$ s; $d_H = V_H t = (100\,m/s)(20.2\,s)$; $d_H = 2020$ m

3. A rifle is fired horizontally. The bullet is found to have traveled 200 meters. The rifle barrel is 1.90 meters from the ground. Disregarding air friction, at what speed must the bullet have been traveling as it left the barrel?

$d_H = 200$ m, $d_V = 1.90$ m; $t = \sqrt{\frac{2d_V}{g}} = \sqrt{\frac{2(1.90\,m)}{9.8\,m/s^2}} = 0.62$ s; $V_H = \frac{d_H}{t} = \frac{200\,m}{0.62\,s}$; $V_H = 321$ m/s

4. A skier leaves the horizontal end of the ramp with a velocity of 25 m/s and lands 70 meters from the base of the ramp. How high is the end of the ramp above the landing area?

$V_H = 25$ m/s, $d_H = 70$ m; $t = \frac{d_H}{V_H} = \frac{70\,m}{25\,m/s} = 2.8$ s; $d_V = \frac{1}{2}gt^2 = \frac{(9.8\,m/s^2)(2.8\,s)^2}{2} = 39.4$ m; $d_V = 38$ m

5. An astronaut stands on the edge of a lunar crater and throws a half-eaten Moon Pie® horizontally with a velocity of 5.00 m/s. The floor of the crater is 100 meters below the astronaut. What horizontal distance will the Moon Pie® travel before hitting the floor of the crater? (Remember that the acceleration due to gravity on the moon is only 1/6 of that on earth.)

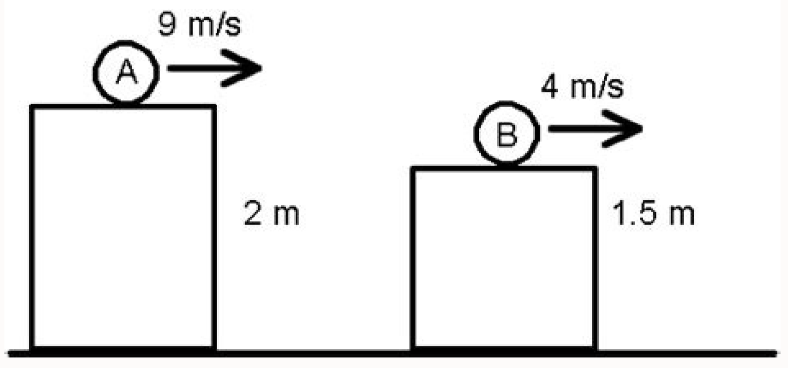
$V_H = 5.00$ m/s, $d_V = 100$ m; $t = \sqrt{\frac{2d_V}{g}} = \sqrt{\frac{2(100\,m)}{9.8\,m/s^2}} = 4.5$ s; $d_H = V_H t = (5.00\,m/s)(4.5\,s) = 22.6$ m; $d_H = 23$ m

size 12{y= { {1} over {2} } \(v rSub { size 8{0y} } +v rSub { size 8{y} } \) t} {} Note that the final vertical velocity, vyvy size 12{v rSub { size 8{y} } } {}, at the highest point is zero. Thus, t = 2 y (v 0y + v y) = 2 (233 m) (67.6 m/s) = 6.90 s. t = 2 y (v 0y + v y) = 2 (233 m) (67.6 m/s) = 6.90 s. alignl { stack { size 12{t= { {2y} over { \(v rSub { size 8{0y} } +v rSub { size 8{y} } \) } } = { {2 times "233"" m"} over { \("67" "." 6" m/s" \) } } } {} # =6 "." "90"" s" {} } } {} Discussion for (b)This time is also reasonable for large fireworks. When you are able to see the launch of fireworks, you will notice several seconds pass before the shell explodes. (Another way of finding the time is by using y=y0+v0yt−12gt2y=y0+v0yt−12gt2 size 12{y=y rSub { size 8{0} } +v rSub { size 8{0y} } t - { {1} over {2} } ital "gt" rSup { size 8{2} } } {}, and solving the quadratic equation for tt size 12{t} {}.)Solution for (c)Because air resistance is negligible, ax=0ax=0 size 12{a rSub { size 8{x} } =0} {} and the horizontal velocity is constant, as discussed above. The horizontal displacement is horizontal velocity multiplied by time as given by x=x0+vxtx=x0+vxt size 12{x=x rSub { size 8{0} } +v rSub { size 8{x} } t} {}, where x0x0 size 12{x rSub { size 8{0} } } {} is equal to zero:x=vxt,x=vxt, size 12{x=v rSub { size 8{x} } t ","} {} where vxvx size 12{v rSub { size 8{x} } } {} is the x-component of the velocity, which is given by vx=v0cosθ0.vx=v0cosθ0. size 12{v rSub { size 8{x} } =v rSub { size 8{0} } "cos"θ rSub { size 8{0} } "." } {} Now,vx=v0cosθ0=(70.0 m/s)(cos 75.0°)=18.1 m/s.vx=v0cosθ0=(70.0 m/s)(cos 75.0°)=18.1 m/s. size 12{v rSub { size 8{x} } =v rSub { size 8{0} } "cos"θ rSub { size 12{0} } = \("70" "." 0" m/s" \) \("cos""75.0°" \) ="18" "." 1" m/s."} {} The time tt size 12{t} {} for both motions is the same, and so xx size 12{t} {} isx=(18.1 m/s)(6.90 s)=125 m.x=(18.1 m/s)(6.90 s)=125 m. size 12{x= \("18" "." 1" m/s" \) \(6 "." "90"" s" \) ="125"" m."} {} Discussion for (c)The horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. Once the shell explodes, air resistance has a major effect, and many fragments will land directly below. In solving part (a) of the preceding example, the expression we found for yy size 12{y} {} is valid for any projectile motion where air resistance is negligible. Call the maximum height y=hy=h size 12{y=h} {}; then, h = v 0 y 2 2 g . h = v 0 y 2 2 g . size 12{y= { {v rSub { size 8{0y} } rSup { size 8{2} } } over {2g} } "." } {} 3.56 This equation defines the maximum height of a projectile and depends only on the vertical component of the initial velocity.

Projectile Motion Worksheet

1. A ball rolls off a desk at a speed of 3.0 m/s and lands 0.40 seconds later.
   a) How far from the base of the desk does the ball land?
   b) How high is the desk?
   c) What is the speed and angle of impact?
2. A slingshot is used to launch a stone horizontally from the top of a 20.0 meter cliff. The stone lands 36.0 meters away.
   a) At what speed was the stone launched?
   b) What is the speed and angle of impact?
3. A canon ball fired horizontally from a cliff has a velocity directed at 60° below horizontal when it hits the ground 3.0 seconds later.
   a) How high is the cliff?
   b) How far from the base of the cliff does the canon ball land?
4. A ball rolls with a speed of 2.0 m/s across a level table that is 1.0 m above the floor. Upon reaching the edge of the table, it follows a parabolic path to the floor. How far along the floor is the landing spot from the table? [0.90 m]
5. A rescue pilot drops a survival kit while her plane is flying at an altitude of 2000.0 m with a forward velocity of 100.0 m/s. If air friction is disregarded, how far in advance of the starving explorer's drop zone should she release the package? [2020 m]
6. A rifle is fired horizontally and travels 200.0 m [E]. The rifle barrel is 1.90 m from the ground. What speed must the bullet have been travelling at? Ignore friction. [321 m/s]
7. A skier leaves the horizontal end of a ramp with a velocity of 25.0 m/s [E] and lands 70.0 m from the base of the ramp. How high is the end of the ramp from the ground? [38.5 m]
8. An astronaut stands on the edge of a lunar crater and throws a half-eaten Twinkie™ horizontally with a velocity of 5.00 m/s. The floor of the crater is 100.0 m below the astronaut. What horizontal distance will the Twinkie™ travel before hitting the floor of the crater? (The acceleration of gravity on the moon is 1/6th that of the Earth). [55.3 m]
9. A baseball player leads off the game and hits a long home run. The ball leaves the bat at an angle of 30.0° from the horizontal with a velocity of 40.0 m/s. How far will it travel in the air? [141 m]

It is important to set up a coordinate system when analyzing projectile motion. One part of defining the coordinate system is to define an origin for the xx size 12{x} {} and yy size 12{y} {} positions. Often, it is convenient to choose the initial position of the object as the origin such that x0=0x0=0 size 12{x rSub { size 8{0} } =0} {} and y0=0y0=0 size 12{y rSub { size 8{0} } =0} {}. It is also important to define the positive and negative directions in the xx size 12{x} {} and yy size 12{y} {} directions.



Typically, we define the positive vertical direction as upwards, and the positive horizontal direction is usually the direction of the object's motion.

When this is the case, the vertical acceleration, gg size 12{g} {}, takes a negative value (since it is directed downwards towards the Earth). However, it is occasionally useful to define the coordinates differently. For example, if you are analyzing the motion of a ball thrown downwards from the top of a cliff, it may make sense to define the positive direction downwards since the motion of the ball is solely in the downwards direction. If this is the case, gg size 12{g} {} takes a positive value. Kilauea in Hawaii is the world's most continuously active volcano. Very active volcanoes characteristically eject red-hot rocks and lava rather than smoke and ash. Suppose a large rock is ejected from the volcano with a speed of 25.0 m/s and at an angle 35.0°35.0° size 12{"35"°} {} above the horizontal, as shown in Figure 3.39. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point.

(a) Calculate the time it takes the rock to follow this path. (b) What are the magnitude and direction of the rock's velocity at impact? Figure 3.39 The trajectory of a rock ejected from the Kilauea volcano. StrategyAgain, resolving this two-dimensional motion into two independent one-dimensional motions will allow us to solve for the desired quantities.

The time a projectile is in the air is governed by its vertical motion alone. We will solve for tt size 12{t} {} first.

Math-Gr12Advanced-UAE Mr. Abdulkader Amro

4- A diver drops from 120 ft above the water (about the height of divers at the Acapulco Cliff Diving competition). What is the diver's velocity at impact?

A- $y=-32t^2+120\,,y'\left(\sqrt{\frac{15}{4}}\right)\approx -87.64ft/s$

B- $y=-16t^2+120\,,y'\left(\sqrt{\frac{15}{2}}\right)\approx -87.64ft/s$

C- $y=-4.9t^2+120\,,y'(\sqrt{24.48})\approx 87.64ft/s$

D- $y=-9.8t^2+120\,,y'(\sqrt{13.48})\approx 87.64ft/s$

5- The Washington Monument is 555 ft and, $5\frac{1}{8}$in high. In a famous experiment, a baseball was dropped from the top of the monument to see if a player could catch it. How fast would the ball be going?

A- $y=-16t^2+555.16\,,y'(5.86)\approx -93.75ft/s$. (If you assume the ball is caught at ground level, the ball will be going 94.27 ft/sec.)

B- $y=16t^2-555.427\,,y'(5.86)\approx -93.75ft/s$. (If you assume the ball is caught at ground level, the ball will be going 94.27 ft/sec.)

C- $y=16t^2-555.16\,,y'(5.86)\approx -93.75ft/s$. (If you assume the ball is caught at ground level, the ball will be going 94.27 ft/sec.)

D- $y=-16t^2+555.427\,,y'(5.86)\approx -93.75ft/s$. (If you assume the ball is caught at ground level, the ball will be going 94.27 ft/sec.)

LIVEWORKSHEETS

While the rock is rising and falling vertically, the horizontal motion continues at a constant velocity. This example asks for the final velocity. Thus, the vertical and horizontal results will be recombined to obtain vv size 12{v} {} and θvθv size 12{θ rSub { size 8{v} } } {} at the final time tt size 12{t} {} determined in the first part of the example.Solution for (a)While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by usingy=y0+v0yt−12gt2 .y=y0+v0yt−12gt2 . size 12{y=y rSub { size 8{0} } +v rSub { size 8{0y} } t - { {1} over {2} } ital "gt" rSup { size 8{2} } "."} {} If we take the initial position y0y0 size 12{y rSub { size 8{0} } } {} to be zero, then the final position is y=−20.0 m.y=−20.0 m. size 12{y= - "20" "." 0" m" "." } {} Now the initial vertical velocity is the vertical component of the initial velocity, found from v0y=v0sinθ0v0y=v0sinθ0 size 12{v rSub { size 8{0y} } =v rSub { size 8{0} } "sin"θ rSub { size 8{0} } } {} = (25.0 m/s25.0 m/s size 12{"25" "." "0 m/s"} {})(sin 35.0°sin 35.0° size 12{"sin 35"°} {}) = 14.3 m/s14.3 m/s size 12{"14" "." "3 m/s"} {}. Substituting known values yields−20.0 m=(14.3 m/s)t−4.90 m/s2t2 .−20.0 m=(14.3 m/s)t−4.90 m/s2t2 . size 12{ - "20" "." 0" m"= \("14" "." 3" m/s" \) t - left (4 "." "90"" m/s" rSup { size 8{2} } right)t rSup { size 8{2} } "."} {} Rearranging terms gives a quadratic equation in tt size 12{t} {}: 4.90 m/s2t2−14.3 m/st−20.0 m=0.4.90 m/s2t2−14.3 m/st−20.0 m=0.

size 12{ left (4 "." "90"" m/s" rSup { size 8{2} } right)t rSup { size 8{2} } - left ("14" "." "3 m/s" right)t - left ("20" "." 0" m" right)=0.} {} This expression is a quadratic equation of the form at2 + bt + c = 0 at2 + bt + c = 0 size 12{ ital "at" rSup { size 8{2} } + ital "bt"+c=0} {} , where the constants are a=4.90 a=4.90 , b=–14.3 b=–14.3 , and c=–20.0. c=–20.0. Its solutions are given by the quadratic formula: t = − b ± b 2 − 4 ac 2a .

t = − b ± b 2 − 4 ac 2a . size 12{t= { { - b +- sqrt {b rSup { size 8{2} } - 4 ital "ac"} } over {"2a"} } "." } {} This equation yields two solutions: t=3.96t=3.96 size 12{t=3 "." "96"} {} and t=–1.03t=–1.03 size 12{t=3 "." "96"} {}.

(It is left as an exercise for the reader to verify these solutions.) The time is t=3.96st=3.96s size 12{t=3 "." "96""s"} {} or –1.03s–1.03s size 12{-1 "." "03""s"} {}. The negative value of time implies an event before the start of motion, and so we discard it.

Thus,t=3.96 s.t=3.96 s. size 12{t=3 "." "96"" s."} {} Discussion for (a)The time for projectile motion is completely determined by the vertical motion. So any projectile that has an initial vertical velocity of 14.3 m/s and lands 20.0 m below its starting altitude will spend 3.96 s in the air.Solution for (b)From the information now in hand, we can find the final horizontal and vertical velocities vxvx size 12{v rSub { size 8{x} } } {} and vyvy size 12{v rSub { size 8{y} } } {} and combine them to find the total velocity vv size 12{v} {} and the angle θ0θ0 size 12{θ rSub { size 8{0} } } {} it makes with the horizontal. Of course, vxvx size 12{v rSub { size 8{x} } } {} is constant so we can solve for it at any horizontal location. In this case, we chose the starting point since we know both the initial velocity and initial angle. Therefore:vx=v0cosθ0=(25.0 m/s)(cos 35°)=20.5 m/s.vx=v0cosθ0=(25.0 m/s)(cos 35°)=20.5 m/s. size 12{v rSub { size 8{x} } =v rSub { size 8{0} } "cos"θ rSub { size 8{0} } = \("25" "." 0" m/s" \) \("cos""35" rSup { size 8{ circ } } \) ="20" "." 5" m/s."} {} The final vertical velocity is given by the following equation: vy=v0y−gt,vy=v0y−gt, size 12{v rSub { size 8{y} } =v rSub { size 8{0y} } - ital "gt,"} {} where v0yv0y size 12{v rSub { size 8{0y} } } {} was found in part (a) to be 14.3 m/s14.3 m/s size 12{"14" "." "3 m/s"} {}. Thus, v y = 14 . 3 m/s − (9 . 80 m/s 2) (3 . 96 s) v y = 14 . 3 m/s − (9 . 80 m/s 2) (3 . 96 s) size 12{v rSub { size 8{y} } ="14" "." 3" m/s" - \(9 "." "80"" m/s" rSup { size 8{2} } \) \(3 "." "96"" s" \) } {} so that vy=−24.5 m/s.vy=−24.5 m/s. size 12{v rSub { size 8{y} } = - "24" "." 5" m/s."} {} To find the magnitude of the final velocity vv size 12{v} {} we combine its perpendicular components, using the following equation: v=vx2+vy2=(20.5 m/s)2+(−24.5 m/s)2,v=vx2+vy2=(20.5 m/s)2+(−24.5 m/s)2, size 12{v= sqrt {v rSub { size 8{x} } rSup { size 8{2} } +v rSub { size 8{y} } rSup { size 8{2} } } = sqrt { \("20" "." 5" m/s" \) rSup { size 8{2} } + \(- "24" "." 5" m/s" \) rSup { size 8{2} } } ","} {} which gives v=31.9 m/s.v=31.9 m/s. size 12{v="31" "." 9" m/s."} {} The direction θvθv size 12{θ rSub { size 8{v} } } {} is found from the equation: θ v = tan − 1 (v y / v x) θ v = tan − 1 (v y / v x) size 12{θ rSub { size 8{v} } ="tan" rSup { size 8{ - 1} } \(v rSub { size 8{y} } /v rSub { size 8{x} } \) } {} so that θv=tan−1(−24.5/20.5)=tan−1(−1.19).θv=tan−1(−24.5/20.5)=tan−1(−1.19). size 12{θ rSub { size 8{v} } ="tan" rSup { size 8{ - 1} } \(- "24" "." 5/"20" "." 5 \) ="tan" rSup { size 8{ - 1} } \(- 1 "." "19" \) "."} {} Thus, θv=−50.1°.θv=−50.1°. size 12{θ rSub { size 8{v} } = - "50" "." 1 rSup { size 12{ circ } "."} } {} Discussion for (b)The negative angle means that the velocity is 50.1°50.1° size 12{"50" "." 1°} {} below the horizontal. This result is consistent with the fact that the final vertical velocity is negative and hence downward—as you would expect because the final altitude is 20.0 m lower than the initial altitude. (See Figure 3.39.) One of the most important things illustrated by projectile motion is that vertical and horizontal motions are independent of each other. Galileo was the first person to fully comprehend this characteristic. He used it to predict the range of a projectile. On level ground, we define range to be the horizontal distance RR size 12{R} {} traveled by a projectile. Galileo and many others were interested in the range of projectiles primarily for military purposes—such as aiming cannons. However, investigating the range of projectiles can shed light on other interesting phenomena, such as the orbits of satellites around the Earth. Let us consider projectile range further. Figure 3.40 Trajectories of projectiles on level ground.

(a) The greater the initial speed v0v0 size 12{v rSub { size 8{0} } } {}, the greater the range for a given initial angle. (b) The effect of initial angle θ0θ0 size 12{θ rSub { size 8{0} } } {} on the range of a projectile with a given initial speed. Note that the range is the same for 15°15° size 12{"15"°} {} and 75°75° size 12{"75"°} {}, although the maximum heights of those paths are different. How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed v0v0 size 12{v rSub { size 8{0} } } {}, the greater the range, as shown in Figure 3.40(a). The initial angle θ0θ0 size 12{θ rSub { size 8{0} } } {} also has a dramatic effect on the range, as illustrated in Figure 3.40(b). For a fixed initial speed, such as might be produced by a cannon, the maximum range is obtained with θ0=45°θ0=45° size 12{θ rSub { size 8{0} } = "45°"} {}.

This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is approximately 38°38° size 12{"38°"} {}. Interestingly, for every initial angle except 45°45° size 12{"45°"} {}, there are two angles that give the same range—the sum of those angles is 90°90° size 12{"90°"} {}. The range also depends on the value of the acceleration of gravity gg size 12{g} {}. The lunar astronaut Alan Shepherd was able to drive a golf ball a great distance on the Moon because gravity is weaker there. The range RR size 12{R} {} of a projectile on level ground for which air resistance is negligible is given by R=v02sin2θ0g,R=v02sin2θ0g, size 12{R= { {v rSub { size 8{0} } rSup { size 8{2} } "sin"2θ rSub { size 8{0} } } over {g} } ","} {} 3.71 where v0v0 size 12{v rSub { size 8{0} } } {} is the initial speed and θ0θ0 size 12{θ rSub { size 8{0} } } {} is the initial angle relative to the horizontal. The proof of this equation is left as an end-of-chapter problem (hints are given), but it does fit the major features of projectile range as described. When we speak of the range of a projectile on level ground, we assume that RR size 12{R} {} is very small compared with the circumference of the Earth. If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path. The range is larger than predicted by the range equation given above because the projectile has farther to fall than it would on level ground. (See Figure 3.41.) If the initial speed is great enough, the projectile goes into orbit.

This possibility was recognized centuries before it could be accomplished. When an object is in orbit, the Earth curves away from underneath the object at the same rate as it falls. The object thus falls continuously but never hits the surface. These and other aspects of orbital motion, such as the rotation of the Earth, will be covered analytically and in greater depth later in this text.Once again we see that thinking about one topic, such as the range of a projectile, can lead us to others, such as the Earth orbits. In Addition of Velocities, we will examine the addition of velocities, which is another important aspect of two-dimensional kinematics and will also yield insights beyond the immediate topic.

Figure 3.41 Hypothetical projectile to satellite. From this theoretical tower, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved. Blast a Buick out of a cannon! Learn about projectile motion by firing various objects. Set the angle, initial speed, and mass. Add air resistance. Make a game out of this simulation by trying to hit a target. EN FR DE ES IT HR SV SR SL NL We and our partners use cookies to Store and/or access information on a device. We and our partners use data for Personalised ads and content, ad and content measurement, audience insights and product development. An example of data being processed may be a unique identifier stored in a cookie. Some of our partners may process your data as a part of their legitimate business interest without asking for consent. To view the purposes they believe they have legitimate interest for, or to object to this data processing use the vendor list link below. The consent submitted will only be used for data processing originating from this website. If you would like to change your settings or withdraw consent at any time, the link to do so is in our privacy policy accessible from our home page.. Continue with Recommended Cookies Loading... Projectile motion problems with descriptive solutions are provided for high school and college students. Every single question is designed to be a complete guide on this topic without referring to your textbook.

There is also a pdf helpful for worksheets with answers.

Overview of projectile motion: Any motion having the following conditions is called a projectile motion.   (i) Follows a parabolic path (trajectory)   (ii) Moves under the influence of gravity. The projectile motion formulas applied to solve two-dimensional projectile motion problems are as follows \begin{gather*} x=(v_0\cos\theta)t+x_0\\\\y=-\frac 12 gt^2+(v_0\sin\theta)t+y_0\\\\ v_y=v_0\sin\theta-gt\\\\v_y^2-(v_0\sin\theta)^2=-2g(y-y_0)\end{gather*} In the next section, we want to know how to solve projectile motion problems in the AP physics exams using these kinematic equations.   Projectile motion problems and answers Problem (1): A person kicks a ball with an initial velocity of $15\,{\rm m/s}$ at an angle of 37° above the horizontal (neglecting the air resistance).

Find (a) the total time the ball is in the air. (b) the horizontal distance traveled by the ball Solution: To solve any projectile motion problems, first of all, adopt a coordinate system and draw the projectile path, putting the initial and final positions, and velocities on it. By doing so, you will be able to solve the relevant projectile equations easily. Hence, we choose the origin of the coordinate system to be at the throwing point, $x_0=0, y_0=0$. (a) Here, the time between throwing and striking the ground is wanted.   In effect, the projectiles have two independent motions: one in the horizontal direction with uniform motion at a constant velocity, i.e., $a_x=0$, and the other in the vertical direction under the effect of gravity, with $a_y=-g$. The kinematic equations that describe the horizontal and vertical distances are as follows \begin{gather*} x=x_0+(\underbrace{v_0\cos \theta}_{v_{0x}})t \\ y=-\frac 12 gt^2+(\underbrace{v_0\sin \theta}_{v_{0y}})t+y_0\end{gather*} By substituting the coordinates of the initial and final points into the vertical equation, we can find the total time the ball is in the air. Setting $y=0$ in the second equation (because the projectile lands at the same level as the throwing point.), we have \begin{align*} y&=-\frac 12 gt^2+(v_0\sin \theta)t+y_0\\\\ 0&=-\frac 12 (9.8)t^2+(15)\sin 37^\circ\,t+0 \end{align*} By rearranging the above expression, we can get two solutions for t: \begin{gather*} t_1=0 \\\\ t_2=\frac{2\times 15\sin37^\circ}{9.8}=1.84\,{\rm s}\end{gather*} The first time is for the starting moment, and the second is the total time the ball was in the air. (b) As mentioned above, the projectile motion is made up of two independent motions with different positions, velocities, and accelerations, and two distinct kinematic equations describe those motions. Between any two desired points in the projectile path, the time needed to move horizontally to reach a specific point is the same time needed to fall vertically to that point. This is an important observation in solving projectile motion problems. Therefore, time is the only common quantity in the horizontal and vertical motions of a projectile. In this problem, the time obtained in part (a) can be substituted in the horizontal kinematic equation, to find the distance traveled as below \begin{align*} x&=x_0+(v_0\cos \theta)t \\ &=0+(15)\cos 37^\circ\,(1.84) \\ &=22.08\quad {\rm m}\end{align*} Be sure to check the following questions. If you are getting ready for the AP Physics exam, probably these problems on the kinematics equation (or these AP Physics Kinematics Problems) are also useful to you. Problem (2): A ball is thrown into the air at an angle of 60° above the horizontal with an initial velocity of $40\,{\rm m/s}$ from a 50-m-high building.

Find (a) The time to reach the top of its path. (b) The maximum height the ball reached from the base of the building. Solution: In the previous question, we found that the projectile is a motion composed of two vertical and horizontal motions. We learned about how to find distances in both directions using relevant kinematic equations. There is also another set of kinematic equations that discuss the velocities in vertical and horizontal directions as follows \begin{gather*} v_x=v_{0x}=v_0\cos\theta \\ v_y=v_{0y}-gt=v_0\sin\theta-gt\end{gather*} As you can see, the horizontal component of the velocity, v_x, is constant throughout the motion, but the vertical component varies with time. As an important note, keep in mind that in the problems about projectile motions, at the highest point of the trajectory, the vertical component of the velocity is always zero, i.e., $v_y=0$. To solve the first part of the problem, specify two points as the initial and final points, then solve the relevant kinematic equations between those points. Here, setting $v_y=0$ in the second equation and solving for the unknown time t, we have \begin{align*} v_y&=v_{0y}-gt \\\\ &=v_0\sin\theta-gt \\\\ 0&=(40)(\sin 60^\circ)-(9.8)(t) \\\\ \Rightarrow t&=\boxed{3.53\, {\rm s}}\end{align*} Thus, the time taken to reach the maximum height of the trajectory (path) is 3.53 seconds from the moment of the launch. We call this maximum time t_{max}. (b) Let the origin be at the throwing point so $x_0=y_0=0$ in the kinematic equations. In this part, the vertical distance traveled to the maximum point is requested. By substituting t_{max} into the vertical distance projectile equation, we can find the maximum height as below \begin{align*} y-y_0&=-\frac 12 gt^2 +(v_0 \sin \theta)t\\ \\ y_{max}&=-\frac 12 (9.8)(3.53)^2+(40 \sin 60^\circ)(3.53)\\\\ &=61.22\quad {\rm m}\end{align*} Therefore, the maximum height the ball is reached from the base of the building is \[H=50+y_{max}=111.22\quad {\rm m}\] For further reading about uniform motion along the horizontal direction, see speed, velocity, and acceleration problems. Problem (3): A person standing on the edge of a $50-\rm m$-high cliff throws a stone horizontally with a speed of $18\,\rm m/s$. (a) What is the initial position of the stone? (b) What are the components of the initial velocity? (c) What are the x- and y-components of the velocity of the stone at any arbitrary time t? (d) How long will it take the stone to strike the bottom of the cliff? (e) With what angle and speed does the stone strike the ground below the cliff? Solution: As mentioned repeatedly, as a first step to solving a projectile motion problem, choose a relevant coordinate system. (a) Usually, we place the origin of the coordinate system at the point where the projectile is thrown. In this case, the coordinate of the initial position is $x_0=0\, , \, y_0=0$. If we had chosen the coordinate at the base of the cliff and placed the origin at that point, the position of the initial point would have been $x_0=0\, , \, y_0=50\,\rm m$. (b) The stone is thrown horizontally, $\theta=0$, so there is no vertical velocity component. Consequently, their initial speed components are $v_{0x}=18\,\rm m/s$ and $v_{0y}=0$. (c) Recall that the components of the velocity of a projectile vary with time according to the following formula. Substitute the given values into it, gives \begin{align*} v_x&=v_{0x}=18\,\rm m/s \\\\ v_y&=v_{0y}-gt\\&=-9.8t \end{align*} (d) If we take the top of the cliff as the origin of our coordinate system, $x_0=y_0=0$, the stone strikes the ground $50\,\rm m$ below our chosen origin, so the coordinates of that point would be $x_0=?\, ,\, y=-50\,\rm m$. Use the vertical displacement kinematic equation below, substitute the numerical values into that, and solve for time t get \begin{gather*} y-y_0=-\frac 12 gt^2+v_{0y}t \\\\ -50-0=-\frac 12 (9.8)t^2-0 \\\\ \Rightarrow \quad \boxed{t=3.2\,\rm s} \end{gather*} (e) First, find the velocity components just before the stone hits the ground. We know that it takes about $3.2\,\rm s$ for the stone to reach the bottom of the cliff. Thus, put this time value into the formulas of velocity components at any time t. \begin{align*} v_x&=v_{0x}=18\,\rm m/s \\\\ v_y&=v_{0y}-gt\\&=-9.8\times 3.2 \\&=-31.36\,\rm m/s \end{align*} Notice that the negative sign here indicates that the vertical velocity component just before hits the ground points downward, as expected. Recall from the section of vector practice problems that have the components of a vector, its magnitude is simply found using the Pythagorean theorem as follows \begin{align*} v&=\sqrt{v_x^2+v_y^2} \\\\ &=\sqrt{18^2+(-31.36)^2} \\ &=\boxed{36.15\,\rm m/s} \end{align*} The angle of impact with the ground, having the velocity components, is also obtained as below \begin{align*} \tan\theta&=\frac{v_y}{v_x} \\\\ &=\frac{-31.36}{18} \\\\ &=-1.74 \end{align*} Taking the inverse tangent of both sides, gives \[\theta=\tan^{-1}(-1.74)=\boxed{-60.1^\circ} \] Therefore, the stone hit the ground at an angle of about 60° with a speed of $36.15\,\rm m/s$. The negative indicates that the angle is below horizontal, which is to be expected. Download Now and get 50% off!: 550 solved high school physics midterm and final exams ($4).

Problem (4): A box slides off a frictionless tabletop with a constant speed of $1.1\,\rm m/s$, and $0.35\,\rm s$ later strikes the floor. (a) How high is the tabletop from the floor? (b) What is the horizontal distance to that point where the book strikes the floor? (c) How fast and at what angle does the book strike the floor? Solution: The total time in the air, $t=0.35\,\rm s$, and the initial speed with which the book leaves the table horizontally, i.e., $v_{0x}=1.1\,\rm m/s$, and $v_{0y}=0$. Contrary to the earlier question, in this case, for practice, we place the origin at the bottom of the tabletop. Thus, the coordinates of the initial position would be $x_0=0\, ,\, y_0=h=?$ and the coordinate of the landing point would be $x=?
\, , \, y=0$. (a) If we substitute the given values into the vertical displacement kinematic equation, we will have \begin{gather*} y-y_0=-\frac 12 gt^2+v_{0y}t \\\\ 0-h=-\frac 12 (9.8)(1.1)^2+0 \\\\ \Rightarrow \quad \boxed{h=5.93\,\rm m} \end{gather*} Thus, the table is 3-m-high. (b) Recall that projectile motion consists of two uniform horizontal and vertical accelerated motions. The time taken by the projectile in the vertical direction is equal to the horizontal time to the landing point. Between the start and final points in the horizontal direction of the projectile path, we can write $\Delta x=v_{0x}t$. Substituting the given values into is get \begin{gather*} x-0=1.1\times (0.35) \\ \Rightarrow \quad \boxed{x=0.385\,\rm m} \end{gather*} where $\Delta x=x-x_0$. Therefore, the book strikes the floor about $39\,\rm cm$ ahead of the base of the table. (c) This part is straightforward. Substitute the given numerical values into the velocity vector components formulas as below to find the velocity components just before the book strikes the floor. \begin{align*} v_x&=v_{0x}=1.1\,\rm m/s \\\\ v_y&=v_{0y}-gt \\\\ &=0-(9.8)(0.35) \\&=-3.43\,\rm m/s \end{align*} The square root of the sum of the velocity components squared gives the velocity of the book at the instant of hitting \begin{align*} v&=\sqrt{v_x^2+v_y^2} \\\\ &=\sqrt{1.1^2+(-3.43)^2} \\\\ &=\boxed{3.6\,\rm m/s} \end{align*} and angle of impact with the ground is found as below \begin{align*} \theta&=\tan^{-1}\left(\frac{v_y}{v_x}\right) \\\\ &=\tan^{-1}\left(\frac{-3.43}{1.1}\right) \\\\ &=\tan^{-1}(-3.11) \\\\&=\boxed{-72.17^\circ} \end{align*} Overall, the book hit the ground at an angle of about 72° with a speed of nearly $3.6\,\rm m/s$.

Problem (5): A cannonball is fired from a cliff with a speed of 800 m/s at an angle of 30° below the horizontal. How long will it take to reach 150 m below the firing point? Solution: First, choose the origin to be at the firing point, so $x_0=y_0=0$. Now, list the known values below   (i) Projectile's initial velocity = $800\,{\rm m/s}$ (ii) Angle of projectile: -30°, the negative sign is because the throw is below horizontal.   (iii) y-coordinate of the final point, 150 m below the origin, $y=-150\,{\rm m}$.   In this problem, the time it takes for the cannonball to reach 100 m below the starting point is required. Since the displacement to that point is known, we apply the vertical displacement projectile formula to find the needed time as below \begin{align*} y-y_0&=-\frac 12 gt^2+(v_0\sin\theta)t\\\\-150&=-\frac 12 (9.8)t^2+(800)\sin(-30^\circ)t\\\\ \Rightarrow & \ 4.9t^2+400t-150=0\end{align*} The above quadratic equation has two solutions, $t_1=0.37\,{\rm s}$ and $t_2=-82\,{\rm s}$. It is obvious that the second time is not acceptable. Therefore, the cannonball takes 0.37 seconds to reach 150 meters below the firing point. Problem (6): Someone throws a stone into the air from ground level. The horizontal component of velocity is 25 m/s and it takes 3 s for the stone to come back to the same height as before. Find (a) The range of the stone. (b) The initial vertical component of velocity (c) The angle of projection. Solution: The known value is   (i) The initial horizontal component of velocity, $v_{0x}=25\,{\rm m/s}$.   (ii) The time between the initial and final points, which are at the same level, $t=3\,{\rm s}$. (a) The range of projectile motion is defined as the horizontal distance between the launch point and impact at the same elevation. Because the horizontal motion in projectiles is a motion with constant velocity, the distance traveled in this direction is obtained as $x=v_{0x}t$, where v_{0x} is the initial component of the velocity. If you put the total time the projectile is in the air into this formula, you get the range of the projectile. In this problem, the stone is thrown from the ground level and after 3 s reaches the same height. Thus, this is the total time of the projectile. Hence, the range of the stone is found as below \begin{align*} x&=v_{0x}t\\&=(25)(3)\\&=75\,{\rm m}\end{align*} (b) The initial vertical component of the projectile's velocity appears in two equations, $v_y=v_{0y}-gt$ and $y-y_0=-\frac 12 gt^2+v_{0y}t$. Using the second formula is more straightforward because the stone reaches the same height, so its vertical displacement between the initial and final points is zero, i.e., $y-y_0=0$. Setting this into the vertical distance projectile equation, we get \begin{align*} y-y_0&=-\frac 12 gt^2+v_{0y}t\\\\ 0&=-\frac 12 (9.8)(3)^2+v_{0y}(3) \\\\ \Rightarrow v_{0y}&=14.7\quad{\rm m/s}\end{align*} To use the first formula, we need some extra facts about projectile motion in the absence of air resistance, as below   (i) The vertical velocity is zero at the highest point of the path of the projectile, i.e., $v_y=0$.   (ii) If the projectile lands at the same elevation from which it was launched, then the time it takes to reach the highest point of the trajectory is half the total time between the initial and final points.   The second note, in the absence of air resistance, is only valid. In this problem, the total flight time is 3 s because air resistance is negligible, so 1.5 seconds are needed for the stone to reach the maximum height of its path. Therefore, using the second equation, we can find v_{0y} as below \begin{align*} v_y&=v_{0y}-gt\\0&=v_{0y}-(9.8)(1.5) \\\Rightarrow v_{0y}&=14.7\quad {\rm m/s}\end{align*} (c) The projection angle is the angle at which the projectile is thrown into the air and performs a two-dimensional motion. Once the components of the initial velocity are available, using trigonometry, we can find the angle of projection as below \begin{align*} \theta&=\tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right)\\\\&=\tan^{-1}\left(\frac{14.7}{25}\right)\\\\&=+30.45^\circ\end{align*} Therefore, the stone is thrown into the air at an angle of about 30° above the horizontal. Problem (7): A ball is thrown at an angle of 60° with an initial speed of 200 m/s. (Neglect the air resistance.) (a) How long is the ball in the air? (b) Find the maximum horizontal distance traveled by the ball. (c) What is the maximum height reached by the ball? Solution: We choose the origin to be the ball's initial position so that $x_0=y_0=0$. The given data is   (i) The projection angle: 60°.   (ii) Initial speed : $v_0=200\,{\rm m/s}$. (a) The initial and final points of the ball are at the same level, i.e., $y-y_0=0$. Thus, the total time the ball is in the air is found by setting $y=0$ in the projectile equation $y=-\frac 12 gt^2+v_{0y}t$ and solving for time t as below \begin{align*} y&=-\frac 12 gt^2+(v_0\sin\theta)t\\\\0&=-\frac 12 (9.8)t^2+(200)(\sin 60^\circ)t\\\\\Rightarrow \quad & \boxed{(-4.9t+100\sqrt{3})t=0} \end{align*} The above expression has two solutions for t. One is the initial time, $t_1=0$, and the other is computed as $t_2=35.4\,{\rm s}$. Hence, the ball has been in the air for about 35 s. (b) The horizontal distance is called the range of the projectile. By inserting the above time (total flight time) into the horizontal distance projectile equation $x=v_{0x}t$, we can find the desired distance traveled. \begin{align*} x&=(v_0\cos\theta)t\\&=(200)(\cos 60^\circ)(35.4)\\&=3540 \quad {\rm m}\end{align*} Therefore, the ball hits the ground 3540 meters away from the throwing point. (c) Using the projectile equation $v_y^2-v_{0y}^2=-2g(y-y_0)$, setting $v_y=0$ at the highest point of the path, and solving for the vertical distance y, the maximum height is found as follows \begin{align*} v_y^2-v_{0y}^2&=-2g(y-y_0)\\0-(200\sin 60^\circ)^2&=-2(9.8)y\\\Rightarrow y&=1531\quad {\rm m}\end{align*} Another method: As mentioned above, the ball hits the ground at the same level as before, so by knowing the total flight time and halving it, we can find the time it takes the ball to reach the highest point of its trajectory. Therefore, by setting the half of the total flight time in the following projectile kinematic formula and solving for y, we can find the maximum height as \begin{align*} y-y_0&=-\frac 12 gt^2+(v_0\sin\theta)t\\\\y&=-\frac 12 (9.8)(17.7)^2+(200\sin60^\circ)(17.7)\\\\&=1531\quad {\rm m}\end{align*} Hence, the ball reaches 1531 meters above the launch point. Problem (8): What are the horizontal range and maximum height of a bullet fired with a speed of 20 m/s at 30° above the horizontal?

Solution: first, find the total flight time, then insert it into the horizontal displacement projectile equation $x=v_{0x}t$ to find the range of the bullet. Because the bullet lands at the same level as the original, its vertical displacement is zero, $y-y_0=0$, in the following projectile formula we can find the total flight time \begin{align*} y-y_0&=-\frac 12 gt^2+(v_0\sin\theta)t\\\\0&=-\frac 12 (9.8)t^2+(20)(\sin30^\circ)t\\\\ \Rightarrow & (-4.9t+10)t=0\end{align*} Solving for time t gives two solutions, one is the initial time $t_1=0$, and the other is $t_2=1.02\,{\rm s}$. Thus the total time of flight is 2.04 s. Therefore, the maximum horizontal distance traveled by the bullet, which is defined as the range of the projectile, is calculated as \begin{align*} x&=(v_0\cos\theta)t\\&=(20\cos 30^\circ)(2.04)\\&=35.3\quad {\rm m}\end{align*} Hence, the bullet is landed about 17 m away from the launch point. Because the air resistance is negligible and the bullet lands at the same height as the original, the time it takes to reach the highest point of its path, is always half the total flight time. On the other hand, recall that the vertical component of velocity at the maximum height is always zero, i.e., $v_y=0$. By inserting these two notes into the following projectile equation, we have \begin{align*} y&=-\frac 12 gt^2+(v_0\sin\theta)t\\\\&=-\frac 12 (9.8)(1.02)^2+(20\sin 30^\circ)(1.02)\\\\&=5.1\quad {\rm m}\end{align*} We could also simply use the kinematic equation $v_y^2-v_{0y}^2=-2g(y-y_0)$, to find the maximum height as below \begin{align*} v_y^2-(v_0 \sin \theta)^2 &=-2g(y-y_0) \\ 0-(20\sin 30^\circ)^2 &=-2(9.8)H\\ \Rightarrow H&= 5.1\quad {\rm m} \end{align*} I think the second method is much simpler.

Problem (9): A projectile is fired horizontally at a speed of 8 m/s from an 80-m-high cliff. Find (a) The velocity just before the projectile hits the ground. (b) The angle of impact. Solution: In this problem, the angle of the projectile is zero because it is fired horizontally. The velocity at each point of a projectile trajectory (path) is obtained by the following formula: \[v=\sqrt{v_x^2+v_y^2}\] where v_x and v_y are the horizontal and vertical components of the projectile's velocity at any instant of time. (a) Recall that the horizontal component of the projectile's velocity is always constant and, for this problem, is found as \begin{align*} v_x&=v_0\cos\theta\\&=8\times \cos 0^\circ\\&=8\quad {\rm m/s}\end{align*} To find the vertical component of the projectile velocity at any moment, $v_y=v_0\sin\theta-gt$, we should find the time taken to that point. In this problem, that point is located just before striking the ground, whose coordinates are $y=-80\,{\rm m}, x=?$. Because it is below the origin, which is assumed to be at the firing point, we inserted a minus sign. Because displacement in the vertical direction is known, we can use the projectile formula for vertical distance. By setting $y=-80$ into it and solving for the time t needed the projectile reaches the ground, we get \begin{align*} y&=-\frac 12 gt^2+(v_0\sin\theta)t\\\\-80&=-\frac 12 (9.8)t^2+(8\times \sin 0^\circ)t\\\\\Rightarrow t&=\sqrt{\frac{2(80)}{9.8}}\\\\&=2.86\quad {\rm s}\end{align*} Now insert this time into the y-component of the projectile's velocity to find v_y just before hitting the ground \begin{align*} v_y&=v_0\sin\theta-gt\\ &=8\sin 0^\circ-(9.8)(2.86)\\&=-28\quad {\rm m/s}\end{align*} Now that both components of the velocity are available, we can compute its magnitude as below \begin{align*} v&=\sqrt{v_x^2+v_y^2}\\\\&=\sqrt{8^2+(-28)^2}\\\\&=29.1\quad {\rm m/s}\end{align*} Therefore, the projectile hits the ground at a speed of 29.1 m/s. (b) At any instant of time, the velocity of the projectile makes some angle with horizontal whose magnitude is obtained as the following formula \[\alpha=\tan^{-1}\left(\frac{v_y}{v_x}\right)\] Substituting the above values into this formula we get \[\alpha =\tan^{-1}\left(\frac{-28}{8}\right)=-74^\circ\] Therefore, the projectile hits the ground at an angle of 74° below the horizontal. To find the components of a vector in physics, refer to the following: Vectors problems with solutions or Vector problems in AP Physics 1. Problem (10): From a cliff of 100-m high, a ball is kicked at 30° above the horizontal with a speed of $20\,{\rm m/s}$. How far from the base of the cliff did the ball hit the ground? (Assume $g=10\,{\rm m/s^2}$).   Solution: Again, similar to any projectile motion problem, we first select a coordinate system and then draw the path of the projectile as shown in the figure below, We choose the origin to be at the kicking point above the cliff, setting $x_0=y_0=0$ in the kinematic equations. The coordinate of the hitting point to the ground is $y=-100\,{\rm m}, x = ?$. A negative is inserted because the final point is below the origin. Now, we find the common quantity in projectile motions- that is, the time between the initial and final points, called total flight time. To find the total time the ball was in the air, we can use the vertical equation and solve for the unknown t as follows \begin{align*} y&=-\frac 12 gt^2 +(v_0\sin \theta)t \\\\ -100&=-\frac 12 (10) t^2+(20\sin 30^\circ)t\\\\&\Rightarrow \quad \boxed{t^2-2t-20=0} \end{align*} The solutions of a quadratic equation $at^2+bt+c=0$ are found by the formula below \[t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\] By matching the above constant coefficients with the standard quadratic equation, we can find the total time as below \[t=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-20)}}{2(1)}\] After simplifying, two solutions are obtained, $t_1=5.6\,{\rm s}$ and $t_2=-3.6\,{\rm s}$. It is obvious that time cannot be negative in physics, so the acceptable answer is t_1. It is the time it takes the ball to travel in the vertical direction. On the other hand, it is also the time it takes the ball to travel the horizontal distance between kicking and hitting points. We insert this time into the horizontal equation to find the horizontal distance traveled, which is known as the range of the projectile. \begin{align*} x&=(v_0\cos \theta)t\\ &=(20)(\cos 30^\circ)(5.6)\\&=97\quad {\rm m}\end{align*} In the following, some projectile motion problems for the AP Physics 1 exam are presented. AP physics MCQ: A cannonball is fired from the top of a $200\,{\rm m}$ cliff with a speed of $60\,{\rm m/s}$ at an angle of 60° above the horizontal. Where does the ball land closest to which of the following choices? (Neglect air resistance.)} (a) 90m   (b) 402 m   (c) 151 m   (d) 200 m Solution: The horizontal distance from the point where the cannonball fired is wanted, $x=?$. The appropriate kinematic equation for this is \[x=(v_0\cos\theta)t\] The only thing we need is the total time the ball is in the air before landing. If we take the firing point as the origin, then the cannonball lands $200\,{\rm m}$ below the origin. So, the ball's vertical displacement is $\Delta y=-200\,{\rm m}$. Set this in the vertical displacement equation, $\Delta y=-\frac 12 gt^2+(v_0\sin\theta)t$, and solve for t. Keep reading the pdf version of this page. AP physics MCQ: Water comes out of the head of a hose at the speed of $6\,{\rm m/s}$.

At what angle(s) should we point the hose so that the water stream strikes the ground $2.5\,{\rm m}$ away? (a) 68° (b) 22° (c) $68^\circ\, ,\, 22^\circ$ (d) 32° Solution: The initial velocity and maximum horizontal distance are given, and we want the angle θ. The water hits the ground at the same elevation as the nozzle, so its vertical displacement is zero, $\Delta y=0$. Substitute this into the kinematic equation below and get the first required equation \begin{align*} \Delta y &=-\frac 12 gt^2 +(v_0 \sin\theta)t\\\\ 0&=-\frac 12 (10)t^2 +(6\sin\theta)t \\\\&=-5t^2 + 6t\sin\theta \\\\&=t(-5t+6\sin\theta) \end{align*} The above relation gives us the first equation \[\boxed{6\sin\theta=5t}\] Now, we use the horizontal displacement formula for projectile and set $x=2.5\,{\rm m}$.

Thus, \begin{gather*} x=(v_0\cos\theta)t\\\\ 2.5=(6\cos\theta)t \\\\ \Rightarrow \boxed{6t\cos\theta=2.5} \end{gather*} Dividing these two boxed equations, we have \begin{gather*} \frac{6\sin\theta}{6t\cos\theta}=\frac{5t}{2.5} \\\\ \boxed{\tan\theta=2t^2\, ,\quad (I)}\end{gather*} This is the only equation that we can find with the given information. Note that t in the all above equations is the total time the water has been in the air. Keep reading the pdf version of this article. Author: Dr. Ali Nemati Date Published: 6/15/2021 Updated: Sep 16, 2021 report this ad © 2015 All rights reserved. by Physexams.com Courses Exams Flash Cards