MATHS -9 (CH-02- POLYNOMIALS)

MATHS -9 (CH-02-2.1- POLYNOMIALS)

Question 1:

Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$ (iii) $3\sqrt{t} + t\sqrt{2}$

(iv) $y + \frac{2}{y}$ (v) $x^{10} + y^3 + t^{50}$

Answer 1:

(i) $4x^2 - 3x + 7$

Yes, this expression is a polynomial in one variable *x*.

(ii) $y^2 + \sqrt{2}$

Yes, this expression is a polynomial in one variable *y*.

(iii) $3\sqrt{t} + t\sqrt{2}$

No. It can be observed that the exponent of variable *t* in term $3\sqrt{t}$ is $\frac{1}{2}$, which is not a whole number. Therefore, this expression is not a polynomial.

(iv) $y + \frac{2}{y}$

No. It can be observed that the exponent of variable *y* in term $\frac{2}{y}$ is −1, which is not a whole number. Therefore, this expression is not a polynomial.

(v) $x^{10} + y^3 + t^{50}$

No. It can be observed that this expression is a polynomial in 3 variables *x*, *y*, and *t*. Therefore, it is not a polynomial in one variable.

Question 2:

Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$ (ii) $2 - x^2 + x^3$

(iii) $\frac{\pi}{2}x^2 + x$ (iv) $\sqrt{2}x - 1$

Answer 4:

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) $5x^3 + 4x^2 + 7x$

This is a polynomial in variable x and the highest power of variable x is 3. Therefore, the degree of this polynomial is 3.

(ii) $4 - y^2$

This is a polynomial in variable y and the highest power of variable y is 2. Therefore, the degree of this polynomial is 2.

(iii) $5t - \sqrt{7}$

This is a polynomial in variable t and the highest power of variable t is 1. Therefore, the degree of this polynomial is 1.

(iv) 3

This is a constant polynomial. Degree of a constant polynomial is always 0.

Question 4:

Write the degree of each of the following polynomials:
(i) $5x^3+4x^2+7x$ (ii) $4-y^2$
(iii) $5t-\sqrt{7}$ (iv) 3

Question 5:

Classify the following as linear, quadratic and cubic polynomial:
(i) x^2+x (ii) $x-x^3$ (iii) $y+y^2+4$ (iv) $1+x$ (v) $3t$
(vi) r^2 (vii) $7x^3$

Answer 5:

Linear polynomial, quadratic polynomial, and cubic polynomial has its degrees as 1, 2, and 3 respectively.
(i) x^2+x is a quadratic polynomial as its degree is 2.
(ii) $x-x^3$ is a cubic polynomial as its degree is 3.
(iii) $y+y^2+4$ is a quadratic polynomial as its degree is 2.
(iv) $1+x$ is a linear polynomial as its degree is 1.
(v) $3t$ is a linear polynomial as its degree is 1.
(vi) r^2 is a quadratic polynomial as its degree is 2.
(vii) $7x^3$ is a cubic polynomial as its degree is 3.

MATHS -9 (CH-02-2.2- POLYNOMIALS)

Question 1:

Find the value of the polynomial $5x-4x^2+3$ at
(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

Answer 1:

(i) $p(x)=5x-4x^2+3$
$p(0)=5(0)-4(0)^2+3$
$=3$
(ii) $p(x)=5x-4x^2+3$
$p(-1)=5(-1)-4(-1)^2+3$
$=-5-4(1)+3=-6$
(iii) $p(x)=5x-4x^2+3$
$p(2)=5(2)-4(2)^2+3$
$=10-16+3=-3$

Question 2:

Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials:
(i) $p(y) = y^2 - y + 1$ (ii) $p(t) = 2 + t + 2t^2 - t^3$
(iii) $p(x) = x^3$ (iv) $p(x) = (x - 1)(x + 1)$

Answer 2:

(i) $p(y) = y^2 - y + 1$
$p(0) = (0)^2 - (0) + 1 = 1$
$p(1) = (1)^2 - (1) + 1 = 1$
$p(2) = (2)^2 - (2) + 1 = 3$
(ii) $p(t) = 2 + t + 2t^2 - t^3$
$p(0) = 2 + 0 + 2(0)^2 - (0)^3 = 2$
$p(1) = 2 + (1) + 2(1)^2 - (1)^3$
$= 2 + 1 + 2 - 1 = 4$
$p(2) = 2 + 2 + 2(2)^2 - (2)^3$
$= 2 + 2 + 8 - 8 = 4$

(iii) $p(x) = x^3$
$p(0) = (0)^3 = 0$
$p(1) = (1)^3 = 1$
$p(2) = (2)^3 = 8$
(iv) $p(x) = (x - 1)(x + 1)$
$p(0) = (0 - 1)(0 + 1) = (-1)(1) = -1$
$p(1) = (1 - 1)(1 + 1) = 0(2) = 0$
$p(2) = (2 - 1)(2 + 1) = 1(3) = 3$

Question 3:

Verify whether the following are zeroes of the polynomial, indicated against them.
(i) $p(x) = 3x + 1, x = -\frac{1}{3}$ (ii) $p(x) = 5x - \pi, x = \frac{4}{5}$
(iii) $p(x) = x^2 - 1$, $x = 1, -1$ (iv) $p(x) = (x + 1)(x - 2)$, $x = -1, 2$
(v) $p(x) = x^2$, $x = 0$ (vi) $p(x) = lx + m$
(vii) $p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$ (viii) $p(x) = 2x + 1, x = \frac{1}{2}$

Answer 3:

(i) If $x = \frac{-1}{3}$ is a zero of given polynomial $p(x) = 3x + 1$, then $p\left(-\frac{1}{3}\right)$ should be 0.

Here, $p\left(\frac{-1}{3}\right) = 3\left(\frac{-1}{3}\right) + 1 = -1 + 1 = 0$

Therefore, $x = \frac{-1}{3}$ is a zero of the given polynomial.

(ii) If $x = \frac{4}{5}$ is a zero of polynomial $p(x) = 5x - \pi$, then $p\left(\frac{4}{5}\right)$ should be 0.

Here, $p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi = 4 - \pi$

As $p\left(\frac{4}{5}\right) \neq 0,$

Therefore, $x = \frac{4}{5}$ is not a zero of the given polynomial.

(iii) If $x = 1$ and $x = -1$ are zeroes of polynomial $p(x) = x^2 - 1$, then $p(1)$ and $p(-1)$ should be 0.
Here, $p(1) = (1)^2 - 1 = 0$, and
$p(-1) = (-1)^2 - 1 = 0$
Hence, $x = 1$ and -1 are zeroes of the given polynomial.
(iv) If $x = -1$ and $x = 2$ are zeroes of polynomial $p(x) = (x+1)(x-2)$, then $p(-1)$ and $p(2)$ should be 0.
Here, $p(-1) = (-1 + 1)(-1 - 2) = 0(-3) = 0$, and
$p(2) = (2 + 1)(2 - 2) = 3(0) = 0$
Therefore, $x = -1$ and $x = 2$ are zeroes of the given polynomial.

(v) If $x = 0$ is a zero of polynomial $p(x) = x^2$, then $p(0)$ should be zero.
Here, $p(0) = (0)^2 = 0$
Hence, $x = 0$ is a zero of the given polynomial.
(vi) If $x = \frac{-m}{l}$ is a zero of polynomial $p(x) = lx + m$, then $p\left(\frac{-m}{l}\right)$ should be 0.

Here, $p\left(\frac{-m}{l}\right) = l\left(\frac{-m}{l}\right) + m = -m + m = 0$

Therefore, $x = -\frac{m}{l}$ is a zero of the given polynomial.

(vii) If $x = \frac{-1}{\sqrt{3}}$ and $x = \frac{2}{\sqrt{3}}$ are zeroes of polynomial $p(x) = 3x^2 - 1$, then

$p\left(\frac{-1}{\sqrt{3}}\right)$ and $p\left(\frac{2}{\sqrt{3}}\right)$ should be 0.

Here, $p\left(\frac{-1}{\sqrt{3}}\right) = 3\left(\frac{-1}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{1}{3}\right) - 1 = 1 - 1 = 0$, and

$p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{4}{3}\right) - 1 = 4 - 1 = 3$

Hence, $x = \frac{-1}{\sqrt{3}}$ is a zero of the given polynomial. However, $x = \frac{2}{\sqrt{3}}$ is not a zero of the given polynomial.

(viii) If $x = \frac{1}{2}$ is a zero of polynomial $p(x) = 2x + 1$, then $p\left(\frac{1}{2}\right)$ should be 0.

Here, $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2$

As $p\left(\frac{1}{2}\right) \neq 0$,

Therefore, $x = \frac{1}{2}$ is not a zero of the given polynomial.

Question 4:

Find the zero of the polynomial in each of the following cases:
(i) $p(x) = x + 5$ (ii) $p(x) = x - 5$ (iii) $p(x) = 2x + 5$
(iv) $p(x) = 3x - 2$ (v) $p(x) = 3x$ (vi) $p(x) = ax$, $a \neq 0$
(vii) $p(x) = cx + d$, $c \neq 0$, c, are real numbers.

Answer 4:

Zero of a polynomial is that value of the variable at which the value of the polynomial is obtained as 0.
(i) $p(x) = x + 5$
$p(x) = 0$
$x + 5 = 0$
$x = -5$
Therefore, for $x = -5$, the value of the polynomial is 0 and hence, $x = -5$ is a zero of the given polynomial.
(ii) $p(x) = x - 5$
$p(x) = 0$
$x - 5 = 0$
$x = 5$
Therefore, for $x = 5$, the value of the polynomial is0 and hence, $x = 5$ is a zero of the given polynomial.

(iii) $p(x) = 2x + 5$
$p(x) = 0$
$2x + 5 = 0$
$2x = -5$
$x = -\frac{5}{2}$
Therefore, for $x = -\frac{5}{2}$, the value of the polynomial is 0 and hence, $x = \frac{-5}{2}$ is a zero of the given polynomial.
(iv) $p(x) = 3x - 2$
$p(x) = 0$
$3x - 2 = 0$
$x = \frac{2}{3}$
Therefore, for $x = \frac{2}{3}$, the value of the polynomial is 0 and hence, $x = \frac{2}{3}$ is a zero of the given polynomial.

(v) $p(x) = 3x$
$p(x) = 0$
$3x = 0$
$x = 0$
Therefore, for $x = 0$, the value of the polynomial is 0 and hence, $x = 0$ is a zero of the given polynomial.
(vi) $p(x) = ax$
$p(x) = 0$
$ax = 0$
$x = 0$
Therefore, for $x = 0$, the value of the polynomial is 0 and hence, $x = 0$ is a zero of the given polynomial.

(vii) $p(x) = cx + d$
$p(x) = 0$
$cx + d = 0$
$x = \frac{-d}{c}$
Therefore, for $x = \frac{-d}{c}$, the value of the polynomial is 0 and hence, $x = \frac{-d}{c}$ is a zero of the given polynomial.

MATHS -9 (CH-02- 2.3-POLYNOMIALS)

Question 1:

Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i) $x + 1$ (ii) $x - \frac{1}{2}$ (iii) x

(iv) $x + \pi$ (v) $5 + 2x$

Answer 1:

(i) $x + 1$

By long division,

$$\begin{array}{r|l} & x^2 + 2x + 1 \\ x+1 & x^3 + 3x^2 + 3x + 1 \\ & x^3 + x^2 \\ & - \quad - \\ \hline & 2x^2 + 3x + 1 \\ & 2x^2 + 2x \\ & - \quad - \\ \hline & x + 1 \\ & x + 1 \\ & - \quad - \\ \hline & 0 \end{array}$$

Therefore, the remainder is 0.

(ii) $x-\frac{1}{2}$

By long division,

$$\begin{array}{r|l} & x^2+\frac{7}{2}x+\frac{19}{4} \\ \hline x-\frac{1}{2} & x^3+3x^2+3x+1 \\ & x^3-\frac{x^2}{2} \\ & -\quad + \\ \hline & \frac{7}{2}x^2+3x+1 \\ & \frac{7}{2}x^2-\frac{7}{4}x \\ & -\quad + \\ \hline & \frac{19}{4}x+1 \\ & \frac{19}{4}x-\frac{19}{8} \\ & -\quad + \\ \hline & \frac{27}{8} \\ \hline \end{array}$$

Therefore, the remainder is $\frac{27}{8}$.

(iii) x

By long division,

$$\begin{array}{r|l} & x^2+3x+3 \\ \hline x & x^3+3x^2+3x+1 \\ & x^3 \\ & - \\ \hline & 3x^2+3x+1 \\ & 3x^2 \\ & - \\ \hline & 3x+1 \\ & 3x \\ & - \\ \hline & 1 \\ \hline \end{array}$$

Therefore, the remainder is 1.

(iv) $x + \pi$

By long division,

$$\begin{array}{r|l} & x^2 + (3-\pi)x + (3 - 3\pi + \pi^2) \\ x+\pi & x^3 + 3x^2 + 3x + 1 \\ & x^3 + \pi x^2 \\ & - \quad - \\ \hline & (3-\pi)x^2 + 3x + 1 \\ & (3-\pi)x^2 + (3-\pi)\pi x \\ & - \qquad - \\ \hline & [3 - 3\pi + \pi^2]x + 1 \\ & [3 - 3\pi + \pi^2]x + (3 - 3\pi + \pi^2)\pi \\ & - \qquad - \\ \hline & [1 - 3\pi + 3\pi^2 - \pi^3] \\ \hline \end{array}$$

Therefore, the remainder is $-\pi^3 + 3\pi^2 - 3\pi + 1.$

(v) $5 + 2x$

By long division,

$$\begin{array}{r|l} & \frac{x^2}{2} + \frac{x}{4} + \frac{7}{8} \\ 2x+5 & x^3 + 3x^2 + 3x + 1 \\ & x^3 + \frac{5}{2}x^2 \\ & - \quad - \\ \hline & \frac{x^2}{2} + 3x + 1 \\ & \frac{x^2}{2} + \frac{5x}{4} \\ & - \quad - \\ \hline & \frac{7x}{4} + 1 \\ & \frac{7}{4}x + \frac{35}{8} \\ & - \quad - \\ \hline & -\frac{27}{8} \\ \hline \end{array}$$

Therefore, the remainder is $-\frac{27}{8}$

MATHS -9 (CH-02-2.4- POLYNOMIALS)

Question 1:

Determine which of the following polynomials has $(x + 1)$ a factor:

(i) $x^3 + x^2 + x + 1$ (ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$ (iv) $x^3 - x^2 - \left(2+\sqrt{2}\right)x + \sqrt{2}$

Answer 1:

(i) If $(x + 1)$ is a factor of $p(x) = x^3 + x^2 + x + 1$, then $p(-1)$ must be zero, otherwise $(x + 1)$ is not a factor of $p(x)$.

$p(x) = x^3 + x^2 + x + 1$

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1 = 0$

Hence, $x + 1$ is a factor of this polynomial.

(ii) If $(x + 1)$ is a factor of $p(x) = x^4 + x^3 + x^2 + x + 1$, then $p(-1)$ must be zero, otherwise $(x + 1)$ is not a factor of $p(x)$.

$p(x) = x^4 + x^3 + x^2 + x + 1$

$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 - 1 + 1 - 1 + 1 = 1$

As $p(-1) \neq 0$,

Therefore, $x + 1$ is not a factor of this polynomial.

(iii) If $(x + 1)$ is a factor of polynomial $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$, then $p(-1)$ must be 0, otherwise $(x + 1)$ is not a factor of this polynomial.

$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1$

As $p(-1) \neq 0$,

Therefore, $x + 1$ is not a factor of this polynomial.

(iv) If $(x + 1)$ is a factor of polynomial $p(x) = x^3 - x^2 - \left(2+\sqrt{2}\right)x + \sqrt{2}$, then $p(-1)$ must be 0, otherwise $(x + 1)$ is not a factor of this polynomial.

$$p(-1) = (-1)^3 - (-1)^2 - \left(2+\sqrt{2}\right)(-1) + \sqrt{2}$$

$$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$$

$$= 2\sqrt{2}$$

As $p(-1) \neq 0$,

Therefore, $(x + 1)$ is not a factor of this polynomial.

Question 2:

Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

Answer 2:

(i) If $g(x) = x + 1$ is a factor of the given polynomial $p(x)$, then $p(-1)$ must be zero.

$p(x) = 2x^3 + x^2 - 2x - 1$

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= 2(-1) + 1 + 2 - 1 = 0$

Hence, $g(x) = x + 1$ is a factor of the given polynomial.

(ii) If $g(x) = x + 2$ is a factor of the given polynomial $p(x)$, then $p(-2)$ must be 0.

$p(x) = x^3 + 3x^2 + 3x + 1$

$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= - 8 + 12 - 6 + 1$

$= -1$

As $p(-2) \neq 0$,

Hence, $g(x) = x + 2$ is not a factor of the given polynomial.

(iii) If $g(x) = x - 3$ is a factor of the given polynomial $p(x)$, then $p(3)$ must be 0.

$p(x) = x^3 - 4x^2 + x + 6$

$p(3) = (3)^3 - 4(3)^2 + 3 + 6$

$= 27 - 36 + 9 = 0$

Hence, $g(x) = x - 3$ is a factor of the given polynomial.

Question 3:

Find the value of k, if $x - 1$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = x^2 + x + k$ (ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$ (iv) $p(x) = kx^2 - 3x + k$

Answer 3:

If $x - 1$ is a factor of polynomial $p(x)$, then $p(1)$ must be 0.

(i) $p(x) = x^2 + x + k$

$p(1) = 0$

$\Rightarrow (1)^2 + 1 + k = 0$

$\Rightarrow 2 + k = 0$

$\Rightarrow k = -2$

Therefore, the value of k is -2.

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

$p(1) = 0$

$\Rightarrow 2(1)^2 + k(1) + \sqrt{2} = 0$

$\Rightarrow 2 + k + \sqrt{2} = 0$

$\Rightarrow k = -2 - \sqrt{2} = -(2 + \sqrt{2})$

Therefore, the value of k is $-(2 + \sqrt{2})$.

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

$p(1) = 0$

$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$

$\Rightarrow k - \sqrt{2} + 1 = 0$

$\Rightarrow k = \sqrt{2} - 1$

Therefore, the value of k is $\sqrt{2} - 1$.

(iv) $p(x) = kx^2 - 3x + k$

$\Rightarrow p(1) = 0$

$\Rightarrow k(1)^2 - 3(1) + k = 0$

$\Rightarrow k - 3 + k = 0$

$\Rightarrow 2k - 3 = 0$

$\Rightarrow k = \frac{3}{2}$

Therefore, the value of k is $\frac{3}{2}$.

Question 4:

Factorise:
(i) $12x^2 - 7x + 1$ (ii) $2x^2 + 7x + 3$
(iii) $6x^2 + 5x - 6$ (iv) $3x^2 - x - 4$

Answer 4:

(i) $12x^2 - 7x + 1$
We can find two numbers such that $pq = 12 \times 1 = 12$ and $p + q = -7$. They are $p = -4$ and $q = -3$.
Here, $12x^2 - 7x + 1 = 12x^2 - 4x - 3x + 1$
$= 4x(3x - 1) - 1(3x - 1)$
$= (3x - 1)(4x - 1)$

(ii) $2x^2 + 7x + 3$
We can find two numbers such that $pq = 2 \times 3 = 6$ and $p + q = 7$.
They are $p = 6$ and $q = 1$.
Here, $2x^2 + 7x + 3 = 2x^2 + 6x + x + 3$
$= 2x(x + 3) + 1(x + 3)$
$= (x + 3)(2x + 1)$
(iii) $6x^2 + 5x - 6$
We can find two numbers such that $pq = -36$ and $p + q = 5$.
They are $p = 9$ and $q = -4$.
Here,
$6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$
$= 3x(2x + 3) - 2(2x + 3)$
$= (2x + 3)(3x - 2)$

(iv) $3x^2 - x - 4$
We can find two numbers such that $pq = 3 \times (-4) = -12$
and $p + q = -1$.
They are $p = -4$ and $q = 3$.
Here,
$3x^2 - x - 4 = 3x^2 - 4x + 3x - 4$
$= x(3x - 4) + 1(3x - 4)$
$= (3x - 4)(x + 1)$

Question 5:

Factorise:
(i) $x^3 - 2x^2 - x + 2$ (ii) $x^3 + 3x^2 - 9x - 5$
(iii) $x^3 + 13x^2 + 32x + 20$ (iv) $2y^3 + y^2 - 2y - 1$

Answer 5:

(i) Let $p(x) = x^3 - 2x^2 - x + 2$

All the factors of 2 have to be considered. These are ± 1, ± 2.

By trial method,

$p(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$

$= -1 - 2 + 1 + 2 = 0$

Therefore, $(x + 1)$ is factor of polynomial $p(x)$.

Let us find the quotient on dividing $x^3 - 2x^2 - x + 2$ by $x + 1$.

By long division,

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\
x+1 & x^3 - 2x^2 - x + 2 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & -3x^2 - x + 2 \\
 & -3x^2 - 3x \\
 & + \quad + \\
\hline
 & 2x + 2 \\
 & 2x + 2 \\
 & - \quad - \\
\hline
 & 0
\end{array}
$$

It is known that,

Dividend = Divisor × Quotient + Remainder

$\therefore x^3 - 2x^2 - x + 2 = (x + 1)(x^2 - 3x + 2) + 0$

$= (x + 1)[x^2 - 2x - x + 2]$

$= (x + 1)[x(x - 2) - 1(x - 2)]$

$= (x + 1)(x - 1)(x - 2)$

$= (x - 2)(x - 1)(x + 1)$

(ii) Let $p(x) = x^3 - 3x^2 - 9x - 5$

All the factors of 5 have to be considered. These are ±1, ± 5.

By trial method,

$p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$

$= -1 - 3 + 9 - 5 = 0$

Therefore, $x + 1$ is a factor of this polynomial.

Let us find the quotient on dividing $x^3 + 3x^2 - 9x - 5$ by $x + 1$.

By long division,

$$\begin{array}{r l} & x^2 - 4x - 5 \\ x+1 \,\big) & x^3 - 3x^2 - 9x - 5 \\ & x^3 + x^2 \\ & - \quad - \\ \hline & -4x^2 - 9x - 5 \\ & -4x^2 - 4x \\ & + \quad + \\ \hline & -5x - 5 \\ & -5x - 5 \\ & + \quad + \\ \hline & 0 \\ \hline \end{array}$$

It is known that,

Dividend = Divisor × Quotient + Remainder

$\therefore x^3 - 3x^2 - 9x - 5 = (x + 1)(x^2 - 4x - 5) + 0$

$= (x + 1)(x^2 - 5x + x - 5)$

$= (x + 1)[(x(x - 5) + 1(x - 5)]$

$= (x + 1)(x - 5)(x + 1)$

$= (x - 5)(x + 1)(x + 1)$

(iii) Let $p(x) = x^3 + 13x^2 + 32x + 20$

All the factors of 20 have to be considered. Some of them are ±1, ± 2, ± 4, ± 5 ......

By trial method,

$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= - 1 + 13 - 32 + 20$

$= 33 - 33 = 0$

As $p(-1)$ is zero, therefore, $x + 1$ is a factor of this polynomial $p(x)$.

Let us find the quotient on dividing $x^3 + 13x^2 + 32x + 20$ by $(x + 1)$.

By long division,

$$x^2 + 12x + 20$$
$$x+1 \overline{) x^3 + 13x^2 + 32x + 20}$$
$$x^3 + x^2$$
$$- \quad -$$
$$12x^2 + 32x$$
$$12x^2 + 12x$$
$$- \quad -$$
$$20x + 20$$
$$20x + 20$$
$$- \quad -$$
$$0$$

It is known that,

Dividend = Divisor × Quotient + Remainder

$x^3 + 13x^2 + 32x + 20 = (x + 1)(x^2 + 12x + 20) + 0$

$= (x + 1)(x^2 + 10x + 2x + 20)$

$= (x + 1)[x(x + 10) + 2(x + 10)]$

$= (x + 1)(x + 10)(x + 2)$

$= (x + 1)(x + 2)(x + 10)$

(iv) Let $p(y) = 2y^3 + y^2 - 2y - 1$

By trial method,

$p(1) = 2(1)^3 + (1)^2 - 2(1) - 1$

$= 2 + 1 - 2 - 1 = 0$

Therefore, $y - 1$ is a factor of this polynomial.

Let us find the quotient on dividing $2y^3 + y^2 - 2y - 1$ by $y - 1$.

$$2y^2 + 3y + 1$$
$$y-1 \overline{) 2y^3 + y^2 - 2y - 1}$$
$$2y^3 - 2y^2$$
$$- \quad +$$
$$3y^2 - 2y - 1$$
$$3y^2 - 3y$$
$$- \quad +$$
$$y - 1$$
$$y - 1$$
$$0$$

$p(y) = 2y^3 + y^2 - 2y - 1$

$= (y - 1)(2y^2 + 3y + 1)$

$= (y - 1)(2y^2 + 2y + y + 1)$

$= (y - 1)[2y(y + 1) + 1(y + 1)]$

$= (y - 1)(y + 1)(2y + 1)$

MATHS -9 (CH-02-2.5- POLYNOMIALS)

Question 1:

Use suitable identities to find the following products:

(i) $(x+4)(x+10)$ (ii) $(x+8)(x-10)$

(iii) $(3x+4)(3x-5)$ (iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

(v) $(3-2x)(3+2x)$

Answer 1:

(i) By using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$,

$$(x+4)(x+10)=x^2+(4+10)x+4\times 10$$
$$=x^2+14x+40$$

(ii) By using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$,

$$(x+8)(x-10)=x^2+(8-10)x+(8)(-10)$$
$$=x^2-2x-80$$

(iii) $(3x+4)(3x-5)=9\left(x+\frac{4}{3}\right)\left(x-\frac{5}{3}\right)$

By using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$,

$$9\left(x+\frac{4}{3}\right)\left(x-\frac{5}{3}\right)=9\left[x^2+\left(\frac{4}{3}-\frac{5}{3}\right)x+\left(\frac{4}{3}\right)\left(-\frac{5}{3}\right)\right]$$
$$=9\left[x^2-\frac{1}{3}x-\frac{20}{9}\right]$$
$$=9x^2-3x-20$$

(iv) By using the identity $(x+y)(x-y)=x^2-y^2$,

$$\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)=\left(y^2\right)^2-\left(\frac{3}{2}\right)^2$$
$$=y^4-\frac{9}{4}$$

(v) By using the identity $(x+y)(x-y)=x^2-y^2$,

$$(3-2x)(3+2x)=(3)^2-(2x)^2$$
$$=9-4x^2$$

Question 2:

Evaluate the following products without multiplying directly:
(i) 103 × 107 (ii) 95 × 96 (iii) 104 × 96

Answer 2:

(i) 103 × 107 = (100 + 3) (100 + 7)
= $(100)^2$ + (3 + 7) 100 + (3) (7)
[By using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$, where
x = 100, a = 3, and b = 7]
= 10000 + 1000 + 21
= 11021
(ii) 95 × 96 = (100 − 5) (100 − 4)
= $(100)^2$ + (− 5 − 4) 100 + (− 5) (− 4)
[By using the identity $(x+a)(x+b)=x^2+(a+b)x+ab$, where
x = 100, a = −5, and b = −4]
= 10000 − 900 + 20
= 9120
(iii) 104 × 96 = (100 + 4) (100 − 4)
= $(100)^2 - (4)^2$ $\left[(x+y)(x-y)=x^2-y^2\right]$
= 10000 − 16
= 9984

Question 3:

Factorise the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - \frac{y^2}{100}$

Answer 3:

(i) $9x^2 + 6xy + y^2 = (3x)^2 + 2(3x)(y) + (y)^2$

$= (3x + y)(3x + y) \quad [x^2 + 2xy + y^2 = (x + y)^2]$

(ii) $4y^2 - 4y + 1 = (2y)^2 - 2(2y)(1) + (1)^2$

$= (2y - 1)(2y - 1) \quad [x^2 - 2xy + y^2 = (x - y)^2]$

(iii) $x^2 - \frac{y^2}{100} = x^2 - \left(\frac{y}{10}\right)^2$

$= \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right) \quad [x^2 - y^2 = (x + y)(x - y)]$

Question 4:

Expand each of the following, using suitable identities:

(i) $(x + 2y + 4z)^2$ (ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$ (iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$ (vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Answer 4:

It is known that,

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(i) $(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

(ii) $(2x - y + z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

(iii) $(-2x + 3y + 2z)^2$

$= (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x) \quad = 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$

(iv) $(3a-7b-c)^2$

$=(3a)^2+(-7b)^2+(-c)^2+2(3a)(-7b)+2(-7b)(-c)+2(-c)(3a)=9a^2+49b^2+c^2-42ab+14bc-6ac$

(v) $(-2x+5y-3z)^2$

$=(-2x)^2+(5y)^2+(-3z)^2+2(-2x)(5y)+2(5y)(-3z)+2(-3z)(-2x)=4x^2+25y^2+9z^2-20xy-30yz+12xz$

(vi) $\left[\frac{1}{4}a-\frac{1}{2}b+1\right]^2$

$=\left(\frac{1}{4}a\right)^2+\left(-\frac{1}{2}b\right)^2+(1)^2+2\left(\frac{1}{4}a\right)\left(-\frac{1}{2}b\right)+2\left(-\frac{1}{2}b\right)(1)+2\left(\frac{1}{4}a\right)(1)$

$=\frac{1}{16}a^2+\frac{1}{4}b^2+1-\frac{1}{4}ab-b+\frac{1}{2}a$

Question 5:

Factorise:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Question 5:

Factorise:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Answer 5:

It is known that,

$(x+y+z)^2=x^2+y^2+z^2+2xy+2yz+2zx$

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

$=(2x)^2+(3y)^2+(-4z)^2+2(2x)(3y)+2(3y)(-4z)+2(2x)(-4z)$

$=(2x+3y-4z)^2$

$=(2x+3y-4z)(2x+3y-4z)$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

$=\left(-\sqrt{2}x\right)^2+(y)^2+\left(2\sqrt{2}z\right)^2+2\left(-\sqrt{2}x\right)(y)+2(y)\left(2\sqrt{2}z\right)+2\left(-\sqrt{2}x\right)\left(2\sqrt{2}z\right)$

$=\left(-\sqrt{2}x+y+2\sqrt{2}z\right)^2$

$=\left(-\sqrt{2}x+y+2\sqrt{2}z\right)\left(-\sqrt{2}x+y+2\sqrt{2}z\right)$

Question 6:

Write the following cubes in expanded form:

(i) $(2x+1)^3$ (ii) $(2a-3b)^3$

(iii) $\left[\frac{3}{2}x+1\right]^3$ (iv) $\left[x-\frac{2}{3}y\right]^3$

Answer 6:

It is known that,

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

and $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(i) $(2x+1)^3 = (2x)^3 + (1)^3 + 3(2x)(1)(2x+1)$

$= 8x^3 + 1 + 6x(2x+1)$

$= 8x^3 + 1 + 12x^2 + 6x$

$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a-3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a-3b)$

$= 8a^3 - 27b^3 - 18ab(2a-3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

(iii) $\left[\frac{3}{2}x+1\right]^3 = \left[\frac{3}{2}x\right]^3 + (1)^3 + 3\left(\frac{3}{2}x\right)(1)\left(\frac{3}{2}x+1\right)$

$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x+1\right)$

$= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x$

$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$

(vi) $\left[x-\frac{2}{3}y\right]^3 = x^3 - \left(\frac{2}{3}y\right)^3 - 3(x)\left(\frac{2}{3}y\right)\left(x-\frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2xy\left(x-\frac{2}{3}y\right)$

$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$

Question 7:

Evaluate the following using suitable identities:

(i) $(99)^3$ (ii) $(102)^3$ (iii) $(998)^3$

Answer 7:

It is known that,

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

and $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(i) $(99)^3 = (100 - 1)^3$

$= (100)^3 - (1)^3 - 3(100)(1)(100 - 1)$

$= 1000000 - 1 - 300(99)$

$= 1000000 - 1 - 29700$

$= 970299$

(ii) $(102)^3 = (100 + 2)^3$
$= (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$
$= 1000000 + 8 + 600(102)$
$= 1000000 + 8 + 61200$
$= 1061208$
(iii) $(998)^3 = (1000 - 2)^3$
$= (1000)^3 - (2)^3 - 3(1000)(2)(1000 - 2)$
$= 1000000000 - 8 - 6000(998)$
$= 1000000000 - 8 - 5988000$
$= 1000000000 - 5988008$
$= 994011992$

Question 8:

Factorise each of the following:
(i) $8a^3 + b^3 + 12a^2b + 6ab^2$ (ii) $8a^3 - b^3 - 12a^2b + 6ab^2$
(iii) $27 - 125a^3 - 135a + 225a^2$ (iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$
(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Answer 8:

It is known that,
$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$
and $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$
(i) $8a^3 + b^3 + 12a^2b + 6ab^2$
$= (2a)^3 + (b)^3 + 3(2a)^2 b + 3(2a)(b)^2$
$= (2a+b)^3$
$= (2a+b)(2a+b)(2a+b)$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$
$= (2a)^3 - (b)^3 - 3(2a)^2 b + 3(2a)(b)^2$
$= (2a-b)^3$
$= (2a-b)(2a-b)(2a-b)$

(iii) $27-125a^3-135a+225a^2$

$=(3)^3-(5a)^3-3(3)^2(5a)+3(3)(5a)^2$

$=(3-5a)^3$

$=(3-5a)(3-5a)(3-5a)$

(iv) $64a^3-27b^3-144a^2b+108ab^2$

$=(4a)^3-(3b)^3-3(4a)^2(3b)+3(4a)(3b)^2$

$=(4a-3b)^3$

$=(4a-3b)(4a-3b)(4a-3b)$

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

$=(3p)^3-\left(\frac{1}{6}\right)^3-3(3p)^2\left(\frac{1}{6}\right)+3(3p)\left(\frac{1}{6}\right)^2$

$=\left(3p-\frac{1}{6}\right)^3$

$=\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)$

Question 9:

Verify:

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$

(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

Answer 9:

(i) It is known that,

$(x+y)^3=x^3+y^3+3xy(x+y)$

$x^3+y^3=(x+y)^3-3xy(x+y)$

$=(x+y)\left[(x+y)^2-3xy\right]$

$=(x+y)(x^2+y^2+2xy-3xy)$

$=(x+y)(x^2+y^2-xy)$

$=(x+y)(x^2-xy+y^2)$

(ii) It is known that,

$(x-y)^3=x^3-y^3-3xy(x-y)$

$x^3-y^3=(x-y)^3+3xy(x-y)$

$=(x-y)\left[(x-y)^2+3xy\right]$

$=(x-y)(x^2+y^2-2xy+3xy)$

$=(x-y)(x^2+y^2+xy)$

$=(x-y)(x^2+xy+y^2)$

Question 10:

Factorise each of the following:

(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

[**Hint:** See question 9.]

Answer 10:

(i) $27y^3 + 125z^3$

$= (3y)^3 + (5z)^3$

$= (3y + 5z)\left[(3y)^2 + (5z)^2 - (3y)(5z)\right]$ $\quad \left[\because a^3 + b^3 = (a+b)(a^2 + b^2 - ab)\right]$

$= (3y + 5z)\left[9y^2 + 25z^2 - 15yz\right]$

(ii) $64m^3 - 343n^3$

$= (4m)^3 - (7n)^3$

$= (4m - 7n)\left[(4m)^2 + (7n)^2 + (4m)(7n)\right]$ $\quad \left[\because a^3 - b^3 = (a-b)(a^2 + b^2 + ab)\right]$

$= (4m - 7n)\left[16m^2 + 49n^2 + 28mn\right]$

Question 11:

Factorise: $27x^3 + y^3 + z^3 - 9xyz$

Answer 11:

It is known that,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$\therefore 27x^3 + y^3 + z^3 - 9xyz$

$= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$= (3x + y + z)\left[(3x)^2 + y^2 + z^2 - (3x)(y) - (y)(z) - z(3x)\right]$

$= (3x + y + z)\left[9x^2 + y^2 + z^2 - 3xy - yz - 3xz\right]$

Question 12:

Verify that $x^3+y^3+z^3-3xyz=\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$

Answer 12:

It is known that,

$$x^3+y^3+z^3-3xyz=(x+y+z)\left(x^2+y^2+z^2-xy-yz-zx\right)$$
$$=\frac{1}{2}(x+y+z)\left[2x^2+2y^2+2z^2-2xy-2yz-2zx\right]$$
$$=\frac{1}{2}(x+y+z)\left[\left(x^2+y^2-2xy\right)+\left(y^2+z^2-2yz\right)+\left(x^2+z^2-2zx\right)\right]$$
$$=\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$$

Question 13:

If $x+y+z=0$, show that $x^3+y^3+z^3=3xyz$.

Answer 13:

It is known that,

$$x^3+y^3+z^3-3xyz=(x+y+z)\left(x^2+y^2+z^2-xy-yz-zx\right)$$

Put $x+y+z=0$,

$$x^3+y^3+z^3-3xyz=(0)\left(x^2+y^2+z^2-xy-yz-zx\right)$$
$$x^3+y^3+z^3-3xyz=0$$
$$x^3+y^3+z^3=3xyz$$

Question 14:

Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

(ii) $(28)^3+(-15)^3+(-13)^3$

Answer 14:

(i) $(-12)^3+(7)^3+(5)^3$

Let $x = -12$, $y = 7$, and $z = 5$

It can be observed that,

$x + y + z = -12 + 7 + 5 = 0$

It is known that if $x + y + z = 0$, then

$x^3+y^3+z^3=3xyz$

$\therefore (-12)^3+(7)^3+(5)^3=3(-12)(7)(5)$

$= -1260$

(ii) $(28)^3+(-15)^3+(-13)^3$

Let $x = 28$, $y = -15$, and $z = -13$

It can be observed that,

$x + y + z = 28 + (-15) + (-13) = 28 - 28 = 0$

It is known that if $x + y + z = 0$, then

$x^3+y^3+z^3=3xyz$

$\therefore (28)^3+(-15)^3+(-13)^3=3(28)(-15)(-13)$

$=16380$

Question 15:

Give possible expressions for the length and breadth of each of thefollowing rectangles, in which their areas are given:

Area: $25a^2-35a+12$	Area: $35y^2+13y-12$
I	II

Answer 15:

Area = Length × Breadth

The expression given for the area of the rectangle has to be factorised. One of its factors will be its length and the other will be its breadth.

(i) $25a^2 - 35a + 12 = 25a^2 - 15a - 20a + 12$

$= 5a(5a-3) - 4(5a-3)$

$= (5a-3)(5a-4)$

Therefore, possible length = $5a - 3$

And, possible breadth = $5a - 4$

(ii) $35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$

$= 7y(5y+4) - 3(5y+4)$

$= (5y+4)(7y-3)$

Therefore, possible length = $5y + 4$

And, possible breadth = $7y - 3$

Question 16:

What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

I	II
Volume: $3x^2 - 12x$	Volume: $12ky^2 + 8ky - 20k$

Answer 16:

Volume of cuboid = Length × Breadth × Height

The expression given for the volume of the cuboid has to be factorised. One of its factors will be its length, one will be its breadth, and one will be its height.

(i) $3x^2 - 12x = 3x(x-4)$

One of the possible solutions is as follows.

Length = 3, Breadth = x, Height = $x - 4$

(ii) $12ky^2 + 8ky - 20k = 4k(3y^2 + 2y - 5)$

$= 4k[3y^2 + 5y - 3y - 5]$

$= 4k[y(3y+5) - 1(3y+5)]$

$= 4k(3y+5)(y-1)$

One of the possible solutions is as follows.

Length = $4k$, Breadth = $3y + 5$, Height = $y - 1$