



BRANCHING OUT:

Designing High School
Math Pathways for Equity

By Phil Daro and Harold Asturias

JUST  EQUATIONS

ACKNOWLEDGMENTS

The authors are grateful to the individuals who helped make this report possible. Robert Q. Berry III, Amy Getz, Erica Heinzmann, Hayin Kimner, Mayra Lara, Vincent Stewart, and Andrea Venezia provided feedback on earlier drafts. In addition, thanks are due to Jenn BeVard for project management, Gretchen Kell for copy editing, and Christopher Artalejo-Price for graphic design. Pamela Burdman, whose clear voice and vision have inspired and guided improvements in mathematics policies and practices, conceived and nurtured this report.

ABOUT JUST EQUATIONS

Just Equations re-conceptualizes the role of mathematics in fostering education equity. Working across educational segments, Just Equations advances evidence-based strategies to ensure that math policies serve as a foundation, not a filter, for college and career success. Just Equations is a project of the Opportunity Institute, in partnership with LearningWorks, Policy Analysis for California Education (PACE), the Education Trust-West, and the Campaign for College Opportunity, with funding from The James Irvine Foundation and College Futures Foundation.

ABOUT THE AUTHORS

Phil Daro has directed large-scale projects related to mathematics learning, standards development, testing, and professional development efforts. Most recently, he was one of the principal authors of the Common Core State Standards in mathematics, and he continues to work on implementation and policy issues related to the Common Core. He also works with the Strategic Education Research Partnership, which leads mathematics and science learning partnerships between practitioners from the Oakland and San Francisco school districts and researchers from institutions including Stanford University and the University of California, Berkeley. Previously, he directed the California Mathematics Project for the University of California system. He is a recipient of the Walter Denham Award from the California Mathematics Council and the Ross Taylor/Glenn Gilbert National Leadership Award from the National Council of Supervisors of Mathematics.

Harold Asturias is director of the Center for Mathematics Excellence and Equity (CeMEE) at the Lawrence Hall of Science, UC Berkeley's public science center. He has experience providing professional development in the areas of standards and assessment in math for large urban districts and smaller rural districts. He has also designed and implemented professional development and instructional materials for K–12 math teachers who teach English language learners.

FOREWORD

Math class plays an outsize role in students' lives for at least two reasons: Ideally, math learning fosters the quantitative reasoning skills students need to use in high school and college, in their careers, and in civic life. But apart from the learning it affords, math also has the gatekeeping power to determine, often in arbitrary ways, whether students can access further opportunities. Tragically, this use of math to sort students can contribute to negative experiences and math anxiety, which detract from the very learning that all students need and deserve. This is particularly damaging to students who are already disadvantaged by the education system, such as low-income students, students of color, and those from families without a history of college-going.

As I discuss in my 2018 report, *The Mathematics of Opportunity*, the “architecture of math opportunity” is “undergirded by misconceptions about math ability, scaffolded by educational inequities and stereotypes, and reinforced by math’s use as a marker or pedigree to confer access to opportunities.” Changing that architecture requires working across educational systems to address four “equity dimensions” of math education: content, instruction, assessment, and the policies that determine students’ readiness for subsequent educational opportunities (including high school graduation and college admission).

The time is now ripe to do that work. Postsecondary institutions around the country have engaged in serious efforts to reform their math requirements, as highlighted in *Multiple Paths Forward*, a report Just Equations published last year with partners at WestEd, a San Francisco-based education research organization. Colleges are (1) offering new pathways in areas like statistics and quantitative reasoning for students with interests outside of the STEM disciplines served by traditional math pathways, and (2) de-emphasizing remedial math tests and courses in favor of corequisites and other just-in-time approaches.

These postsecondary reforms have paved the way for reconsidering the math required for college admission, as well as the math preparation high schools provide. Just Equations’ recently-published equity principles for redesigning math opportunity include ensuring that all students have access to rigorous, relevant, and diversified pathways that align with their aspirations.

In the context of these postsecondary pathways reforms, Just Equations commissioned Phil Daro and Harold Asturias to delve more deeply into what redesigned high school pathways might look like. Drawing on their experience in mathematics education, as well as on relevant research, Daro and Asturias analyze the weaknesses in existing math sequences and propose that high schools “branch out” and supplement existing STEM-oriented math pathways with new, rigorous offerings for students with other interests.

Their discussion builds on some ideas in the 2018 report by the National Council of Teachers of Mathematics (NCTM), *Catalyzing Change in High School Mathematics*, which pointed to the need to end tracking in math class, ensure equitable and evidence-based instruction, and design new pathways. Specifically, *Catalyzing Change* called for four-year math pathways with students in a common pathway for the first two to three years of high school, followed by a year or two in pathways tailored to students’ “needs, goals, interests, and aspirations.” Branching Out also complements and informs the work of the Charles A. Dana Center’s Launch Years Initiative consensus panels, which are developing a “new paradigm” for college and career readiness in math to meet the needs of the workforce and to prioritize the success of historically underserved students.

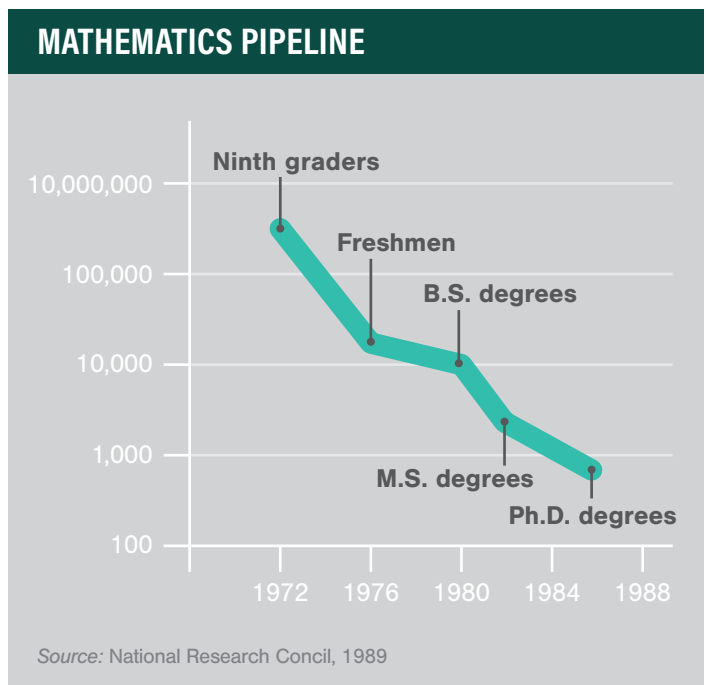
Pamela Burdman
Director, Just Equations



Mathematics course requirements in high school and college are a barrier to opportunity for large numbers of students. These barriers are not relevant to many of the opportunities they prevent students from pursuing. Each year, for example, requirements for algebra remediation deny hundreds of thousands of students the chance to progress in college, regardless of whether algebra is used in their intended fields of study. Since the burden falls heaviest on historically disadvantaged students, efforts to improve equity in mathematics outcomes are not likely to succeed without focusing on this mismatch.

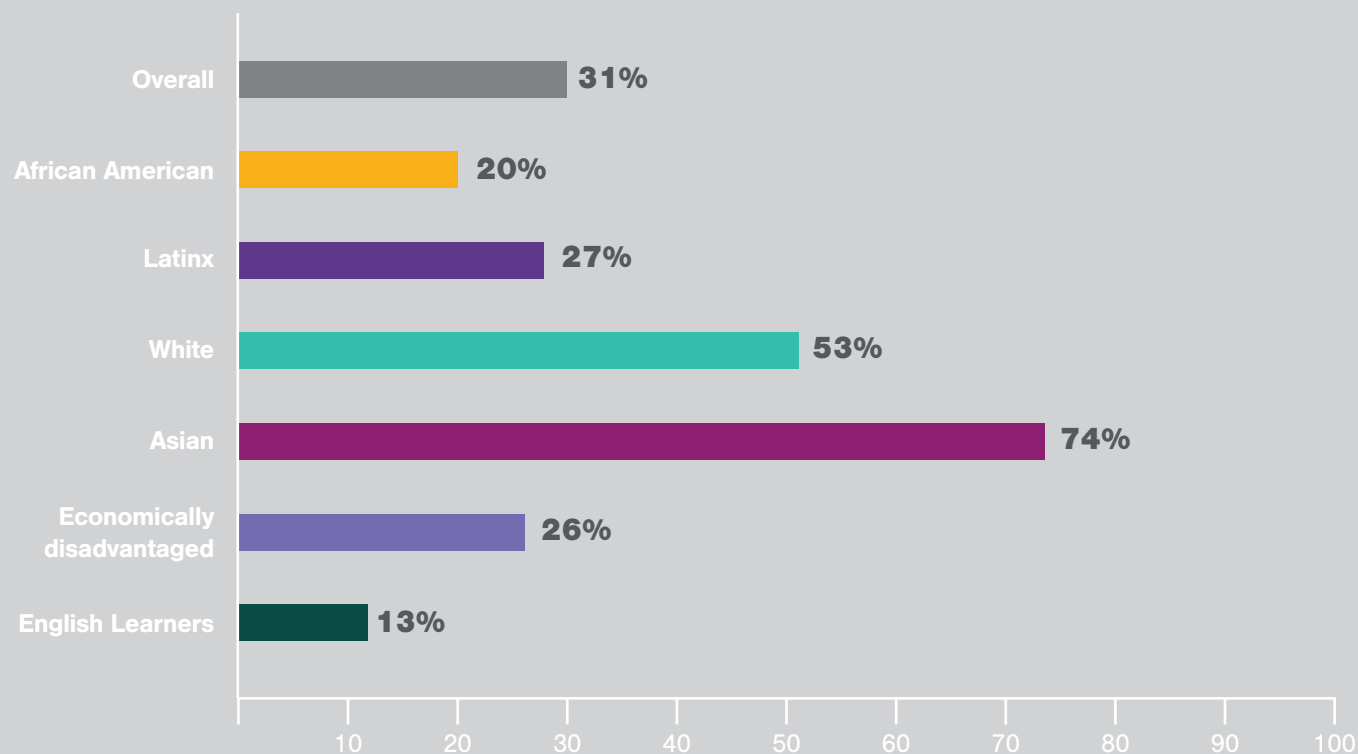
Despite decades of attention to the fate of students attempting to navigate mathematics pathways, improvements have been slight. In 1989, a report on the future of mathematics education, *Everybody Counts*, highlighted the dilemma: “From high school through graduate school, the half-life of students in the mathematics pipeline is about one year; on average, we lose half the students from mathematics each year. When mathematics acts as a filter, it not only filters students out of careers, but frequently out of school itself.” (National Research Council, 1989, p. 7) (See: Mathematics Pipeline)

Drawing a direct tie between limited opportunity to learn and high dropout rates among African American and Latinx students, the report’s authors called for making mathematics “a pump, not a filter” in the American education pipeline.



HIGH SCHOOL MATH PROFICIENCY

Percentage of 11th graders in 2018 who met or exceeded standards on the California Assessment of Student Performance and Progress



Source: California Department of Education, 2018
<https://www.cde.ca.gov/nr/ne/yr18/yr18rel62.asp>

More than two decades later, that had yet to occur, as demonstrated by an extensive study of transcripts of California students who were high school seniors in 2010: Measured by scores on state tests, only 29 percent of the students had reached proficiency in Algebra 2, and just one-third had done so in Algebra 1. Existing course sequences were failing to prepare more than two-thirds of the students for college. (Finkelstein, Fong, Tiffany-Morales, Shields, and Huang, 2012)

And by 2018, problems with the pipeline through math courses and tests were persisting. Despite the implementation of new state standards and tests, just 31 percent of students were meeting or exceeding standards in math, compared with 56 percent in English. Furthermore, there were sizeable racial and ethnic disparities, with white and Asian students demonstrating proficiency at more than twice the rate of African American and Latinx students. (California Department

of Education, 2018) (See: High School Math Proficiency) While it could be that the tests themselves weren't accurately measuring students' math achievement, that in itself would be indicative of the problem. Rather than view students' performance through a deficit lens, it's time to acknowledge that it's the traditional math expectations that are flawed.

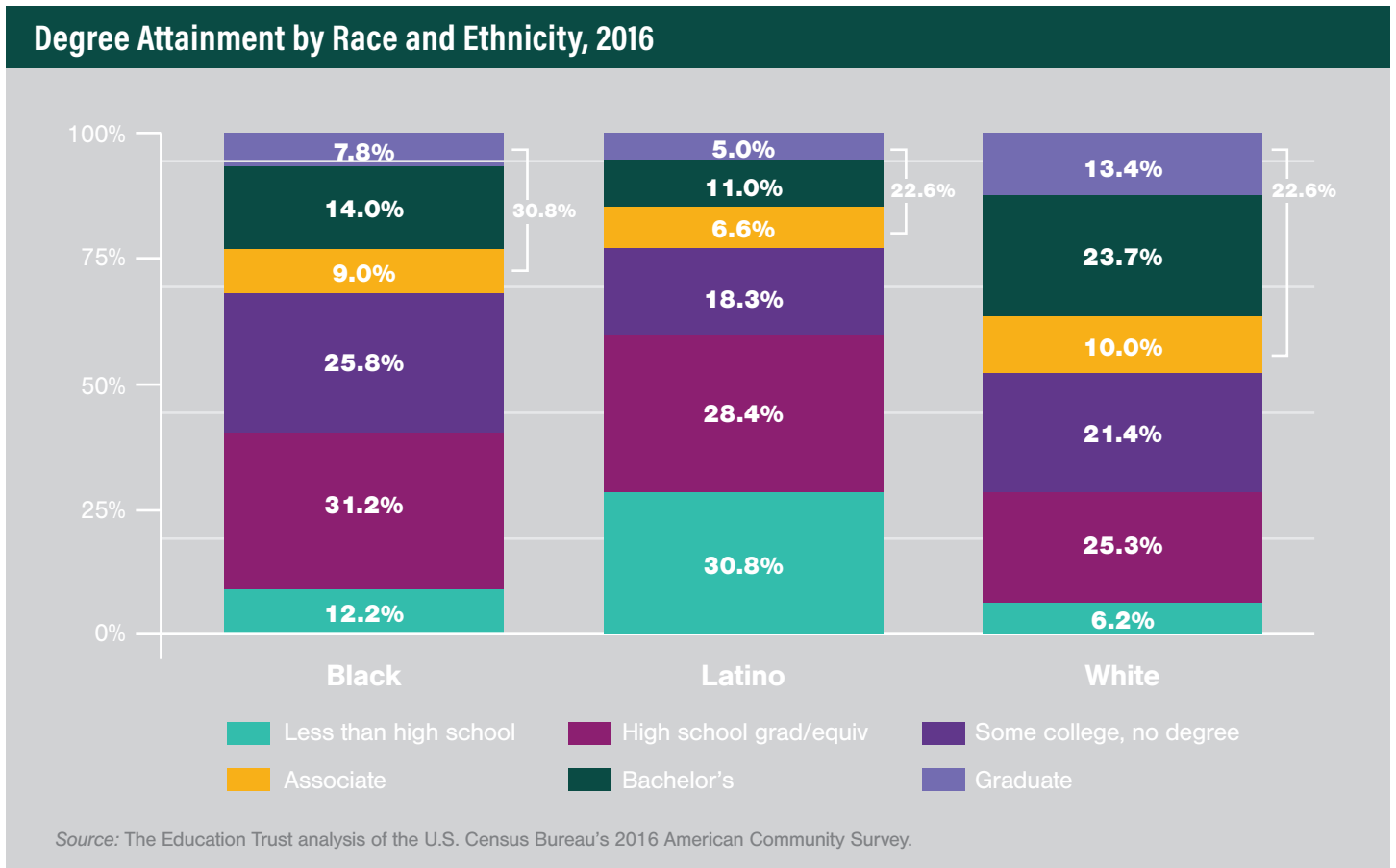
The cost of this dysfunctional math pipeline, in terms of inequitable college attainment, has been great, according to an analysis of census data by The Education Trust. By 2016, the percentage of white adults holding a college degree (an associate degree or higher) was only 47 percent. The percentage of African American and Latinx adults with a degree in the same year was even lower. Only slightly more than 22 percent of Latinx adults and 31 percent of African American adults had earned some form of college degree by the end of 2016. (Nichols and Schak, 2018, p.

1; Schak and Nichols, 2018, p. 1) (See: Degree Attainment by Race & Ethnicity)

Mathematics education needs to support students' transitions to and through college, whether they're pursuing STEM (science, technology, engineering, and math) disciplines or other promising majors that prepare students for careers in other fields like law, politics, design, and the media. It also needs to be relevant for students who pursue careers directly after high school, without attending college.

But reform efforts have yet to address a fundamental aspect of the problem: Too many potential STEM students, especially Latinx and African American students, are being filtered out of opportunities. At the same time, too many whites, Asians, Latinxs, and African Americans are being blocked from pursuing other careers by irrelevant math hurdles.

To ensure more equitable outcomes, this troubled system clearly needs to be redesigned.



THE REDESIGN IMPERATIVE

The need to take a tough look at high school math sequences has been clear for more than a decade. “The mathematics pathways to college must be analyzed, evaluated and redesigned using criteria grounded in the purpose of preparation, not selection,” noted Daro’s 2008 essay, *Mathematics for Whom?*

Schools typically set out to offer courses in a single pathway, the pathway through algebra that leads to calculus and college majors in STEM fields. Some students make their way successfully through this pathway, even though many of them do not pursue STEM majors. But many more get stuck in detours, repeats, or watered-down versions of STEM pathway courses.

The California transcript study exposed the fallacy that students take a common pathway through math courses. In fact, the most common pathway—from basic math in seventh grade through calculus in 12th—was pursued by only 3.3 percent of students. And the 20 most common pathways were pursued by fewer than a third of students. Of those 20 pathways, 12 involved retaking a course. In fact, 50 percent of students repeated a course, with more than one-third repeating Algebra 1. (Finkelstein et al., 2012) When only a few students proceed through a pathway as designed, there must be a problem with the pathway. (See: A Common Math Pathway?)

For most students, traditional math offerings have not formed into rational pathways that lead to worthwhile postsecondary opportunities. Without offering a high quality alternative pathway, schools nevertheless routinely discourage the STEM aspirations of adolescents, especially those from underrepresented groups. Many students who don’t successfully navigate the pathway to calculus sink into a bog of remediation and ineligibility from which few escape. The existence of these dead-end detours speaks to the need for redesign, especially since most students in them are not even aware that such routes may be leading them away from the opportunities they seek.

A COMMON MATH PATHWAY?

U.S. students traditionally were expected, at a minimum, to take a four-course math sequence starting in eighth grade: pre-algebra, Algebra 1, geometry, and Algebra 2. In many states, these courses were required for high school graduation as well as for admission to public university. (California has never required Algebra 2 for high school graduation, though some school districts in California do require it.)

The scope of Algebra 2 has changed over time. Historically, it focused on solution methods for a variety of types of equations. Later, as calculus became a primary entry point for college mathematics, the study of functions was added to the course. But, since much vestigial content on solving equations was retained, Algebra 2 became unwieldy for teachers and students. There was too much content and too little focus.

Algebra 2’s failure to prepare students to study calculus led to the development of precalculus, which focused more explicitly on functions. That left Algebra 2 in the curriculum, with its confusion of topics and explosion of notational devices. Thus, for students aspiring to take calculus (or, in some cases, statistics) in high school, the norm became a six-course sequence, which pushed pre-algebra into seventh grade. For many other students, Algebra 2 was a terminal course. Despite the existence of just one pathway, students’ exposure to math varied widely.

With the introduction of the Common Core State Standards, most states spread traditional Algebra 1 content across grades eight and nine. The new eighth grade course was called Math 8. This made time available in grade nine for more difficult topics and for expanding the study of functions. Some schools follow the international “integrated” approach, in which the content of Algebra 1, geometry and Algebra 2 are combined over three years, rather than offered in separate courses.

BRANCHING OUT: BEYOND A SINGLE PATHWAY



Education systems must be able to respond as effectively to a future musician who is uninterested in traditional math courses as they would to a student who wants to be an engineer, but hasn't had a chance to take advanced math courses.

Why limit students to one pathway? With the presumed common pathway failing to serve the majority of students, it is time to design alternatives that work for many more students. Students need high-quality options, not just STEM versus non-STEM.

They need pathway options that prepare them for a postsecondary world that branches into exciting careers, such as working as a journalist, elected official, high school principal, marketing executive, attorney, game designer, first responder, movie producer, or stockbroker. We call these BRANCH¹ fields.

Students prepare for these BRANCH fields through programs in law, the humanities, the arts, social sciences, and health and human services.

Designing new BRANCH math pathways can accomplish at least four goals:

1. STEM-interested students will be able to learn the mathematics that prepares them for STEM careers.
2. BRANCH-interested students will be able to learn the mathematics that prepares them for BRANCH careers without being blocked by irrelevant requirements.
3. Latinx and African American students will have ample opportunities to thrive in college, including in STEM fields, as will female students of all ethnicities.
4. Students who initially choose a BRANCH pathway will be able to switch to a STEM pathway during high school or college, and vice versa, if their interests change.

¹ Though BRANCH is not an acronym, we have chosen to use all capitals to indicate that these pathways should be comparable and equally rigorous to STEM pathways.

To accomplish these goals, the existing policies, traditions, beliefs, and expectations that have allowed us to fail so many students must be examined, challenged, and updated. In particular, we need to eliminate barriers to opportunity based on income, race, ethnicity, gender, and any other factors beyond the control of the student, to move from a deficit model to an asset-based narrative. We want each student's experience with mathematics in school to lead to worthwhile opportunities that reflect the student's aspirations and include flexibility to change direction.

Education systems must be able to respond as effectively to a future musician who is uninterested in traditional math courses as they would to a student who wants to be an engineer, but hasn't had a chance to take advanced math courses. Systems need to help each student achieve his or her goal, rather than deny both opportunities, as the current system too often does.

High schools need to take responsibility for establishing a small set of math pathways that each leads to fulfilling opportunities. There are numerous challenges in making such a change, including the work to design the pathways themselves. Two particularly vexing factors need to be addressed up front, because they are outside the control of high schools and have the power to inhibit potential reforms:

1. **Pathway choices**, which can have serious and long-lasting implications for students' future opportunities, must be made during adolescence. The history of tracking in math highlights the need for deliberate attention to equity in supporting students in making choices.
2. **Postsecondary policies** governing how institutions select and place incoming students exercise a strong influence on high school math curriculum.

Both of these challenges are complex to address, because there is no single fix, and change involves schools and colleges, counselors and teachers, parents and students, classrooms and systems². Not tackling them in an intersegmental and equity-minded way only risks making them more intractable. Classroom, school, district, and state policies have long created opportunity gaps, and as socio-economic advantages accumulate for some, disadvantages accumulate for others. (See: Race, Math, and the Matthew Effect)

Long-standing assumptions about math learning have magnified existing racial and socio-economic inequities. (Burdman, 2018) These must be confronted, and new policies designed to reverse their effects.



PATHWAY CHOICES VS. TRACKING

If the best way to ensure students have genuine opportunities to prepare for their futures is by offering multiple math pathways, who decides which pathway a student pursues, and when should that choice be made? The crux of the difficulty is this: The time ideally comes during high school, when students are still young and may not have decided on a career path. But having others make choices for students is not a solution.

Students' options frequently have been limited by traditional tracking policies, as well as by inadequate course offerings. In effect, decisions are being made for students, but rarely in transparent and accountable ways. Let's be clear: Existing high school math sequences, and the way students are traditionally assigned to them, are part of the problem. They are based on the presumption of one primary pathway in which only some students can succeed.

² A forthcoming analysis by the Charles A. Dana Center (2019) includes a compelling statement about these challenges.

RACE, MATH, AND THE MATTHEW EFFECT

The observation that equity gaps widen over time, dubbed the “Matthew Effect” by sociologist Robert K. Merton (1968), applies readily in mathematics. The rich get richer and the poor get poorer, in terms of math learning—a phenomenon that has been described as “Born to Win, Schooled to Lose” (Carnevale, Fasules, Quinn, and Campbell, 2019). Consider this analogy for how opportunity gaps accumulate and achievement gaps widen:

Two little girls join the soccer team. They have nearly identical athletic skills. Both are good runners, but Maria runs a half step faster than Gail. When the first soccer lesson begins, there’s one ball. They both run forward, and Maria gets to the ball first and kicks it. After they’ve done this 20 times, Maria has practiced kicking the ball 19 times, and Gail has had only one opportunity. A week later, Maria has learned a lot more about kicking the ball than Gail.

The slight difference in speed has widened into a much bigger difference in opportunity that, in turn, has widened the gap in kicking skills. After a while, Gail thinks she’s not good at soccer. She thinks Maria is good at soccer. Gail’s sense of soccer identity and belonging lead to diminished motivation and effort. Meanwhile, Maria’s excitement and motivation grow. There is no mystery as to why Maria will improve at a much faster rate than Gail. Her opportunities to learn in the moment, and over an increasing accumulation of moments, are greater—much greater.

It does not have to be this way.

Across the field, another soccer team is practicing, and that team will wind up winning the championship. That team is well coached. They have 20 little girls and 20 balls. All the girls kick the ball often and get feedback from the trajectory of the kicked ball. The coaches systematically give feedback to each girl, not just the girls standing near them, or the girl who attracts their attention.

In our math classrooms, similar unnecessary differences in opportunity cause Matthew Effects. When the teacher poses a problem, some students raise their hands, saying “Ooh, ooh, ooh!” wanting to be called on. Time and again, they get the teachers’ attention, feedback, and encouragement. Meanwhile, many other students’ hands stay down as they build the mistaken identity that they cannot learn math. When teachers move too quickly, covering topic after topic, some students get left behind, not receiving feedback or instruction that makes sense to them. Day after day, the opportunity gap accumulates, and aspirations—especially STEM aspirations—erode, particularly for students of color and low-income students.

We must shift the pedagogic culture of mathematics instruction from “quick and snappy” to “curious and thoughtful.”

Such inequities exist not just within classrooms, but also at the level of district and school policies and practices. An example is the assignment of students to courses and the assignment of teachers to courses. The lower a student’s socioeconomic status, the more likely he or she is to be taught by a first-year teacher, a substitute teacher on any given day, or a teacher with less expertise in the subject. (Weisberg, Sexton, Mulhern, & Keeling, 2009; Boyd, Lankford, Loeb, Wyckoff, 2005; and Ronfeldt, Loeb, Wyckoff, 2013) Since teacher assignment and student assignment are variables that adults running the system can control, these Matthew Effects can be eliminated through responsible, albeit brave, changes in policy and practice.

Perhaps the most obvious and cruelest example of the Matthew Effect is the failure to offer all students a realistic opportunity to learn the mathematics needed for advanced high school courses, then tracking the same students away from those courses because they are assessed as lacking the mathematics they never had a chance to learn. Tracking doesn’t solve the failure to prepare many students. It multiplies the damage.

We do not need “placement” and its assumptions about student “ability” in order to create options. Such practices are inherently vulnerable to secrecy and bias. As noted by the NCTM (2018), placement of students into tracks has had tragic consequences, particularly for students of color. Instead, students can be offered options based on their own aspirations and interests. It’s similar to students

Students can be offered options based on their own aspirations ... rather than being placed

choosing, rather than being placed in, their college majors. With appropriate guidance and information, students implementing their own choices may work harder than students who have been placed³.

When you involve students in choosing among options, the bias and barriers of placement are swapped for a new challenge: recruiting and supporting historically excluded students in STEM and BRANCH courses. Teachers and schools will have to actively support students in developing STEM aspirations. Recruiting students from racial, ethnic, gender, or social class groups not well represented in STEM is essential.

Supporting students’ positive academic identity and agency cannot happen without deliberate work on the part of educators to address implicit bias, assumptions about student capabilities, and the ways that math traditionally reinforces privilege. Students’ identities as learners and their sense of belonging in a goal-oriented academic community are heavily influenced by the explicit and implicit messages they get from teachers and schools.

Even if schools change the context and support for students to make choices, the question remains about when pathways should diverge. On the one hand, maturation levels and the desire to keep options open suggest we want to wait as long as possible. At the same time, the longer we wait, the more time is lost for developing the expertise to pursue specialized fields. The longer we wait to offer viable BRANCH pathways, the longer the content and focus of common courses will be under pressure to prepare STEM students, subjecting BRANCH students to unnecessary difficulty and inadvertently closing educational opportunities to them. Rather than direct all students through STEM preparation, students should have the opportunity to specialize in BRANCH options. Later, if they want to change course from one pathway to another, their experience working in depth in a specialization will likely benefit them more than the shallow survey courses that are often the only alternative to STEM-oriented mathematics courses.

Most systems defer the pathway decision until after high school. Delay is not the solution, because this ends up closing opportunities by default, allowing more and more students to flounder with no clear pathway. We must keep the option for students to pursue STEM pathways open as long as possible, while also offering alternative rigorous pathways that work for students aspiring to other postsecondary goals.

New branches beginning in 10th or 11th grade would offer viable pathways for all students, whether or not they are focused on STEM fields. In the Pathway Options section on page 16, we examine some designs employed by school systems where pathways diverge after 10th grade. These designs allow STEM-aspiring students to complete AP Calculus during high school, as some currently aspire to do (though the next section will highlight arguments for de-emphasizing high school calculus).

³ Patali et al, 2008. This meta-analysis of 41 studies examined the effect of choice on intrinsic motivation and related outcomes in a variety of settings, with both child and adult samples. Results indicated that providing choice enhanced intrinsic motivation, effort, task performance, and perceived competence, among other outcomes.

The difficulties around the options available to students and the timing of their choices are real. Pathway design must account for such difficulties by:

- creating rigorous pathways that articulate with postsecondary policies and practices and align with a range of student aspirations⁴;
- giving more weight to student aspirations and less to students' perceived preparation levels;
- supporting educators to address the role of bias and privilege in traditional school structures and to dislodge harmful preconceptions about student abilities;
- implementing instructional and support strategies that address uneven prior opportunities and damaged math student identities;
- ensuring that pathway options are communicated early, publicly, and clearly to all stakeholders; and
- establishing summer or semester courses to serve as bridges for students who choose to switch pathways.

Positive academic identity and agency cannot happen without deliberate work on the part of educators to address implicit bias, assumptions about student capabilities, and the ways that math traditionally reinforces privilege.



⁴ These strategies align with *Principles to Guide Lasting Impact: Implementation Guide* developed by the Charles A. Dana Center to guide higher education institutions in designing mathematics pathways.

POSTSECONDARY POLICIES AND PRACTICES

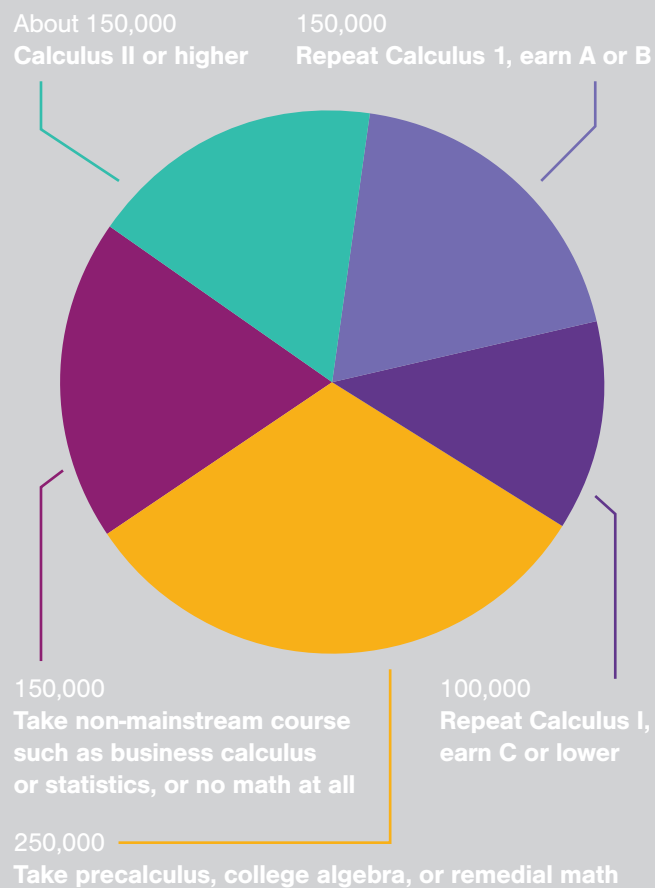
The need for high school mathematics pathways to align with postsecondary policies has been a major cause of paralysis for education leaders, with each segment unable to act as long as the other hasn't. Higher education policies and practices, particularly admissions and readiness policies, have long exerted serious leverage over high school mathematics pathways, as detailed below. Recent reforms should ease some of those pressures.

COLLEGE ADMISSIONS

Admissions policies and practices set strong conditions for high school mathematics pathways. Because universities typically require high school math proficiency at least through Algebra 2, there has been little innovation around the third year of math. A technically-demanding course known for teaching skills such as solving exponential equations and trigonometric identities, Algebra 2 is not relevant to most non-STEM fields. In addition, universities have often awarded a GPA boost for AP, International Baccalaureate (IB), and other honors courses in order to encourage students to take challenging courses in high school.

The fact that all challenging courses in math traditionally have led to calculus also shows the influence of college admissions on high school pathways. Admissions offices use calculus as an indicator of achievement. This emphasis in the high school curriculum creates two problems: First, not all students are interested in fields that require calculus. Second, unlike in other subjects, students must accelerate through a sequence of courses in order to reach AP Calculus. The pressure to take calculus creates pressure on students to accelerate, thereby skipping other valuable mathematics content or compressing the time spent on each topic. It means rushing through one of the highest priorities in high school math, which is developing expertise in the language and reasoning of elementary algebra.

First College Math Course for Those Who Took Calculus in High School



Source: Bressoud, 2017

Since not all students have the opportunity to accelerate, acceleration also operates as a form of tracking.

The Mathematics Association of America (MAA) and the NCTM issued a joint statement in 2012 declaring calculus a college course that needn't be taught in high school. To avoid the problems associated with acceleration, these organizations asked high schools to stop emphasizing calculus and instead prioritize developing students' expertise in algebra, geometry, and trigonometry. (Bressoud, 2012)

Mathematician David Bressoud’s longitudinal analyses (2017) of AP Calculus students’ college course-taking reveal that, for most students who take calculus in high school, the course does not serve a purpose other than as a signal to admissions offices of elite institutions. (See: First College Math After High School Calculus) As long as a subset of students and parents believe calculus in high school gives them an admissions advantage, they will pressure schools to offer the pathway through calculus. But this situation serves few students well: The emphasis on calculus disadvantages students who don’t take it. And many students who take calculus are ill-served by the pressure to forgo other learning opportunities, as explained by noted math instructor Daniel Teague:

Before a student can learn calculus in a manner that has some significant residual, they must want to learn calculus... When the goal is not to develop a deep and abiding understanding and facility with the tools of calculus, but to pass the course with a good grade... the learning can be quite superficial. (Bressoud, 2017, p. 5)

READINESS

How higher education institutions define readiness for general education math courses also impacts

pathway options in high school. The need for students to complete college prerequisites before taking college-level courses that meet the quantitative reasoning requirement for the baccalaureate degree has driven the massive remediation system in community colleges and some public universities. This system has thwarted the aspirations of a very large number of students, including many who were not drawn to STEM majors. Just 19 percent of bachelor’s degrees and 8 percent of associate degrees awarded in 2016 were in STEM fields. (National Center for Education Statistics, 2019) (See: Postsecondary STEM majors and Algebra 2)

Historically, colleges have used mastery of high school Algebra 2 as the prerequisite for a general education math or quantitative reasoning class. This practice is not related to mathematical content, but rather to Algebra 2’s status as the third course in the high school sequence. But, as the California transcript study showed, it has not performed this role well. It is a more difficult course than any before it, and less coherent and focused than any after it on the path to calculus. Using Algebra 2 in this way overlooks more valuable content and prioritizes technically difficult topics of lower importance.



POSTSECONDARY STEM MAJORS AND ALGEBRA 2

Proportion of associate degree graduates in STEM fields:

8%

Proportion of B.A. holders using Algebra 2 or beyond:

18-31%

Proportion of bachelor’s degree graduates in STEM fields:

19%

Number of community college programs that required Algebra 2 mastery of entering students

1/441

Sources: NCES, 2019; NCEE, 2013; Georgetown University Center on Education and the Workforce, 2013

NEW POSTSECONDARY POLICY CONTEXT

Despite these challenges, new higher education policies are improving the structure and substance of options available to students with varied aspirations, thus enhancing the conditions for redesigning high school math pathways. The “rigidity” of higher education requirements can no longer be used as an excuse by K-12 for refusing to make beneficial changes in high school pathways.

For example, the California State University (CSU) system no longer requires Algebra 2 as a prerequisite for general education math or quantitative reasoning courses. Instead, the prerequisite needs to be “reflective only of skills and knowledge required in the course.” (California State University, 2017) Most statistics classes, for example, wouldn’t have an intermediate algebra prerequisite (though CSU does require Algebra 2 for freshman admission). The CSU has also eliminated traditional remedial courses in favor of just-in-time support. General education math courses, whether College Algebra, Introduction to Statistics or, Logic and Computing, should offer concurrent or corequisite options that align with the courses to support student success.

Numerous studies have shown that aligning prerequisite content with general education courses and offering corequisites instead of remedial courses can vastly improve students’ success in general education math courses. In statistics pathway programs developed by organizations such as the Carnegie Foundation for the Advancement of Teaching and the California Acceleration Project, students have passed their general education math courses at rates three to four times higher than students assigned to traditional algebra-based remedial courses. (Burdman, Booth, et al., 2018) Likewise, researchers at the City University of New York (CUNY) found that students who took a corequisite statistics course instead of traditional algebra remediation were 50 percent more likely to graduate—and did better in more advanced quantitative courses. (Logue, Douglas, and Watanabe-Rose, 2018)



These reforms open up avenues that would have been difficult for high schools to consider in the past. For example, high schools in California and nationally have been developing new senior year courses outside of the traditional calculus pathway. But these changes also point to others that colleges should consider:

- Most universities still require Algebra 2 for freshman admission. A broader definition of quantitative reasoning by higher education institutions would open the door for high schools to offer a third-year course with greater relevance to the majority of fields. While some public universities say they accept courses “equivalent” to Algebra 2, it’s not clear how such policies are implemented.
- AP Calculus should be reconsidered as an admissions screen. AP Statistics, a course students can reach without accelerating through other math courses, should be of equal value. In addition, admissions offices could consider offering a GPA boost only for honors courses that don’t require acceleration. That is, acceleration could disqualify a course from the honors bonus. Short of that, transparency on the merits of various AP courses, in itself, would be helpful.

ENVISIONING SOLUTIONS

There are many better solutions than our current broken system. Any effective system for moving students through middle and high school mathematics will have the following elements:

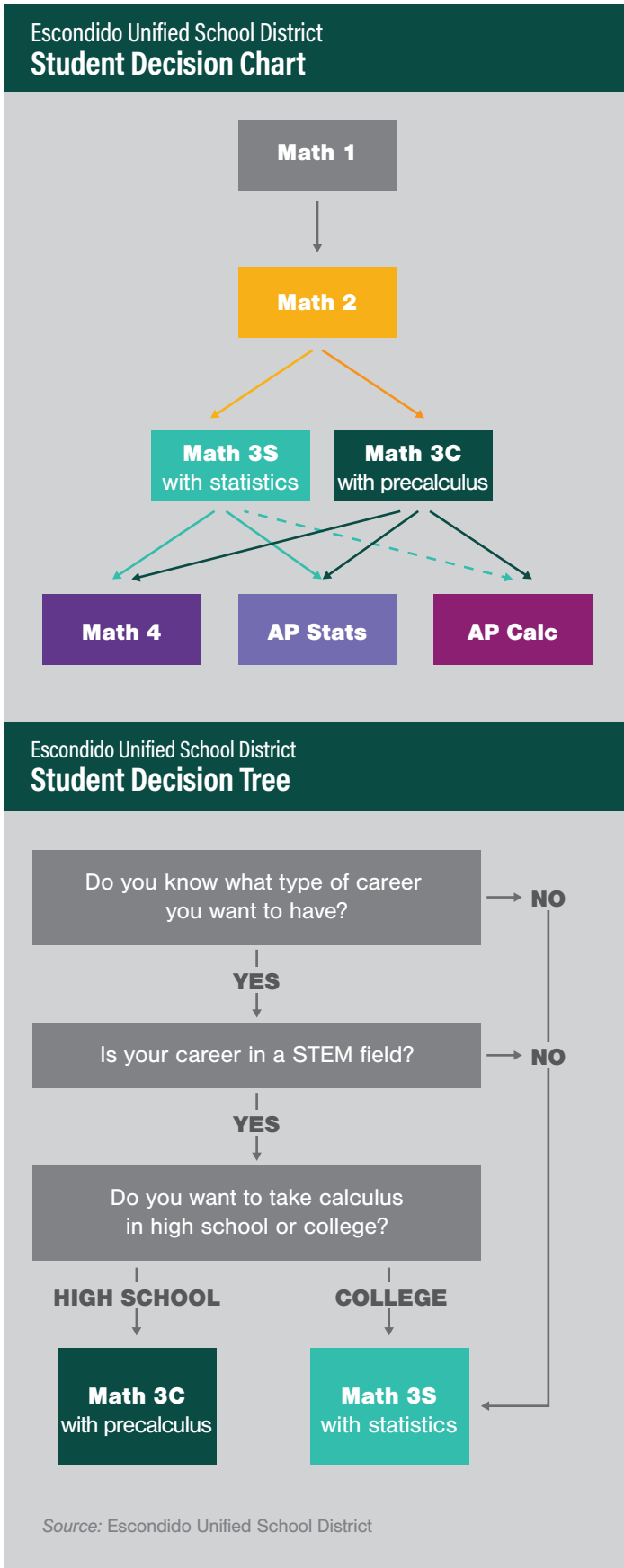
- pathways as options that lead to postsecondary opportunities, with some flexibility to switch pathways (from BRANCH to STEM or vice versa);
- relevance of pathway content, expertise, and goals;
- recruitment of students to pathways; and
- support for students within pathways.

PATHWAY OPTIONS

Better-engineered pathways for students are not hard to imagine. To begin, we shift from thinking of pathways as tracks based on levels of student ability to thinking of pathways in terms of the valuable postsecondary opportunities they offer. Some districts and state systems already are making innovative changes to pathway options. As illustrated by Oregon’s education systems, as well as by two California school districts (Escondido and San Francisco), there is more than one way to engineer pathways that offer students worthwhile options.

Internationally, many countries with better math outcomes than ours, including Finland and Singapore, have common courses through ninth grade, followed by specialized programs beginning in the equivalent of 10th grade, with math tailored to students’ specialties and interests. STEM aspirants often take two math courses in 10th grade. (By contrast, in the U.S. system, students taking two math courses are often those who struggle with math).

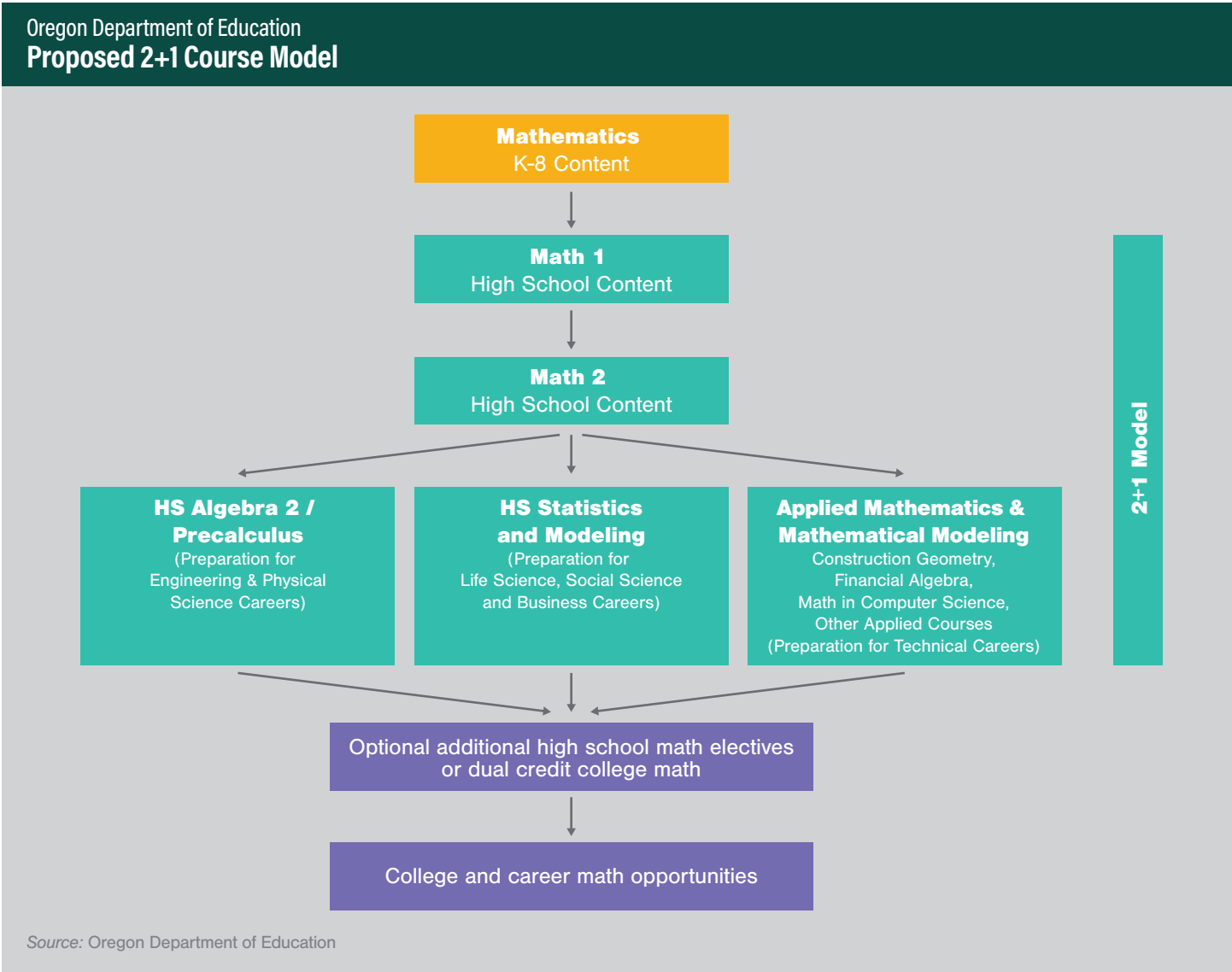
Though the illustrated pathways represent different solutions, all three align with recommendations by the NCTM (2018) in *Catalyzing Change in High School Mathematics*: to offer four-year math pathways that include two to three years in a common, shared pathway.



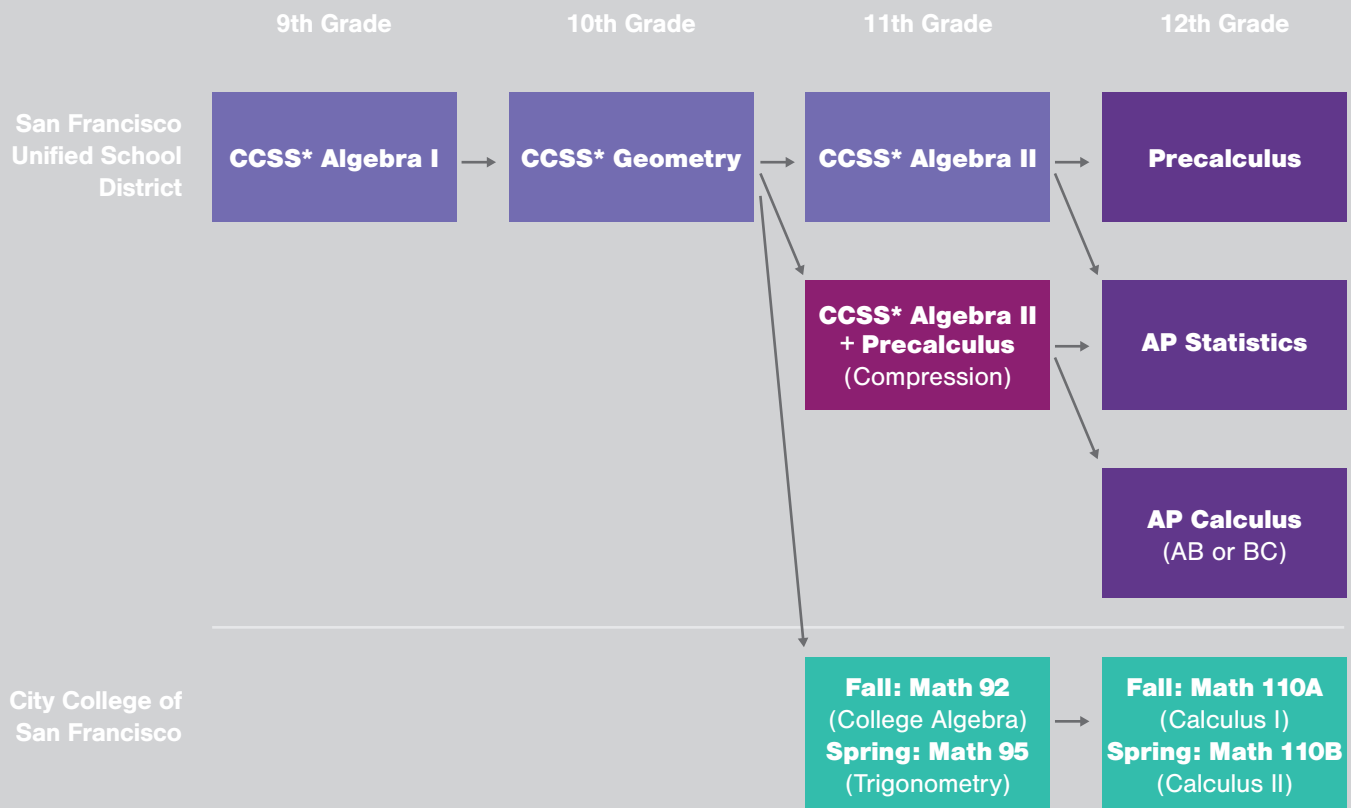
In this country, the most practical time for pathways to diverge is after 10th grade. This allows for two versions of the 11th grade course formerly known as Algebra 2—a STEM version that can include much of precalculus, and a broader BRANCH version that emphasizes content such as statistics and modeling. The STEM students benefit by taking math with classmates who share their STEM ambitions. The BRANCH students benefit from a course aligned with their interests and from being surrounded by students with similar aspirations.

Typically, a high school will be able to offer a STEM pathway and one or two BRANCH pathways that lead to worthwhile postsecondary

To begin, we shift from thinking of pathways as tracks based on levels of student ability to thinking of pathways in terms of the valuable postsecondary opportunities they offer.



San Francisco Unified School District Mathematics Pathways



*Common Core State Standards

Source: San Francisco Unified School District

opportunities. Good solutions will offer high school students options, rather than place them into tracks or point them toward dead ends. Options such as bridge courses or corequisites should be made available to students who wish to switch pathways during high school or college.

BRANCH PATHWAY CONTENT

No art teacher would provide students with a pull-down menu of specific ways to create paintings of different kinds—portraits of men, portraits of women, portraits of children, paintings showing buildings, paintings of rural scenes, paintings of skies, etc. No, they teach the students how to paint (which includes helping them learn how to see). From which grounding, the student can create and develop their own “strategies” to produce paintings of various kinds. (Devlin, 2019)

If math is a “way of thinking,” as Keith Devlin used the art analogy above to illustrate, a good starting point for considering the content of high school math pathways is the “Essential Concepts” identified by the NCTM (2018). These are concepts that all students should be expected to learn in a common pathway. The importance of these essential concepts continues as students progress into STEM or BRANCH pathways. Because the newly-developed 9th and 10th grade common courses may have less emphasis on Algebra 2 topics than current courses, 11th and 12th grade STEM courses will need some revision. For the STEM pathway to include calculus in grade 12, some compression of Algebra 2 into precalculus will be needed. This will improve the focus and coherence of the STEM pathway.

Content of the BRANCH pathways requires more innovation and articulation, since they will be new. BRANCH courses should focus on relevant mathematics for students in areas such as the humanities and social sciences, most importantly:

- an initial course suitable for 11th grade (without acceleration) as an alternative to traditional Algebra 2. The emphasis should be on interesting, relevant, and challenging applications of middle-level mathematics, not a forced march through advanced math topics. Studies of how math is used outside of STEM show that students most need high-level use of middle-level mathematics. (National Center on Education and the Economy, 2013) The course should develop enough algebraic expertise to meet the third course requirement for admission to public universities in most states.

- a fourth-year course that goes more deeply into the topics of the initial course.
- an honors or AP version of the fourth-year course that is comparable to an introductory college-level course. Such an AP course would not require students to accelerate through the middle and high school math sequence to reach.

The BRANCH courses should be open to STEM students to take in addition to their required STEM courses. Course goals should include: development of facility with symbolic notation; use of functions to model real-world situations; development of understanding of visual representations; and development of problem-solving, teamwork, and communication skills. (See: BRANCH Course Goals)

BRANCH COURSE GOALS

DEVELOPMENT OF FACILITY WITH SYMBOLIC NOTATION

Symbolic notation, in which letters are used to represent numerical quantities, should be treated as a form of language development, rather than remediation. Importantly, this facility provides a foundation for subsequent quantitative courses and leaves open the option of switching to a STEM pathway. Facility with symbolic expressions and equations also is a powerful thinking and communication tool with general utility in many fields and in life. Students in the BRANCH pathway may have high levels of anxiety about symbolic notation, in particular. Reducing anxiety requires actively engaging students in learning, rather than fostering a culture of testing and correction.

USE OF FUNCTIONS TO MODEL REAL-WORLD SITUATIONS

Emphasis should be on formulating and using functions, including the graphs that represent them, to model real-world situations. The courses should focus on extended applications, modeling, and the interpretation of results, rather than on technical exercises.

DEVELOPMENT OF UNDERSTANDING OF VISUAL REPRESENTATIONS IN COMPLICATED APPLICATIONS

Flow charts, decision trees, tables, 2D diagrams of 3D objects, digital visualizations, scale drawings and other visual tools are common features of work in many fields. Many of these visual tools are mathematical representations, or they rely on mathematics to construct or interpret. Greater understanding of the construction and use of diagrams would be beneficial for many students.

DEVELOPMENT OF PROBLEM-SOLVING, TEAMWORK, AND COMMUNICATION SKILLS

By using mathematics to model real-life problems, students learn to apply mathematics they already know, as well as learn new math concepts, while developing sustained reasoning to make sense of the world. Mathematics modeling uses mathematics to represent aspects of the world for the purpose of making decisions, predictions, and plans. It can help answer “messy” questions, such as, “How do we determine the average rainfall in a state? Where’s the best place to locate a fire station? What is a fair voting system? How can I hang pictures along a staircase so they look straight?” (COMAP & SIAM, p. 7)



Several design principles are important:

1. BRANCH mathematics courses should not teach watered-down versions of STEM topics, but instead topics with their own heft and potential relevance in BRANCH fields. Fields like mathematical modeling, data science, statistics, probability, digital graphics, decision theory, robotics, and game design are both practical and alluring for students. The design challenge that philanthropic and government funding must address is how to use these topics to develop math skills, as well as how to use mathematics to make sense of these topics.
2. Success in the courses should depend as little as possible on students' incoming facility with the technical demands of mathematics, so assumptions about their facility should be minimized.
3. In addition to specific mathematical concepts and techniques, coursework should include a complement of projects and mini projects. The mix should resemble a science, English, or social studies course, rather than the typical hurried march through math topics and exercises with which only some students can keep up. Although the students will have had experience with projects in other subjects, many math teachers have little experience with project work, so this requires professional development. The purpose of these courses is the development of useful expertise with relevant mathematics, rather than superficial (and probably temporary) skill with a long list of advanced topics. Projects should allow for wide variation in entering students' proficiency in mathematics. Unlike math problems requiring an identical solution from each student, project work puts students in the role of 'makers' who manage their work, see the varied and concrete results of their efforts, and learn from each other.
4. New senior year courses in emerging areas of math and quantitative reasoning should be developed. (See: Emerging Courses & Pathways) Improving the quality and variety of senior year math courses is a better way to engage students in math learning than simply requiring them to take math. Four years of math should be encouraged, not required, for high school graduation. Ultimately, students' decisions about taking math in their senior year should hinge on their individual interests and prior math experience. Although the senior year is a good time for students to stay engaged in mathematical thinking and brush up on unfinished learning, ensuring quality math instruction in the first three years of high school is a higher priority than simply increasing the number of courses students take.

EMERGING COURSES & PATHWAYS

Although statistics is the most common alternative to traditional math courses in many states, other quantitative reasoning courses are emerging. These include new math courses being taught in high school (often in collaboration with universities), as well as postsecondary quantitative reasoning courses that could be adapted for high school juniors or seniors:

INTRODUCTION TO DATA SCIENCE

Developed with the support of faculty at the University of California, Los Angeles, and with funding from the National Science Foundation, this course was piloted by the Los Angeles Unified School District. The course teaches students elementary statistics, as well as the programming language R, which students use to record data about their lives, such as their sleep or snacking habits. Designed initially as a senior year course, but also offered to some juniors to fulfill the Algebra 2 requirement, the course is now offered by more than 20 of Los Angeles' 100 high schools. Another 15 districts are replicating the course.

DISCRETE MATHEMATICS

In partnership with San Diego State University, Sweetwater Union High School District has developed and piloted a course, Discrete Mathematics, which offers a broad collection of math content with multiple real-world applications. The course, designed for seniors, includes topics such as two-player games, counting (or combinatorics), cryptography, and graph theory. These topics are designed

to be interesting and relevant, while deepening students' quantitative reasoning and ability to apply math learning in everyday contexts.

QUANTITATIVE REASONING

The Charles A. Dana Center at the University of Texas, as well as the Carnegie Math Pathways initiative, have developed and assisted higher education systems in developing innovative quantitative reasoning courses. The learning outcomes in these general education courses include using the concepts of numeracy to investigate quantitative relationships and solve problems, making decisions by analyzing mathematical models, and using the language of statistics and probability to investigate and draw conclusions from real-world contexts.⁴

MATHEMATICS AND CULTURE

A few universities offer courses in ethnomathematics that could be modified for high school students. For example, Sacramento State University offers "Mathematical Practices Across Cultures," which meets the campus's general education quantitative reasoning requirement. The course is described as an "introduction to diverse mathematical thought, action and practices across cultures." Students study topics including mathematical modeling and the relationship of culture to diverse forms of quantitative reasoning and problem-solving, as well as how time and space are perceived by diverse traditions.



⁴ See, for example, the quantitative reasoning learning outcomes developed by Ohio math faculty, with support from the Dana Center, as described in Burdman, Booth, et al., p. 35.

RECRUITMENT

Providing options does not relieve schools of their responsibility for increasing enrollment in STEM pathways—and in college, in general—by students from underrepresented groups. To avoid math pathways being used as a new tracking mechanism, schools will need to proactively recruit students into STEM pathways from populations historically underrepresented in STEM fields, including Latinx and African American students and women of all ethnicities. Likewise, they will need to promote the benefits of the new BRANCH pathways for students with interests in areas such as law, the arts, humanities, and social sciences. Since BRANCH courses will, by design, lead to desirable college and career opportunities, it's important that their value be effectively communicated. That way, parents and families will no longer believe that the pathway to calculus is the only route to success.

Schools will need new recruiting tools and strategies for the responsibility of correcting the historic biases of the system. The recruitment of students begins in the pedagogy and culture of the classroom. High school students' identities as learners are heavily influenced by how the adults in their school treat them and by their own beliefs about their agency. They need adults who believe in their learning capacity, as well as experiences that develop their capacity to participate in, and negotiate, mathematical sense-making with peers. They need to recognize their own and each others' ways of thinking as being mathematically valid. (Boaler, William, and Zevenbergen, 2000) And they need to believe their intellectual abilities aren't fixed, but can be developed.

Recent research on mindset interventions shows the possibility. For example, David Yeager's recent study of over 6,000 ninth graders found that a less than one-hour, online intervention on growth mindset resulted in improved grades for lower-achieving students, as well as increased enrollment in higher-level math courses. The observed effects were sustained in schools where the expectations of instructors and peers were consistent with the growth mindset message. (Yeager et al., 2019) Programs such as Academic



Youth Development at the Charles A. Dana Center at the University of Texas at Austin and Stanford University's YouCubed also incorporate mindset research. (Hochanadel and Finamore, 2015) Such programs help students see themselves as mathematical thinkers and doers. The result is students who believe they can improve by seeking challenging learning experiences and persevering with them.

Math and science teachers have to play a primary role in recruiting students from diverse backgrounds, including some who may have unconventional academic records. Recruiting teachers from diverse backgrounds to play this role is also an important equity strategy.

Beyond the classroom, explicit recruitment is required. Imagine a mathematics or science teacher looking a student in the eye and saying, in his or her own way, "I see a scientist." Unfortunately, schools routinely discourage the STEM aspirations and identity of adolescents from underrepresented groups, so structures are needed to change this. Each teacher should have a recruitment goal, and the math department chair, especially, should lead a campaign each year. Specific students should be recruited by specific adults.

SUPPORT

Students may require support to succeed in their chosen pathways. Ideally, support begins with in the culture of the classroom. How to provide homework and assignment support for students with unfinished learning from prior courses is a fundamental design challenge, given constraints on time and resources. Sometimes changes in school structures and policies are necessary, such as changing bell schedules to facilitate supplemental support. Support can take several forms:

SUPPLEMENTAL SUPPORT

Many students can benefit from additional learning opportunities. For some, supplementing their regular coursework with tutoring that focuses on help with regular course assignments and assessments can be the difference between success and failure. The power of tutoring is that the tutor sees the student in the act of working on problems, gives feedback, and asks questions. One highly efficient use of resources is **peer tutoring** and other “near-peer” supports, like organized study groups. Another form of supplemental support, provided during regular grade-level instruction, is “just-in-time” support, in which students get help with an assignment while they are working on it. The focus is the unfinished learning that’s needed for the present assignment, not whole topic remediation. Support or corequisite classes are another possible approach, if they are effectively designed to help students succeed in the regular course and are structured to ensure that students don’t miss out on other educational opportunities.

PEDAGOGIC STRUCTURES

Enhancements to pedagogy are also important—in particular, **instructional routines** such as Think-Pair-Share, Draft-Feedback-Revise, and Notice and Wonder. The emphasis should be on making student thinking visible, revising draft thinking by comparing different ways of thinking and representing to bring out the underlying mathematics that is the target of the lesson. This revision of thinking deepens, clarifies, and corrects the students’ grasp of the relevant mathematics. These routines also help students develop agency and learn to collaborate. Mathematics

teachers should develop expertise in orchestrating productive discussions. Teachers need to manage discussions so students get opportunities to be listened to and to receive productive feedback, and it’s important to distribute these opportunities in ways that support students who can most benefit. **Menus or stations** allow students to work—independently, alone, or in groups—on a set of pre-planned activities, such as math games, mini projects, graphing software practice, or fluency exercises. The independent work frees the instructor to guide small groups of four or five students in just-in-time math interventions.



GRADING APPROACHES

Grading policies in mathematics classes have to be brought more in line with grading in most high school courses. A grading system should recognize how grading itself can reinforce the sort of “fixed mindset” that interferes with students’ motivation. Grading policies should protect students from falling into a hole out of which they cannot climb. The rules for getting a B should be different than the rules for getting an A. There need to be opportunities to improve a grade through revision. The pedagogic use of the **draft-feedback-revise** cycle naturally accommodates initial differences in students’ prior mathematics experiences and leads to a greater effort on the part of students to learn during the revision phase of the cycle.

GETTING FROM HERE TO THERE

Current high school math pathways are not sufficiently coherent to effectively cultivate quantitative reasoning skills for large numbers of students, creating an imperative for redesign. It is time to ensure that students have rigorous pathway options that lead to postsecondary opportunities in line with their career aspirations—and that they are supported to succeed in those pathways. It's also important that factors known at birth—like race, ethnicity, class, and gender—don't predetermine students' journeys through math.

Designing and enacting such changes is no simple challenge. This isn't a single organization developing and implementing a strategic plan. We have a constellation of systems and agencies with multiple layers of governance: schools, districts, departments of education, math departments, admissions offices, and college and



university systems—as well as state and federal agencies that fund them. It is hard to imagine getting an aggregation of institutions that prize their independence to act in concert toward a common goal, not to mention have the resources to achieve those goals. One key is to maintain a singular focus on the coherence of students' pathways through these institutions.

One key is to maintain a singular focus on the coherence of students' pathways through these institutions.

To synchronize change across so many levels, we envision this work taking place in three stages:

Analysis. Study current math practices and policies to identify those that create and perpetuate disparate opportunities to achieve.

Development. Design and implement new pathway options (and related policies and practices) to reduce disparate opportunities to achieve.

Refinement. Evaluate and update new pathways (and related policies and practices) to ensure that they reduce disparate opportunities to achieve.

Multiple players have critical roles to play, as outlined in *Key Steps for Branching Out*. New postsecondary math pathways and parallel changes to postsecondary admissions and placement practices help set the conditions for high-quality high school math pathways that prepare students for college and career. Likewise, research will be essential in developing and evaluating new pathways. Ultimately, the success of new high school math pathways will depend on how effectively and equitably they prepare students to enter a range of college majors and career fields.

KEY STEPS FOR BRANCHING OUT

K-12 SCHOOLS, DISTRICTS, AND SYSTEMS

1. Conduct equity audits of existing math pathway policies and practices—e.g., teacher assignments, classroom practices, grading, policies, and bell schedules—to uncover “Matthew Effect” mechanisms that widen opportunity gaps.
2. Shift from tracking students by “ability” to offering pathways as options for students, while implementing strategies to dislodge preconceived notions of student abilities.
3. Design ninth and 10th grade courses that prioritize content important for BRANCH pathways, while shifting more technical STEM-applicable content into junior and senior year STEM courses.
4. Design junior and senior year BRANCH courses, including an AP mathematics course that seniors can take without accelerating through the curriculum, as well as junior and senior year STEM courses that prepare students for calculus in high school or college.
5. Support teachers, counselors, administrators, students, and families to understand the new options.

POSTSECONDARY INSTITUTIONS AND SYSTEMS

1. Adopt changes to admissions and placement criteria that support the redesign of high school math pathways (including accepting BRANCH pathway courses and reducing the emphasis on acceleration for AP Calculus).
2. Ensure that prerequisites for general education quantitative reasoning courses are relevant to BRANCH courses and majors.
3. Design, implement, and evaluate corequisite courses that can (a) support student success in required courses and (b) serve as bridges between STEM and BRANCH pathways for students who choose to switch pathways.
4. Conduct equity audits of math pathways and corequisite math courses to ensure they are meeting equity goals by diminishing racial and socio-economic gaps.

RESEARCH COMMUNITY

1. Partner with higher education institutions to evaluate the effectiveness of new postsecondary pathways in preparing students for their chosen fields and in reducing equity gaps in college success, including enrollment disparities in STEM majors.
2. Design, develop, and research practices, tools, and systems that replace “Matthew Effect” mechanisms with practices, tools, and systems that reduce equity gaps.
3. Partner with K-12 to develop and evaluate new common ninth and 10th grade courses as well as new 11th and 12th grade BRANCH and STEM courses.
4. Work with systems to design, develop, and evaluate student support systems and practices, including high school and college corequisite math courses.



REFERENCES

- Boaler, J., William, D., & Zevenbergen, R. (2000). *The construction of identity in secondary mathematics education*, International Mathematics Education and Society Conference, Montechoro, Portugal, 2000.
- Boyd, D., Lankford H., Loeb, S., & Wyckoff, J. (2005). Explaining the short careers of high-achieving teachers in schools with low-performing students. *American Economic Review*, 95(2), 166-171.
- Bressoud, D. (2012, April 2). MAA/NCTM joint position on calculus. [Blog post]. Retrieved from: <http://launchings.blogspot.com/2012/04/maanctm-joint-position-on-calculus.html>
- Bressoud, D. (Ed.). (2017). *The role of calculus in the transition from high school to college mathematics*. Retrieved from Mathematical Association of America website: https://www.maa.org/sites/default/files/RoleOfCalc_rev.pdf
- Burdman, P. (2018). *The mathematics of opportunity: Rethinking the role of math in educational equity*. Berkeley, CA: Just Equations.
- Burdman, P., Booth, K., Thorn, C., Bahr, P. R., McNaughtan, J., & Jackson, G. (2018). *Multiple paths forward: Diversifying mathematics as a strategy for college success*. San Francisco, CA: WestEd & Just Equations.
- California Department of Education. (2018). *State schools chief Tom Torlakson announces results of California assessment of student performance and progress online tests*. [News release]. Retrieved from: <https://www.cde.ca.gov/nr/ne/yr18/yr18rel62.asp>
- California State University. (2017). *Memorandum from Chancellor Timothy P. White re: Exec. Order No. 1100*. Retrieved from: <https://www.calstate.edu/eo/EO-1110.html>
- Carnevale, A., Fasules, M., Quinn, M., & Campbell, K. (2019). *Born to win, schooled to lose: Why equally talented students don't get equal chances to be all they can be*. Retrieved from Georgetown University Center on Education and the Workforce website: <https://cew.georgetown.edu/cew-reports/schooled2lose/>
- Center on Education and the Workforce, Georgetown University. (2013). (Calculations of O*NET 17.0 and American Community Survey data). Unpublished.
- Charles A. Dana Center at the University of Texas at Austin (2019). *Launch Years: The opportunity to remove barriers to success in mathematics*. Publication forthcoming. Austin, TX: Author.
- Charles A. Dana Center at the University of Texas at Austin. *Principles to guide lasting impact: Implementation guide*. Retrieved from: <https://dcmathpathways.org/dcmp/dcmp-model>
- Consortium for Mathematics and its Applications and Society for Industrial and Applied Mathematics. (2016). *Guidelines for assessment and instruction in mathematical modeling education*. Philadelphia, PA: Society for Industrial and Applied Mathematics.
- Daro, P. (2008). *Mathematics for whom? The top of high school meets the bottom of college*. The Opportunity Equation. Retrieved from: <http://www.learningworksca.org/wp-content/uploads/2012/02/025-Phil-Daro-Mathematics-for-Whom.pdf>
- Devlin, K. (2019, July 1). Mathematics is a way of thinking: How can we best teach it? [Blog post]. Retrieved from Mathematical Association of America website: <https://www.mathvalues.org/masterblog/2019/6/27/mathematics-is-a-way-of-thinking-how-can-we-best-teach-it>

Finkelstein, N., Fong, A., Tiffany-Morales, J., Shields, P., & Huang, M. (2012). *College bound in middle school & high school?: How math course sequences matter*. Retrieved from Center for the Future of Teaching and Learning at WestEd website: https://www.wested.org/online_pubs/resource1274.pdf

Hochanadel, A., & Finamore, D. (2015). Fixed and growth mindset in education and how grit helps students persist in the face of adversity. *Journal of International Education Research*, 11(1), 47-50.

Logue, A. W., Douglas, D., & Watanabe-Rose, M. (2019). *Corequisite mathematics remediation: Results over time and in different contexts*. Retrieved from CUNY Academic Works website: https://academicworks.cuny.edu/gc_pubs/540

Ma, J., Pender, M., & Welch, M. (2016). *Education pays 2016: The benefits of higher education for individuals and society*. College Board Trends in Higher Education Series.

Merton, R. K. (1968). The Matthew Effect in science. *Science*, 159(3810), 56–63. doi:10.1126/pmid.17737466

National Center on Education and the Economy. (2013). *What does it really mean to be college and work ready? The mathematics and English literacy required of first year community college students*. Retrieved from: <http://ncee.org/college-and-work-ready/>

National Council of Teachers of Mathematics. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*. Washington, D.C.: Author.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, D.C.: National Academies Press.

Nichols, A. H., & Schak, J. O. (2018). *Degree attainment for black adults: National and state trends*. Washington, D.C.: Education Trust.

Patall, E. A., Cooper, H., & Robinson, J. C. (2008). The effects of choice on intrinsic motivation and related outcomes: A meta-analysis of research findings. *Psychological Bulletin*, 134(2), 270-300. Retrieved from: <http://dx.doi.org/10.1037/0033-2909.134.2.270>

Ronfeldt, M., Loeb, S., & Wyckoff, J. (2013). How teacher turnover harms student achievement. *American Educational Research Journal*, 50(1), 4-36.

Schak, J. O., & Nichols, A. (2018). *Degree attainment for Latino adults: National and state trends*. Washington, D.C.: Education Trust.

Undergraduate Degree Fields. (2019, May). Retrieved from National Center for Education Statistics website: https://nces.ed.gov/programs/coe/indicator_cta.asp

Weisberg, D., Sexton, S., Mulhern, J., & Keeling, D. (2009). *The Widget Effect: Our national failure to acknowledge and act on differences in teacher effectiveness*. New York, NY: The New Teacher Project.

Yeager, D. S., Hanselman, P., Walton, G. M., Murray, J. S., Crosnoe, R., Muller, C., & Dweck, C. S. (2019). A national experiment reveals where a growth mindset improves achievement. *Nature*. Retrieved from: <https://doi.org/10.1038/s41586-019-1466-y>

This work is licensed under the Creative Commons Attribution-NonCommercial 4.0 International License. To view a copy of this license, visit <http://creativecommons.org/licenses/by-nc/4.0/> or send a letter to Creative Commons, PO Box 1866, Mountain View, CA 94042, USA.



JUST // EQUATIONS

justequations.org