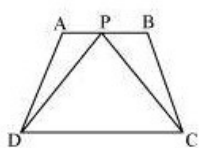


MATHS -9 (CH-09 - PARALLELOGRAMS & TRIANGLES)

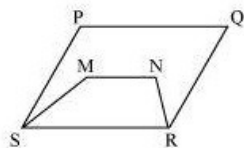
MATHS -9 (CH-09 -9.1- PARALLELOGRAMS & TRIANGLES)

Question 1:

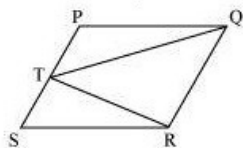
Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.



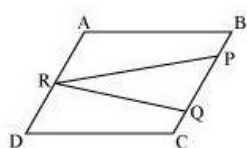
(i)



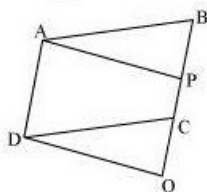
(ii)



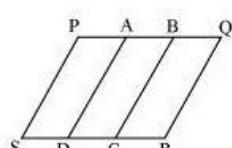
(iii)



(iv)



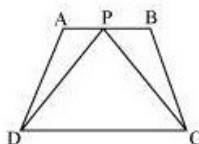
(v)



(vi)

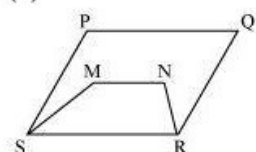
Answer 1:

(i)



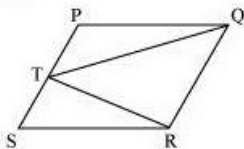
Yes. It can be observed that trapezium ABCD and triangle PCD have a common base CD and these are lying between the same parallel lines AB and CD.

(ii)



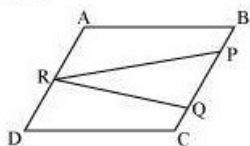
No. It can be observed that parallelogram PQRS and trapezium MNRS have a common base RS. However, their vertices, (i.e., opposite to the common base) P, Q of parallelogram and M, N of trapezium, are not lying on the same line.

(iii)



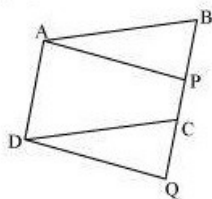
Yes. It can be observed that parallelogram PQRS and triangle TQR have a common base QR and they are lying between the same parallel lines PS and QR.

(iv)



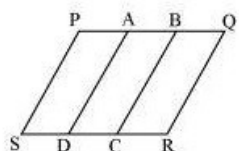
No. It can be observed that parallelogram ABCD and triangle PQR are lying between same parallel lines AD and BC. However, these do not have any common base.

(v)



Yes. It can be observed that parallelogram ABCD and parallelogram APQD have a common base AD and these are lying between the same parallel lines AD and BQ.

(vi)

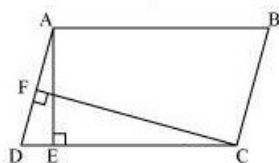


No. It can be observed that parallelogram PBCS and PQRS are lying on the same base PS. However, these do not lie between the same parallel lines.

MATHS -9 (CH-09 -9.2- PARALLELOGRAMS & TRIANGLES)

Question 1:

In the given figure, ABCD is parallelogram, $AE \perp DC$ and $CF \perp AD$. If $AB = 16$ cm, $AE = 8$ cm and $CF = 10$ cm, find AD.



Answer 1:

In parallelogram ABCD, $CD = AB = 16 \text{ cm}$

[Opposite sides of a parallelogram are equal]

We know that

Area of a parallelogram = Base \times Corresponding altitude

Area of parallelogram ABCD = $CD \times AE = AD \times CF$

$16 \text{ cm} \times 8 \text{ cm} = AD \times 10 \text{ cm}$

$$AD = \frac{16 \times 8}{10} \text{ cm} = 12.8 \text{ cm}$$

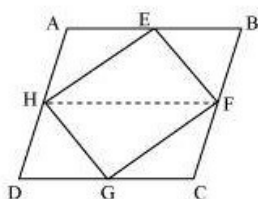
Thus, the length of AD is 12.8 cm.

Question 2:

If E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD show that

$$\text{ar}(\text{EFGH}) = \frac{1}{2} \text{ar}(\text{ABCD})$$

Answer 2:



Let us join HF.

In parallelogram ABCD,

$AD = BC$ and $AD \parallel BC$ (Opposite sides of a parallelogram are equal and parallel)

$AB = CD$ (Opposite sides of a parallelogram are equal)

$$\Rightarrow \frac{1}{2}AD = \frac{1}{2}BC \text{ and } AH \parallel BF$$

$$\Rightarrow AH = BF \text{ and } AH \parallel BF (\because H \text{ and } F \text{ are the mid-points of } AD \text{ and } BC)$$

Therefore, ABFH is a parallelogram.

Since $\triangle HEF$ and parallelogram ABFH are on the same base HF and between the same parallel lines AB and HF,

$$\therefore \text{Area}(\triangle HEF) = \frac{1}{2} \text{Area}(\text{ABFH}) \dots (1)$$

Similarly, it can be proved that

$$\text{Area}(\triangle HGF) = \frac{1}{2} \text{Area}(\text{HDCF}) \dots (2)$$

On adding equations (1) and (2), we obtain

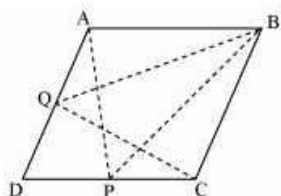
$$\begin{aligned} \text{Area}(\triangle HEF) + \text{Area}(\triangle HGF) &= \frac{1}{2} \text{Area}(\text{ABFH}) + \frac{1}{2} \text{Area}(\text{HDCF}) \\ &= \frac{1}{2} [\text{Area}(\text{ABFH}) + \text{Area}(\text{HDCF})] \end{aligned}$$

$$\Rightarrow \text{Area}(\text{EFGH}) = \frac{1}{2} \text{Area}(\text{ABCD})$$

Question 3:

P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that $\text{ar}(\triangle APB) = \text{ar}(\triangle BQC)$.

Answer 3:



It can be observed that $\triangle BQC$ and parallelogram ABCD lie on the same base BC and these are between the same parallel lines AD and BC.

$$\therefore \text{Area}(\triangle BQC) = \frac{1}{2} \text{Area}(\text{ABCD}) \dots (1)$$

Similarly, $\triangle APB$ and parallelogram ABCD lie on the same base AB and between the same parallel lines AB and DC.

$$\therefore \text{Area}(\triangle APB) = \frac{1}{2} \text{Area}(\text{ABCD}) \dots (2)$$

From equation (1) and (2), we obtain

$$\text{Area}(\triangle BQC) = \text{Area}(\triangle APB)$$

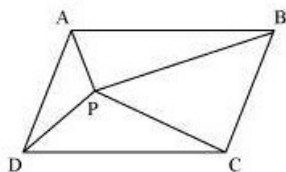
Question 4:

In the given figure, P is a point in the interior of a parallelogram ABCD. Show that

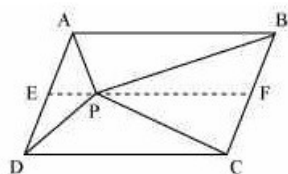
$$(i) \text{ar}(\triangle APB) + \text{ar}(\triangle PCD) = \frac{1}{2} \text{ar}(\text{ABCD})$$

$$(ii) \text{ar}(\triangle APD) + \text{ar}(\triangle PBC) = \text{ar}(\triangle APB) + \text{ar}(\triangle PCD)$$

[Hint: Through P, draw a line parallel to AB]



Answer 4:



(i) Let us draw a line segment EF, passing through point P and parallel to line segment AB.

In parallelogram ABCD,

$AB \parallel EF$ (By construction) ... (1)

ABCD is a parallelogram.

$\therefore AD \parallel BC$ (Opposite sides of a parallelogram)

$\Rightarrow AE \parallel BF$... (2)

From equations (1) and (2), we obtain

$AB \parallel EF$ and $AE \parallel BF$

Therefore, quadrilateral ABFE is a parallelogram.

It can be observed that $\triangle APB$ and parallelogram ABFE are lying on the same base AB and between the same parallel lines AB and EF.

$$\therefore \text{Area} (\triangle APB) = \frac{1}{2} \text{Area} (ABFE) \dots (3)$$

Similarly, for $\triangle PCD$ and parallelogram EFCD,

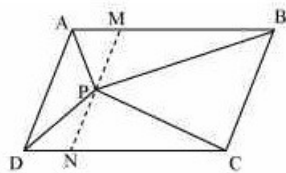
$$\text{Area} (\triangle PCD) = \frac{1}{2} \text{Area} (EFCD) \dots (4)$$

Adding equations (3) and (4), we obtain

$$\text{Area} (\triangle APB) + \text{Area} (\triangle PCD) = \frac{1}{2} [\text{Area} (ABFE) + \text{Area} (EFCD)]$$

$$\text{Area} (\triangle APB) + \text{Area} (\triangle PCD) = \frac{1}{2} \text{Area} (ABCD) \dots (5)$$

(ii)



Let us draw a line segment MN, passing through point P and parallel to line segment AD.

In parallelogram ABCD,

$MN \parallel AD$ (By construction) ... (6)

ABCD is a parallelogram.

$\therefore AB \parallel DC$ (Opposite sides of a parallelogram)

$\Rightarrow AM \parallel DN$... (7)

From equations (6) and (7), we obtain

$MN \parallel AD$ and $AM \parallel DN$

Therefore, quadrilateral AMND is a parallelogram.

It can be observed that $\triangle APD$ and parallelogram AMND are lying on the same base AD and between the same parallel lines AD and MN.

$\therefore \text{Area}(\triangle APD) = \frac{1}{2} \text{Area}(\text{AMND})$... (8)

Similarly, for $\triangle PCB$ and parallelogram MNCB,

$\text{Area}(\triangle PCB) = \frac{1}{2} \text{Area}(\text{MNCB})$... (9)

Adding equations (8) and (9), we obtain

$$\text{Area}(\triangle APD) + \text{Area}(\triangle PCB) = \frac{1}{2} [\text{Area}(\text{AMND}) + \text{Area}(\text{MNCB})]$$

$$\text{Area}(\triangle APD) + \text{Area}(\triangle PCB) = \frac{1}{2} \text{Area}(\text{ABCD}) \quad \dots (10)$$

On comparing equations (5) and (10), we obtain

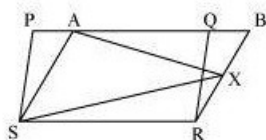
$$\text{Area}(\triangle APD) + \text{Area}(\triangle PBC) = \text{Area}(\triangle APB) + \text{Area}(\triangle PCD)$$

Question 5:

In the given figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that

(i) $\text{ar}(\text{PQRS}) = \text{ar}(\text{ABRS})$

(ii) $\text{ar}(\triangle AXS) = \frac{1}{2} \text{ar}(\text{PQRS})$



Answer 5:

(i) It can be observed that parallelogram PQRS and ABRS lie on the same base SR and also, these lie in between the same parallel lines SR and PB.

$$\therefore \text{Area (PQRS)} = \text{Area (ABRS)} \dots (1)$$

(ii) Consider $\triangle AXS$ and parallelogram ABRS.

As these lie on the same base and are between the same parallel lines AS and BR,

$$\therefore \text{Area } (\triangle AXS) = \frac{1}{2} \text{Area (ABRS)} \dots (2)$$

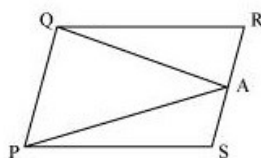
From equations (1) and (2), we obtain

$$\text{Area } (\triangle AXS) = \frac{1}{2} \text{Area (PQRS)}$$

Question 6:

A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Answer 6:



From the figure, it can be observed that point A divides the field into three parts. These parts are triangular in shape – $\triangle PSA$, $\triangle PAQ$, and $\triangle QRA$

$$\text{Area of } \triangle PSA + \text{Area of } \triangle PAQ + \text{Area of } \triangle QRA = \text{Area of } \parallel \text{gm PQRS} \dots (1)$$

We know that if a parallelogram and a triangle are on the same base and between the same parallels, then the area of the triangle is half the area of the parallelogram.

$$\therefore \text{Area } (\triangle PAQ) = \frac{1}{2} \text{Area (PQRS)} \dots (2)$$

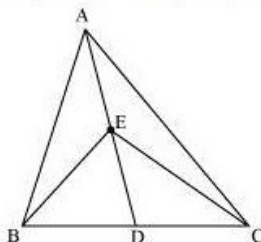
From equations (1) and (2), we obtain

$$\text{Area } (\triangle PSA) + \text{Area } (\triangle QRA) = \frac{1}{2} \text{Area (PQRS)} \dots (3)$$

Clearly, it can be observed that the farmer must sow wheat in triangular part PAQ and pulses in other two triangular parts PSA and QRA or wheat in triangular parts PSA and QRA and pulses in triangular parts PAQ.

Question 1:

In the given figure, E is any point on median AD of a $\triangle ABC$. Show that
 $\text{ar}(\triangle ABE) = \text{ar}(\triangle ACE)$



Answer 1:

AD is the median of $\triangle ABC$. Therefore, it will divide $\triangle ABC$ into two triangles of equal areas.

$$\therefore \text{Area}(\triangle ABD) = \text{Area}(\triangle ACD) \dots (1)$$

ED is the median of $\triangle EBC$.

$$\therefore \text{Area}(\triangle EBD) = \text{Area}(\triangle ECD) \dots (2)$$

On subtracting equation (2) from equation (1), we obtain

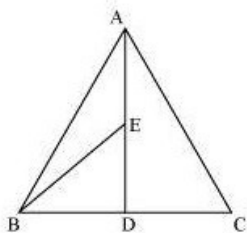
$$\text{Area}(\triangle ABD) - \text{Area}(\triangle EBD) = \text{Area}(\triangle ACD) - \text{Area}(\triangle ECD)$$

$$\text{Area}(\triangle ABE) = \text{Area}(\triangle ACE)$$

Question 2:

In a triangle ABC, E is the mid-point of median AD. Show that $\text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle ABC)$

Answer 2:



AD is the median of $\triangle ABC$. Therefore, it will divide $\triangle ABC$ into two triangles of equal areas.

$$\therefore \text{Area}(\triangle ABD) = \text{Area}(\triangle ACD)$$

$$\Rightarrow \text{Area}(\triangle ABD) = \frac{1}{2} \text{Area}(\triangle ABC) \dots (1)$$

In $\triangle ABD$, E is the mid-point of AD. Therefore, BE is the median.

$$\therefore \text{Area}(\triangle BED) = \text{Area}(\triangle ABE)$$

$$\Rightarrow \text{Area}(\triangle BED) = \frac{1}{2} \text{Area}(\triangle ABD)$$

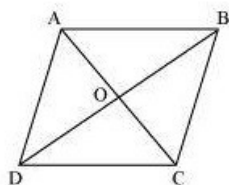
$$\Rightarrow \text{Area}(\triangle BED) = \frac{1}{2} \times \frac{1}{2} \text{Area}(\triangle ABC) \text{ [From equation (1)]}$$

$$\Rightarrow \text{Area}(\triangle BED) = \frac{1}{4} \text{Area}(\triangle ABC)$$

Question 3:

Show that the diagonals of a parallelogram divide it into four triangles of equal area.

Answer 3:



We know that diagonals of parallelogram bisect each other.

Therefore, O is the mid-point of AC and BD.

BO is the median in $\triangle ABC$. Therefore, it will divide it into two triangles of equal areas.

$$\therefore \text{Area}(\triangle AOB) = \text{Area}(\triangle BOC) \dots (1)$$

In $\triangle BCD$, CO is the median.

$$\therefore \text{Area}(\triangle BOC) = \text{Area}(\triangle COD) \dots (2)$$

$$\text{Similarly, Area}(\triangle COD) = \text{Area}(\triangle AOD) \dots (3)$$

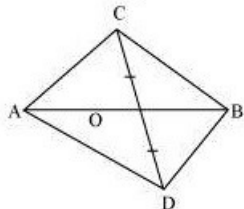
From equations (1), (2), and (3), we obtain

$$\text{Area}(\triangle AOB) = \text{Area}(\triangle BOC) = \text{Area}(\triangle COD) = \text{Area}(\triangle AOD)$$

Therefore, it is evident that the diagonals of a parallelogram divide it into four triangles of equal area.

Question 4:

In the given figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bisected by AB at O, show that $\text{ar}(\triangle ABC) = \text{ar}(\triangle ABD)$.



Answer 4:

Consider $\triangle ACD$.

Line-segment CD is bisected by AB at O. Therefore, AO is the median of $\triangle ACD$.

$$\therefore \text{Area}(\triangle ACO) = \text{Area}(\triangle ADO) \dots (1)$$

Considering $\triangle BCD$, BO is the median.

$$\therefore \text{Area}(\triangle BCO) = \text{Area}(\triangle BDO) \dots (2)$$

Adding equations (1) and (2), we obtain

$$\text{Area}(\triangle ACO) + \text{Area}(\triangle BCO) = \text{Area}(\triangle ADO) + \text{Area}(\triangle BDO)$$

$$\Rightarrow \text{Area}(\triangle ABC) = \text{Area}(\triangle ABD)$$

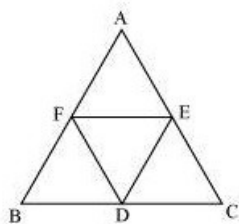
Question 5:

D, E and F are respectively the mid-points of the sides BC, CA and AB of a $\triangle ABC$. Show that
(i) BDEF is a parallelogram.

$$(ii) \text{ar}(\triangle DEF) = \frac{1}{4} \text{ar}(\triangle ABC)$$

$$(iii) \text{ar}(\text{BDEF}) = \frac{1}{2} \text{ar}(\triangle ABC)$$

Answer 5:



(i) In $\triangle ABC$,
E and F are the mid-points of side AC and AB respectively.
Therefore, $EF \parallel BC$ and $EF = \frac{1}{2} BC$ (Mid-point theorem)
However, $BD = \frac{1}{2} BC$ (D is the mid-point of BC)
Therefore, $BD = EF$ and $BD \parallel EF$
Therefore, BDEF is a parallelogram.

(ii) Using the result obtained above, it can be said that quadrilaterals BDEF, DCEF, AFDE are parallelograms.

We know that diagonal of a parallelogram divides it into two triangles of equal area.

$\therefore \text{Area}(\triangle BFD) = \text{Area}(\triangle DEF)$ (For parallelogram BDEF)
 $\text{Area}(\triangle CDE) = \text{Area}(\triangle DEF)$ (For parallelogram DCEF)
 $\text{Area}(\triangle AFE) = \text{Area}(\triangle DEF)$ (For parallelogram AFDE)
 $\therefore \text{Area}(\triangle AFE) = \text{Area}(\triangle BFD) = \text{Area}(\triangle CDE) = \text{Area}(\triangle DEF)$

Also,

$\text{Area}(\triangle AFE) + \text{Area}(\triangle BFD) + \text{Area}(\triangle CDE) + \text{Area}(\triangle DEF) = \text{Area}(\triangle ABC)$
 $\Rightarrow \text{Area}(\triangle DEF) + \text{Area}(\triangle DEF) + \text{Area}(\triangle DEF) + \text{Area}(\triangle DEF) = \text{Area}(\triangle ABC)$
 $\Rightarrow 4 \text{Area}(\triangle DEF) = \text{Area}(\triangle ABC)$
 $\Rightarrow \text{Area}(\triangle DEF) = \frac{1}{4} \text{Area}(\triangle ABC)$

(iii) $\text{Area}(\text{parallelogram BDEF}) = \text{Area}(\triangle DEF) + \text{Area}(\triangle BFD)$
 $\Rightarrow \text{Area}(\text{parallelogram BDEF}) = \text{Area}(\triangle DEF) + \text{Area}(\triangle DEF)$
 $\Rightarrow \text{Area}(\text{parallelogram BDEF}) = 2 \text{Area}(\triangle DEF)$
 $\Rightarrow \text{Area}(\text{parallelogram BDEF}) = 2 \times \frac{1}{4} \text{Area}(\triangle ABC)$
 $\Rightarrow \text{Area}(\text{parallelogram BDEF}) = \frac{1}{2} \text{Area}(\triangle ABC)$

Question 6:

In the given figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that $OB = OD$.

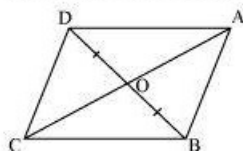
If $AB = CD$, then show that:

(i) $\text{ar}(\triangle DOC) = \text{ar}(\triangle AOB)$

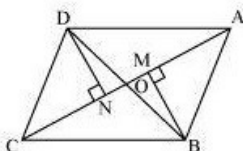
(ii) $\text{ar}(\triangle DCB) = \text{ar}(\triangle ACB)$

(iii) $DA \parallel CB$ or ABCD is a parallelogram.

[Hint: From D and B, draw perpendiculars to AC.]



Answer 6:



Let us draw $DN \perp AC$ and $BM \perp AC$.

(i) In $\triangle DON$ and $\triangle BOM$,

$\angle DNO = \angle BMO$ (By construction)

$\angle DON = \angle BOM$ (Vertically opposite angles)

$OD = OB$ (Given)

By AAS congruence rule,

$\triangle DON \cong \triangle BOM$

$\therefore DN = BM \dots (1)$

We know that congruent triangles have equal areas.

$\therefore \text{Area}(\triangle DON) = \text{Area}(\triangle BOM) \dots (2)$

In $\triangle DNC$ and $\triangle BMA$,

$\angle DNC = \angle BMA$ (By construction)

$CD = AB$ (Given)

$DN = BM$ [Using equation (1)]

$\therefore \triangle DNC \cong \triangle BMA$ (RHS congruence rule)

$\Rightarrow \text{Area}(\triangle DNC) = \text{Area}(\triangle BMA) \dots (3)$

On adding equations (2) and (3), we obtain

$\text{Area}(\triangle DON) + \text{Area}(\triangle DNC) = \text{Area}(\triangle BOM) + \text{Area}(\triangle BMA)$

Therefore, $\text{Area}(\triangle DOC) = \text{Area}(\triangle AOB)$

(ii) We obtained,

$\text{Area}(\triangle DOC) = \text{Area}(\triangle AOB)$

$\Rightarrow \text{Area}(\triangle DOC) + \text{Area}(\triangle OCB) = \text{Area}(\triangle AOB) + \text{Area}(\triangle OCB)$

(Adding $\text{Area}(\triangle OCB)$ to both sides)

$\Rightarrow \text{Area}(\triangle DCB) = \text{Area}(\triangle ACB)$

(iii) We obtained,

$$\text{Area } (\triangle DCB) = \text{Area } (\triangle ACB)$$

If two triangles have the same base and equal areas, then these will lie between the same parallels.

$$\therefore DA \parallel CB \dots (4)$$

In $\triangle DOA$ and $\triangle BOC$,

$$\angle DOA = \angle BOC \text{ (Vertically opposite angles)}$$

$$OD = OB \text{ (Given)}$$

$$\angle ODA = \angle OBC \text{ (Alternate opposite angles)}$$

By ASA congruence rule,

$$\triangle DOA \cong \triangle BOC$$

$$\therefore DA = BC \dots (5)$$

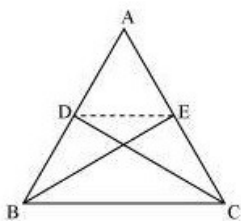
In quadrilateral ABCD, one pair of opposite sides is equal and parallel ($AD = BC$)

Therefore, ABCD is a parallelogram.

Question 7:

D and E are points on sides AB and AC respectively of $\triangle ABC$ such that $\text{ar}(\triangle DBC) = \text{ar}(\triangle EBC)$. Prove that $DE \parallel BC$.

Answer 7:



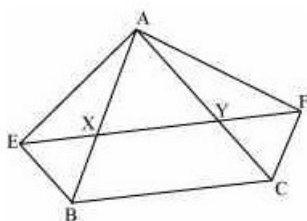
Since $\triangle BCE$ and $\triangle BCD$ are lying on a common base BC and also have equal areas, $\triangle BCE$ and $\triangle BCD$ will lie between the same parallel lines.

$$\therefore DE \parallel BC$$

Question 8:

XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that
 $\text{ar}(\triangle ABE) = \text{ar}(\triangle ACF)$

Answer 8:



It is given that
 $XY \parallel BC \Rightarrow EY \parallel BC$
 $BE \parallel AC \Rightarrow BE \parallel CY$
 Therefore, EBCY is a parallelogram.
 It is given that

$XY \parallel BC \Rightarrow XF \parallel BC$
 $FC \parallel AB \Rightarrow FC \parallel XB$
 Therefore, BCFX is a parallelogram.

Parallelograms EBCY and BCFX are on the same base BC and between the same parallels BC and EF.

$$\therefore \text{Area}(\triangle EBCY) = \text{Area}(\triangle BCFX) \dots (1)$$

Consider parallelogram EBCY and $\triangle AEB$

These lie on the same base BE and are between the same parallels BE and AC.

$$\therefore \text{Area}(\triangle ABE) = \frac{1}{2} \text{Area}(\triangle EBCY) \dots (2)$$

Also, parallelogram BCFX and $\triangle ACF$ are on the same base CF and between the same parallels CF and AB.

$$\therefore \text{Area}(\triangle ACF) = \frac{1}{2} \text{Area}(\triangle BCFX) \dots (3)$$

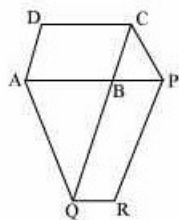
From equations (1), (2), and (3), we obtain

$$\text{Area}(\triangle ABE) = \text{Area}(\triangle ACF)$$

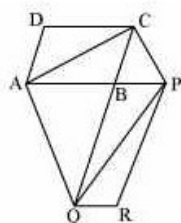
Question 9:

The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see the following figure). Show that
 $\text{ar}(\text{ABCD}) = \text{ar}(\text{PBQR})$.

[Hint: Join AC and PQ. Now compare area (ACQ) and area (APQ)]



Answer 9:



Let us join AC and PQ.

ΔACQ and ΔAPQ are on the same base AQ and between the same parallels AQ and CP.

$$\therefore \text{Area}(\Delta ACQ) = \text{Area}(\Delta APQ)$$

$$\Rightarrow \text{Area}(\Delta ACQ) - \text{Area}(\Delta ABQ) = \text{Area}(\Delta APQ) - \text{Area}(\Delta ABQ)$$

$$\Rightarrow \text{Area}(\Delta ABC) = \text{Area}(\Delta QBP) \dots (1)$$

Since AC and PQ are diagonals of parallelograms ABCD and PBQR respectively,

$$\therefore \text{Area}(\Delta ABC) = \frac{1}{2} \text{Area}(\text{ABCD}) \dots (2)$$

$$\text{Area}(\Delta QBP) = \frac{1}{2} \text{Area}(\text{PBQR}) \dots (3)$$

From equations (1), (2), and (3), we obtain

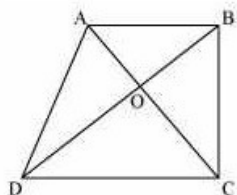
$$\frac{1}{2} \text{Area}(\text{ABCD}) = \frac{1}{2} \text{Area}(\text{PBQR})$$

$$\text{Area}(\text{ABCD}) = \text{Area}(\text{PBQR})$$

Question 10:

Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at O. Prove that $\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$.

Answer 10:



It can be observed that $\triangle DAC$ and $\triangle DBC$ lie on the same base DC and between the same parallels AB and CD.

$$\therefore \text{Area}(\triangle DAC) = \text{Area}(\triangle DBC)$$

$$\Rightarrow \text{Area}(\triangle DAC) - \text{Area}(\triangle DOC) = \text{Area}(\triangle DBC) - \text{Area}(\triangle DOC)$$

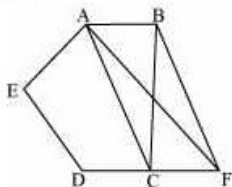
$$\Rightarrow \text{Area}(\triangle AOD) = \text{Area}(\triangle BOC)$$

Question 11:

In the given figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that

(i) $\text{ar}(\triangle ACB) = \text{ar}(\triangle ACF)$

(ii) $\text{ar}(\triangle AEDF) = \text{ar}(\text{pentagon } ABCDE)$



Answer 11:

(i) $\triangle ACB$ and $\triangle ACF$ lie on the same base AC and are between the same parallels AC and BF.

$$\therefore \text{Area}(\triangle ACB) = \text{Area}(\triangle ACF)$$

(ii) It can be observed that

$$\text{Area}(\triangle ACB) = \text{Area}(\triangle ACF)$$

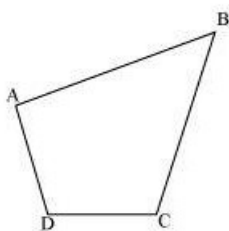
$$\Rightarrow \text{Area}(\triangle ACB) + \text{Area}(\triangle ACDE) = \text{Area}(\triangle ACF) + \text{Area}(\triangle ACDE)$$

$$\Rightarrow \text{Area}(\text{pentagon } ABCDE) = \text{Area}(\triangle AEDF)$$

Question 12:

A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Answer 12:

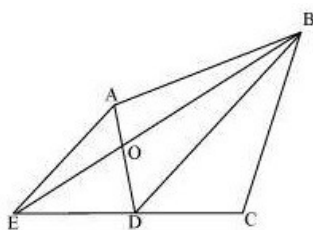


Let quadrilateral ABCD be the original shape of the field.

The proposal may be implemented as follows.

Join diagonal BD and draw a line parallel to BD through point A. Let it meet the extended side CD of ABCD at point E. Join BE and AD. Let them intersect each other at O. Then, portion $\triangle AOB$ can be cut from the original field so that the new shape of the field will be $\triangle BCE$. (See figure)

We have to prove that the area of $\triangle AOB$ (portion that was cut so as to construct Health Centre) is equal to the area of $\triangle DEO$ (portion added to the field so as to make the area of the new field so formed equal to the area of the original field)



It can be observed that $\triangle DEB$ and $\triangle DAB$ lie on the same base BD and are between the same parallels BD and AE.

$$\therefore \text{Area}(\triangle DEB) = \text{Area}(\triangle DAB)$$

$$\Rightarrow \text{Area}(\triangle DEB) - \text{Area}(\triangle DOB) = \text{Area}(\triangle DAB) - \text{Area}(\triangle DOB)$$

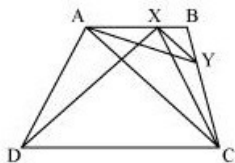
$$\Rightarrow \text{Area}(\triangle DEO) = \text{Area}(\triangle AOB)$$

Question 13:

ABCD is a trapezium with $AB \parallel DC$. A line parallel to AC intersects AB at X and BC at Y. Prove that $\text{ar}(\triangle ADX) = \text{ar}(\triangle ACY)$.

[Hint: Join CX.]

Answer 13:



It can be observed that $\triangle ADX$ and $\triangle ACX$ lie on the same base AX and are between the same parallels AB and DC .

$$\therefore \text{Area}(\triangle ADX) = \text{Area}(\triangle ACX) \dots (1)$$

$\triangle ACY$ and $\triangle ACX$ lie on the same base AC and are between the same parallels AC and XY .

$$\therefore \text{Area}(\triangle ACY) = \text{Area}(\triangle ACX) \dots (2)$$

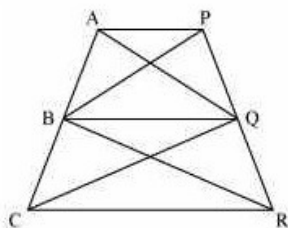
From equations (1) and (2), we obtain

$$\text{Area}(\triangle ADX) = \text{Area}(\triangle ACY)$$

Question 14:

In the given figure, $AP \parallel BQ \parallel CR$. Prove that $\text{ar}(\triangle AQC) = \text{ar}(\triangle PBR)$.

Answer 14:



Since $\triangle ABQ$ and $\triangle PBQ$ lie on the same base BQ and are between the same parallels AP and BQ ,

$$\therefore \text{Area}(\triangle ABQ) = \text{Area}(\triangle PBQ) \dots (1)$$

Again, $\triangle BCQ$ and $\triangle BRQ$ lie on the same base BQ and are between the same parallels BQ and CR .

$$\therefore \text{Area}(\triangle BCQ) = \text{Area}(\triangle BRQ) \dots (2)$$

On adding equations (1) and (2), we obtain

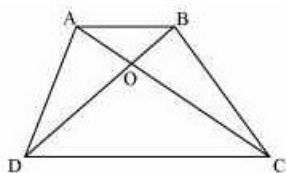
$$\text{Area}(\triangle ABQ) + \text{Area}(\triangle BCQ) = \text{Area}(\triangle PBQ) + \text{Area}(\triangle BRQ)$$

$$\Rightarrow \text{Area}(\triangle AQC) = \text{Area}(\triangle PBR)$$

Question 15:

Diagonals AC and BD of a quadrilateral $ABCD$ intersect at O in such a way that $\text{ar}(\triangle AOD) = \text{ar}(\triangle BOC)$. Prove that $ABCD$ is a trapezium.

Answer 15:



It is given that

$$\text{Area } (\triangle AOD) = \text{Area } (\triangle BOC)$$

$$\text{Area } (\triangle AOD) + \text{Area } (\triangle AOB) = \text{Area } (\triangle BOC) + \text{Area } (\triangle AOB)$$

$$\text{Area } (\triangle ADB) = \text{Area } (\triangle ACB)$$

We know that triangles on the same base having areas equal to each other lie between the same parallels.

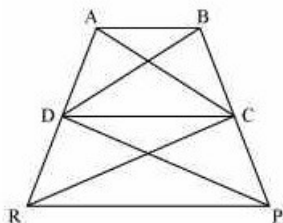
Therefore, these triangles, $\triangle ADB$ and $\triangle ACB$, are lying between the same parallels.

i.e., $AB \parallel CD$

Therefore, ABCD is a trapezium.

Question 16:

In the given figure, $\text{ar}(\text{DRC}) = \text{ar}(\text{DPC})$ and $\text{ar}(\text{BDP}) = \text{ar}(\text{ARC})$. Show that both the quadrilaterals ABCD and DCPR are trapeziums.



Answer 16:

It is given that

$$\text{Area } (\triangle DRC) = \text{Area } (\triangle DPC)$$

As $\triangle DRC$ and $\triangle DPC$ lie on the same base DC and have equal areas, therefore, they must lie between the same parallel lines.

$$\therefore DC \parallel RP$$

Therefore, DCPR is a trapezium.

It is also given that

$$\text{Area } (\triangle BDP) = \text{Area } (\triangle ARC)$$

$$\Rightarrow \text{Area } (\triangle BDP) - \text{Area } (\triangle DPC) = \text{Area } (\triangle ARC) - \text{Area } (\triangle DRC)$$

$$\Rightarrow \text{Area } (\triangle BDC) = \text{Area } (\triangle ADC)$$

Since $\triangle BDC$ and $\triangle ADC$ are on the same base CD and have equal areas, they must lie between the same parallel lines.

$$\therefore AB \parallel CD$$

Therefore, ABCD is a trapezium.

MATHS -9 (CH-09 -9.4- PARALLELOGRAMS & TRIANGLES)

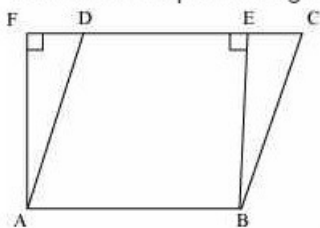
Question 1:

Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Answer 1:

As the parallelogram and the rectangle have the same base and equal area, therefore, these will also lie between the same parallels.

Consider the parallelogram ABCD and rectangle ABEF as follows.



Here, it can be observed that parallelogram ABCD and rectangle ABEF are between the same parallels AB and CF.

We know that opposite sides of a parallelogram or a rectangle are of equal lengths. Therefore,

$AB = EF$ (For rectangle)

$AB = CD$ (For parallelogram)

$\therefore CD = EF$

$\Rightarrow AB + CD = AB + EF \dots (1)$

Of all the line segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.

$\therefore AF < AD$

And similarly, $BE < BC$

$\therefore AF + BE < AD + BC \dots (2)$

From equations (1) and (2), we obtain

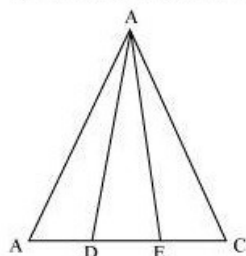
$AB + EF + AF + BE < AD + BC + AB + CD$

Perimeter of rectangle ABEF < Perimeter of parallelogram ABCD

Question 2:

In the following figure, D and E are two points on BC such that $BD = DE = EC$. Show that $\text{ar}(\triangle ABD) = \text{ar}(\triangle ADE) = \text{ar}(\triangle AEC)$.

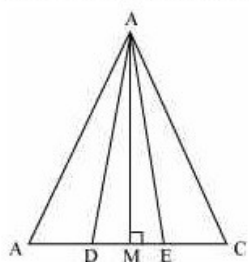
Can you answer the question that you have left in the 'Introduction' of this chapter, whether the field of *Budhia* has been actually divided into three parts of equal area?



[Remark: Note that by taking $BD = DE = EC$, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into n equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide $\triangle ABC$ into n triangles of equal areas.]

Answer 2:

Let us draw a line segment $AM \perp BC$.



We know that,

$$\text{Area of a triangle} = \frac{1}{2} \times \text{Base} \times \text{Altitude}$$

$$\text{Area } (\triangle ADE) = \frac{1}{2} \times DE \times AM$$

$$\text{Area } (\triangle ABD) = \frac{1}{2} \times BD \times AM$$

$$\text{Area } (\triangle AEC) = \frac{1}{2} \times EC \times AM$$

It is given that $DE = BD = EC$

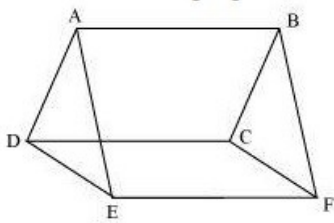
$$\Rightarrow \frac{1}{2} \times DE \times AM = \frac{1}{2} \times BD \times AM = \frac{1}{2} \times EC \times AM$$

$$\Rightarrow \text{Area } (\triangle ADE) = \text{Area } (\triangle ABD) = \text{Area } (\triangle AEC)$$

It can be observed that *Budhia* has divided her field into 3 equal parts.

Question 3:

In the following figure, ABCD, DCFE and ABFE are parallelograms. Show that $\text{ar} (\triangle ADE) = \text{ar} (\triangle BCF)$.



Answer 3:

It is given that ABCD is a parallelogram. We know that opposite sides of a parallelogram are equal.

$$\therefore AD = BC \dots (1)$$

Similarly, for parallelograms DCEF and ABFE, it can be proved that

$$DE = CF \dots (2)$$

$$\text{And, } EA = FB \dots (3)$$

In $\triangle ADE$ and $\triangle BCF$,

$$AD = BC \text{ [Using equation (1)]}$$

$$DE = CF \text{ [Using equation (2)]}$$

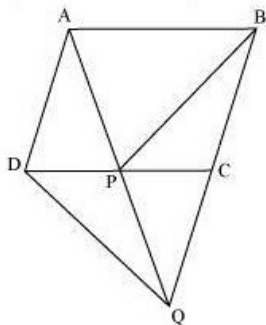
$$EA = FB \text{ [Using equation (3)]}$$

$$\therefore \triangle ADE \cong \triangle BCF \text{ (SSS congruence rule)}$$

$$\Rightarrow \text{Area } (\triangle ADE) = \text{Area } (\triangle BCF)$$

Question 4:

In the following figure, ABCD is parallelogram and BC is produced to a point Q such that $AD = CQ$. If AQ intersect DC at P, show that $\text{ar}(\triangle BPC) = \text{ar}(\triangle DPQ)$.
[Hint: Join AC.]

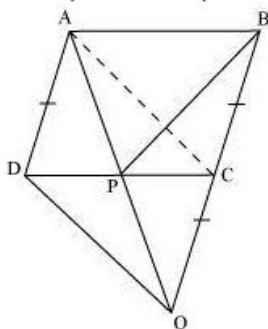


Answer 4:

It is given that ABCD is a parallelogram.

$AD \parallel BC$ and $AB \parallel DC$ (Opposite sides of a parallelogram are parallel to each other)

Join point A to point C.



Consider $\triangle APC$ and $\triangle BPC$

$\triangle APC$ and $\triangle BPC$ are lying on the same base PC and between the same parallels PC and AB.

Therefore,

$$\text{Area}(\triangle APC) = \text{Area}(\triangle BPC) \dots (1)$$

In quadrilateral ACDQ, it is given that

$$AD = CQ$$

Since ABCD is a parallelogram,

$AD \parallel BC$ (Opposite sides of a parallelogram are parallel)

CQ is a line segment which is obtained when line segment BC is produced.

$$\therefore AD \parallel CQ$$

We have,

$$AC = DQ \text{ and } AC \parallel DQ$$

Hence, ACQD is a parallelogram.

Consider $\triangle DCQ$ and $\triangle ACQ$

These are on the same base CQ and between the same parallels CQ and AD. Therefore,

$$\text{Area}(\triangle DCQ) = \text{Area}(\triangle ACQ)$$

$$\Rightarrow \text{Area}(\triangle DCQ) - \text{Area}(\triangle PQC) = \text{Area}(\triangle ACQ) - \text{Area}(\triangle PQC)$$

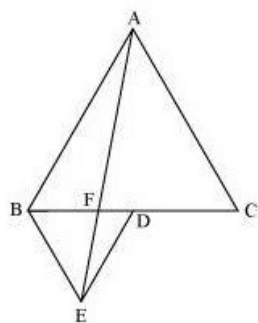
$$\Rightarrow \text{Area}(\triangle DPQ) = \text{Area}(\triangle APC) \dots (2)$$

From equations (1) and (2), we obtain

$$\text{Area}(\triangle BPC) = \text{Area}(\triangle DPQ)$$

Question 5:

In the following figure, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that



$$(i) \text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$$

$$(ii) \text{ar}(\triangle BDE) = \frac{1}{2} \text{ar}(\triangle BAE)$$

$$(iii) \text{ar}(\triangle ABC) = 2 \text{ar}(\triangle BEC)$$

$$(iv) \text{ar}(\triangle BFE) = \text{ar}(\triangle AFD)$$

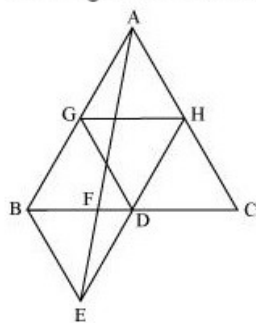
$$(v) \text{ar}(\triangle BFE) = 2 \text{ar}(\triangle FED)$$

$$(vi) \text{ar}(\triangle FED) = \frac{1}{8} \text{ar}(\triangle AFC)$$

Answer 5:

(i) Let G and H be the mid-points of side AB and AC respectively.

Line segment GH is joining the mid-points. Therefore, it will be parallel to third side BC and also its length will be half of the length of BC (mid-point theorem).



$$\Rightarrow GH = \frac{1}{2} BC \text{ and } GH \parallel BD$$

$$\Rightarrow GH = BD = DC \text{ and } GH \parallel BD \text{ (D is the mid-point of BC)}$$

Consider quadrilateral GHDB.

$GH \parallel BD$ and $GH = BD$

Two line segments joining two parallel line segments of equal length will also be equal and parallel to each other.

Therefore, $BG = DH$ and $BG \parallel DH$

Hence, quadrilateral GHDB is a parallelogram.

We know that in a parallelogram, the diagonal bisects it into two triangles of equal area.

Hence, $\text{Area}(\triangle BDG) = \text{Area}(\triangle HGD)$

Similarly, it can be proved that quadrilaterals DCHG, GDHA, and BEDG are parallelograms and their respective diagonals are dividing them into two triangles of equal area.

$$\text{ar}(\triangle GDH) = \text{ar}(\triangle CHD) \text{ (For parallelogram DCHG)}$$

$$\text{ar}(\triangle GDH) = \text{ar}(\triangle HAG) \text{ (For parallelogram GDHA)}$$

$$\text{ar}(\triangle BDE) = \text{ar}(\triangle DBG) \text{ (For parallelogram BEDG)}$$

$$\text{ar}(\triangle ABC) = \text{ar}(\triangle BDG) + \text{ar}(\triangle GDH) + \text{ar}(\triangle DCH) + \text{ar}(\triangle AGH)$$

$$\text{ar}(\triangle ABC) = 4 \times \text{ar}(\triangle BDE)$$

$$\text{Hence, } \text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$$

(ii) Area ($\triangle BDE$) = Area ($\triangle AED$) (Common base DE and $DE \parallel AB$)

$$\text{Area } (\triangle BDE) - \text{Area } (\triangle FED) = \text{Area } (\triangle AED) - \text{Area } (\triangle FED)$$

$$\text{Area } (\triangle BEF) = \text{Area } (\triangle AFD) \quad (1)$$

$$\text{Area } (\triangle ABD) = \text{Area } (\triangle ABF) + \text{Area } (\triangle AFD)$$

$$\text{Area } (\triangle ABD) = \text{Area } (\triangle ABF) + \text{Area } (\triangle BEF) \quad [\text{From equation (1)}]$$

$$\text{Area } (\triangle ABD) = \text{Area } (\triangle ABE) \quad (2)$$

AD is the median in $\triangle ABC$.

$$\text{ar } (\triangle ABD) = \frac{1}{2} \text{ar } (\triangle ABC)$$

$$= \frac{4}{2} \text{ar } (\triangle BDE) \quad (\text{As proved earlier})$$

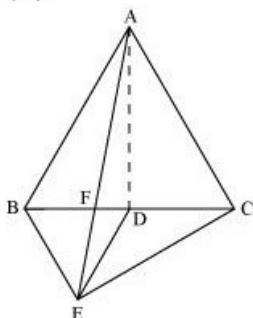
$$\text{ar } (\triangle ABD) = 2 \text{ar } (\triangle BDE) \quad (3)$$

From (2) and (3), we obtain

$$2 \text{ar } (\triangle BDE) = \text{ar } (\triangle ABE)$$

$$\text{Or, ar } (\triangle BDE) = \frac{1}{2} \text{ar } (\triangle ABE)$$

(iii)



$\text{ar } (\triangle ABE) = \text{ar } (\triangle BEC)$ (Common base BE and $BE \parallel AC$)

$$\text{ar } (\triangle ABF) + \text{ar } (\triangle BEF) = \text{ar } (\triangle BEC)$$

Using equation (1), we obtain

$$\text{ar } (\triangle ABF) + \text{ar } (\triangle AFD) = \text{ar } (\triangle BEC)$$

$$\text{ar } (\triangle ABD) = \text{ar } (\triangle BEC)$$

$$\frac{1}{2} \text{ar } (\triangle ABC) = \text{ar } (\triangle BEC)$$

$$\text{ar } (\triangle ABC) = 2 \text{ar } (\triangle BEC)$$

(iv) It is seen that $\triangle BDE$ and $\triangle AED$ lie on the same base (DE) and between the parallels DE and AB.

$$\therefore \text{ar } (\triangle BDE) = \text{ar } (\triangle AED)$$

$$\Rightarrow \text{ar } (\triangle BDE) - \text{ar } (\triangle FED) = \text{ar } (\triangle AED) - \text{ar } (\triangle FED)$$

$$\therefore \text{ar } (\triangle BFE) = \text{ar } (\triangle AFD)$$

(v) Let h be the height of vertex E, corresponding to the side BD in $\triangle BDE$.

Let H be the height of vertex A, corresponding to the side BC in $\triangle ABC$.

In (i), it was shown that $\text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC)$.

$$\therefore \frac{1}{2} \times BD \times h = \frac{1}{4} \left(\frac{1}{2} \times BC \times H \right)$$

$$\Rightarrow BD \times h = \frac{1}{4} (2BD \times H)$$

$$\Rightarrow h = \frac{1}{2} H$$

In (iv), it was shown that $\text{ar}(\triangle BFE) = \text{ar}(\triangle AFD)$.

$\therefore \text{ar}(\triangle BFE) = \text{ar}(\triangle AFD)$

$$= \frac{1}{2} \times FD \times H = \frac{1}{2} \times FD \times 2h = 2 \left(\frac{1}{2} \times FD \times h \right)$$

$$= 2 \text{ar}(\triangle FED)$$

Hence, $\text{ar}(\triangle BFE) = 2 \text{ar}(\triangle FED)$.

(vi) $\text{Area}(\triangle AFC) = \text{area}(\triangle AFD) + \text{area}(\triangle ADC)$

$$= \text{ar}(\triangle BFE) + \frac{1}{2} \text{ar}(\triangle ABC) \quad \left[\text{In (iv), } \text{ar}(\triangle BFE) = \text{ar}(\triangle AFD) ; \text{AD is median of } \triangle ABC \right]$$

$$= \text{ar}(\triangle BFE) + \frac{1}{2} \times 4 \text{ar}(\triangle BDE) \quad \left[\text{In (i), } \text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC) \right]$$

Now, by (v),

$$= \text{ar}(\triangle BFE) + 2 \text{ar}(\triangle BDE) \quad \dots (5)$$

$$\text{ar}(\triangle BFE) = 2 \text{ar}(\triangle FED). \dots (6)$$

$$\text{ar}(\triangle BDE) = \text{ar}(\triangle BFE) + \text{ar}(\triangle FED) = 2 \text{ar}(\triangle FED) + \text{ar}(\triangle FED) = 3 \text{ar}(\triangle FED) \quad \dots (7)$$

Therefore, from equations (5), (6), and (7), we get:

$$\text{ar}(\triangle AFC) = 2 \text{ar}(\triangle FED) + 2 \times 3 \text{ar}(\triangle FED) = 8 \text{ar}(\triangle FED)$$

$$\therefore \text{ar}(\triangle AFC) = 8 \text{ar}(\triangle FED)$$

$$\text{Hence, } \text{ar}(\triangle FED) = \frac{1}{8} \text{ar}(\triangle AFC)$$

Question 6:

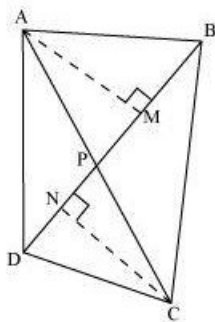
Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that

$$\text{ar}(\text{APB}) \times \text{ar}(\text{CPD}) = \text{ar}(\text{APD}) \times \text{ar}(\text{BPC})$$

[Hint: From A and C, draw perpendiculars to BD]

Answer 6:

Let us draw $AM \perp BD$ and $CN \perp BD$



$$\text{Area of a triangle} = \frac{1}{2} \times \text{Base} \times \text{Altitude}$$

$$\begin{aligned} \text{ar}(\text{APB}) \times \text{ar}(\text{CPD}) &= \left[\frac{1}{2} \times BP \times AM \right] \times \left[\frac{1}{2} \times PD \times CN \right] \\ &= \frac{1}{4} \times BP \times AM \times PD \times CN \end{aligned}$$

$$\begin{aligned} \text{ar}(\text{APD}) \times \text{ar}(\text{BPC}) &= \left[\frac{1}{2} \times PD \times AM \right] \times \left[\frac{1}{2} \times CN \times BP \right] \\ &= \frac{1}{4} \times PD \times AM \times CN \times BP \\ &= \frac{1}{4} \times BP \times AM \times PD \times CN \end{aligned}$$

$$\therefore \text{ar}(\text{APB}) \times \text{ar}(\text{CPD}) = \text{ar}(\text{APD}) \times \text{ar}(\text{BPC})$$

Question 7:

P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that

$$(i) \text{ar}(\text{PRQ}) = \frac{1}{2} \text{ar}(\text{ARC}) \quad (ii) \text{ar}(\text{RQC}) = \frac{3}{8} \text{ar}(\text{ABC})$$

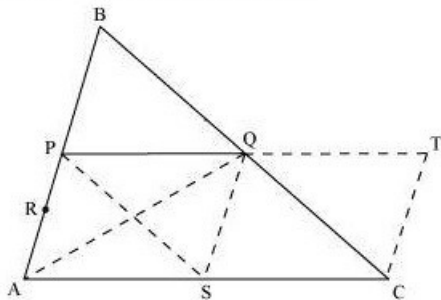
$$(iii) \text{ar}(\text{PBQ}) = \text{ar}(\text{ARC})$$

Answer 7:

Take a point S on AC such that S is the mid-point of AC.

Extend PQ to T such that PQ = QT.

Join TC, QS, PS, and AQ.



In $\triangle ABC$, P and Q are the mid-points of AB and BC respectively. Hence, by using mid-point theorem, we obtain

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC$$

$\Rightarrow PQ \parallel AS$ and $PQ = AS$ (As S is the mid-point of AC)

$\therefore PQSA$ is a parallelogram. We know that diagonals of a parallelogram bisect it into equal areas of triangles.

$$\therefore \text{ar}(\triangle PAS) = \text{ar}(\triangle SQP) = \text{ar}(\triangle PAQ) = \text{ar}(\triangle SQA)$$

Similarly, it can also be proved that quadrilaterals PSCQ, QSCT, and PSQB are also parallelograms and therefore,

$$\text{ar}(\triangle PSQ) = \text{ar}(\triangle CQS) \text{ (For parallelogram PSCQ)}$$

$$\text{ar}(\triangle QSC) = \text{ar}(\triangle CTQ) \text{ (For parallelogram QSCT)}$$

$$\text{ar}(\triangle PSQ) = \text{ar}(\triangle QBP) \text{ (For parallelogram PSQB)}$$

Thus,

$$\text{ar}(\triangle PAS) = \text{ar}(\triangle SQP) = \text{ar}(\triangle PAQ) = \text{ar}(\triangle SQA) = \text{ar}(\triangle QSC) = \text{ar}(\triangle CTQ) = \text{ar}(\triangle QBP) \dots (1)$$

$$\text{Also, ar}(\triangle ABC) = \text{ar}(\triangle PBQ) + \text{ar}(\triangle PAS) + \text{ar}(\triangle PQS) + \text{ar}(\triangle QSC)$$

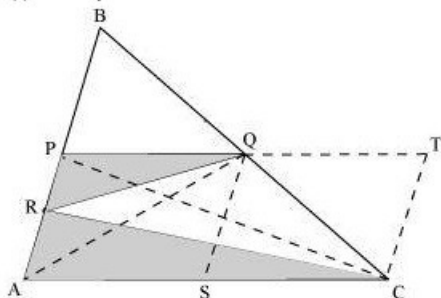
$$\text{ar}(\triangle ABC) = \text{ar}(\triangle PBQ) + \text{ar}(\triangle PBQ) + \text{ar}(\triangle PBQ) + \text{ar}(\triangle PBQ)$$

$$= \text{ar}(\triangle PBQ) + \text{ar}(\triangle PBQ) + \text{ar}(\triangle PBQ) + \text{ar}(\triangle PBQ)$$

$$= 4 \text{ ar}(\triangle PBQ)$$

$$\Rightarrow \text{ar}(\triangle PBQ) = \frac{1}{4} \text{ ar}(\triangle ABC) \dots (2)$$

(i) Join point P to C.



In $\triangle PAQ$, QR is the median.

$$\therefore \text{ar}(\triangle PRQ) = \frac{1}{2} \text{ar}(\triangle PAQ) = \frac{1}{2} \times \frac{1}{4} \text{ar}(\triangle ABC) = \frac{1}{8} \text{ar}(\triangle ABC) \dots (3)$$

In $\triangle ABC$, P and Q are the mid-points of AB and BC respectively. Hence, by using mid-point theorem, we obtain

$$PQ = \frac{1}{2} AC$$

$$AC = 2PQ \Rightarrow AC = PT$$

Also, $PQ \parallel AC \Rightarrow PT \parallel AC$

Hence, PACT is a parallelogram.

$$\text{ar}(\text{PACT}) = \text{ar}(\text{PACQ}) + \text{ar}(\triangle QTC)$$

$$= \text{ar}(\text{PACQ}) + \text{ar}(\triangle PBQ) \text{ [Using equation (1)]}$$

$$\therefore \text{ar}(\text{PACT}) = \text{ar}(\triangle ABC) \dots (4)$$

$$\text{ar}(\triangle ARC) = \frac{1}{2} \text{ar}(\triangle PAC) \quad (\text{CR is the median of } \triangle PAC)$$

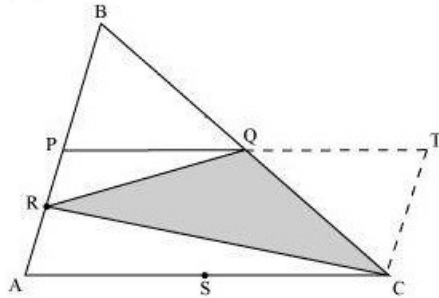
$$= \frac{1}{2} \times \frac{1}{2} \text{ar}(\text{PACT}) \quad (\text{PC is the diagonal of parallelogram PACT})$$

$$= \frac{1}{4} \text{ar}(\triangle \text{PACT}) = \frac{1}{4} \text{ar}(\triangle ABC)$$

$$\Rightarrow \frac{1}{2} \text{ar}(\triangle ARC) = \frac{1}{8} \text{ar}(\triangle ABC)$$

$$\Rightarrow \frac{1}{2} \text{ar}(\triangle ARC) = \text{ar}(\triangle PRQ) \text{ [Using equation (3)]} \dots (5)$$

(ii)



$$\text{ar}(\text{PACT}) = \text{ar}(\Delta \text{PRQ}) + \text{ar}(\Delta \text{ARC}) + \text{ar}(\Delta \text{QTC}) + \text{ar}(\Delta \text{RQC})$$

Putting the values from equations (1), (2), (3), (4), and (5), we obtain

$$\text{ar}(\Delta \text{ABC}) = \frac{1}{8} \text{ar}(\Delta \text{ABC}) + \frac{1}{4} \text{ar}(\Delta \text{ABC}) + \frac{1}{4} \text{ar}(\Delta \text{ABC}) + \text{ar}(\Delta \text{RQC})$$

$$\text{ar}(\Delta \text{ABC}) = \frac{5}{8} \text{ar}(\Delta \text{ABC}) + \text{ar}(\Delta \text{RQC})$$

$$\text{ar}(\Delta \text{RQC}) = \left(1 - \frac{5}{8}\right) \text{ar}(\Delta \text{ABC})$$

$$\text{ar}(\Delta \text{RQC}) = \frac{3}{8} \text{ar}(\Delta \text{ABC})$$

(iii) In parallelogram PACT,

$$\text{ar}(\Delta \text{ARC}) = \frac{1}{2} \text{ar}(\Delta \text{PAC}) \quad (\text{CR is the median of } \Delta \text{PAC})$$

$$= \frac{1}{2} \times \frac{1}{2} \text{ar}(\text{PACT}) \quad (\text{PC is the diagonal of parallelogram PACT})$$

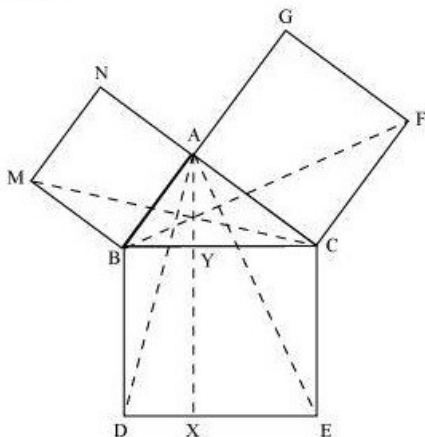
$$= \frac{1}{4} \text{ar}(\Delta \text{PACT})$$

$$= \frac{1}{4} \text{ar}(\Delta \text{ABC})$$

$$= \text{ar}(\Delta \text{PBQ})$$

Question 8:

In the following figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment $AX \perp DE$ meets BC at Y. Show that:



- (i) $\triangle MBC \cong \triangle ABD$
- (ii) $\text{ar}(\text{BYXD}) = 2\text{ar}(\text{MBC})$
- (iii) $\text{ar}(\text{BYXD}) = 2\text{ar}(\text{ABMN})$
- (iv) $\triangle FCB \cong \triangle ACE$
- (v) $\text{ar}(\text{CYXE}) = 2\text{ar}(\text{FCB})$
- (vi) $\text{ar}(\text{CYXE}) = \text{ar}(\text{ACFG})$
- (vii) $\text{ar}(\text{BCED}) = \text{ar}(\text{ABMN}) + \text{ar}(\text{ACFG})$

Note: Result (vii) is the famous *Theorem of Pythagoras*. You shall learn a simpler proof of this theorem in class X.

Answer 8:

- (i) We know that each angle of a square is 90° .
Hence, $\angle ABM = \angle DBC = 90^\circ$
 $\Rightarrow \angle ABM + \angle ABC = \angle DBC + \angle ABC$
 $\Rightarrow \angle MBC = \angle ABD$
 In $\triangle MBC$ and $\triangle ABD$,
 $\angle MBC = \angle ABD$ (Proved above)
 $MB = AB$ (Sides of square ABMN)
 $BC = BD$ (Sides of square BCED)
 $\therefore \triangle MBC \cong \triangle ABD$ (SAS congruence rule)



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(ii) We have

$$\Delta MBC \cong \Delta ABD$$

$$\Rightarrow \text{ar}(\Delta MBC) = \text{ar}(\Delta ABD) \dots (1)$$

It is given that $AX \perp DE$ and $BD \perp DE$ (Adjacent sides of square BDEC)

$\Rightarrow BD \parallel AX$ (Two lines perpendicular to same line are parallel to each other)

ΔABD and parallelogram $BYXD$ are on the same base BD and between the same parallels BD and AX .

$$\therefore \text{ar}(\Delta ABD) = \frac{1}{2} \text{ar}(BYXD)$$

$$\text{ar}(BYXD) = 2 \text{ar}(\Delta ABD)$$

$$\text{Area}(BYXD) = 2 \text{area}(\Delta MBC) \text{ [Using equation (1)]} \dots (2)$$

(iii) ΔMBC and parallelogram $ABMN$ are lying on the same base MB and between same parallels MB and NC .

$$\therefore \text{ar}(\Delta MBC) = \frac{1}{2} \text{ar}(ABMN)$$

$$2 \text{ar}(\Delta MBC) = \text{ar}(ABMN)$$

$$\text{ar}(BYXD) = \text{ar}(ABMN) \text{ [Using equation (2)]} \dots (3)$$

(iv) We know that each angle of a square is 90° .

$$\therefore \angle FCA = \angle BCE = 90^\circ$$

$$\Rightarrow \angle FCA + \angle ACB = \angle BCE + \angle ACB$$

$$\Rightarrow \angle FCB = \angle ACE$$

In ΔFCB and ΔACE ,

$$\angle FCB = \angle ACE$$

$$FC = AC \text{ (Sides of square } ACFG)$$

$$CB = CE \text{ (Sides of square } BCED)$$

$$\Delta FCB \cong \Delta ACE \text{ (SAS congruence rule)}$$



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(v) It is given that $AX \perp DE$ and $CE \perp DE$ (Adjacent sides of square BDEC)

Hence, $CE \parallel AX$ (Two lines perpendicular to the same line are parallel to each other)

Consider $\triangle ACE$ and parallelogram $CYXE$

$\triangle ACE$ and parallelogram $CYXE$ are on the same base CE and between the same parallels CE and AX .

$$\therefore \text{ar}(\triangle ACE) = \frac{1}{2} \text{ar}(CYXE)$$

$$\Rightarrow \text{ar}(CYXE) = 2 \text{ar}(\triangle ACE) \dots (4)$$

We had proved that

$$\therefore \triangle FCB \cong \triangle ACE$$

$$\text{ar}(\triangle FCB) \cong \text{ar}(\triangle ACE) \dots (5)$$

On comparing equations (4) and (5), we obtain

$$\text{ar}(CYXE) = 2 \text{ar}(\triangle FCB) \dots (6)$$

(vi) Consider $\triangle FCB$ and parallelogram $ACFG$

$\triangle FCB$ and parallelogram $ACFG$ are lying on the same base CF and between the same parallels CF and BG .

$$\therefore \text{ar}(\triangle FCB) = \frac{1}{2} \text{ar}(ACFG)$$

$$\Rightarrow \text{ar}(ACFG) = 2 \text{ar}(\triangle FCB)$$

$$\Rightarrow \text{ar}(ACFG) = \text{ar}(CYXE) \text{ [Using equation (6)]} \dots (7)$$

(vii) From the figure, it is evident that

$$\text{ar}(BCED) = \text{ar}(BYXD) + \text{ar}(CYXE)$$

$$\Rightarrow \text{ar}(BCED) = \text{ar}(ABMN) + \text{ar}(ACFG) \text{ [Using equations (3) and (7)]}$$