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DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

BASIC NOTES (CBSE/NCERT)

FACTORISATION (ALGEBRA)

PART -1 OUT OF 1

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∴ Factorisation (Algebra) :-

Q. Factorisation :- When we factorise an algebraic expression, we write it as a product of factors. These factors may be numbers, algebraic variables or algebraic expressions.

Ex - Expression like are already in factor form :-

$3xy$, $5x^2y$, $2x(x+2)$, $5(y+1)$, $2(x+1)(x+2)$ etc.

Q. Find Common factors :- and factorise :-

Ex - Factorise $12a^2b + 15ab^2$

Sol. $12a^2b = 2 \times 2 \times 3 \times a \times a \times b$

$15ab^2 = 3 \times 5 \times a \times b \times b$

The two terms have 3, a and b as common factors.

$$12a^2b + 15ab^2 = (3 \times a \times b \times 2 \times 2 \times a) + (3 \times a \times b \times 5 \times b)$$



$$= 3 \times a \times b [(2 \times 2 \times a) + (5 \times b)]$$

$$= 3ab \times [4a + 5b]$$

$$= 3ab (4a + 5b) \text{ [required factor form]}$$

Try :- $10x^2 - 18x^3 + 14x^4$

$22y - 33z$

$14pq + 35pqr$

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∴ Factorise (Algebra) :-

Q. Regrouping : - Use group of two terms :-

EX - Factorise $6xy - 4y + 6 - 9x$ Sol. $6xy - 4y + 6 - 9x$

Thinking of grouping :-

First two terms have a common factor $2y$

$$\begin{aligned}
 6xy - 4y &= 2 \times 3 \times x \times y - 2 \times 2 \times y \\
 &= 2y (3x - 2) \\
 &= 2y (3x - 2)
 \end{aligned}$$

last two terms have a common factor -3

$$\begin{aligned}
 6 - 9x &= 2 \times 3 - 3 \times 3 \times x \\
 &= 3(2 - 3x)
 \end{aligned}$$



$$\begin{aligned}
 &= 3(2 - 3x) \quad [-\text{common for } (2 - 3x) \text{ is } 3] \\
 &= -3(3x - 2)
 \end{aligned}$$

$$\text{Factor of } = 6xy - 4y + 6 - 9x$$

$$= 2y(3x - 2) - 3(3x - 2)$$

$$= (3x - 2)(2y - 3)$$

Therefore factor of $6xy - 4y + 6 - 9x$ are
 $(3x - 2)$ and $(2y - 3)$

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∴ Factorisation (Algebra) :-

②. Factorise using identities :-

$$\bullet (a+b)^2 = a^2 + 2ab + b^2$$

$$\bullet (a-b)^2 = a^2 - 2ab + b^2$$

$$\bullet (a+b)(a-b) = a^2 - b^2$$

Ex - $x^2 + 8x + 16$

Sol. $16 = 4 \times 4 = (4)^2$

$$8x = 2 \cdot 4 \cdot x$$

$$x^2 + 2 \cdot x \cdot 4 + 4^2 \quad \left[\text{using } (a+b)^2 = a^2 + 2ab + b^2 \right]$$

$$(x+4)^2$$

$$(x+4)(x+4)$$

Ex - $49p^2 - 36$

Sol. $49p^2 - 36$

$$49p^2 = 7 \times 7 \times p^2 = 7^2 \times p^2 = (7p)^2$$

$$36 = 2 \times 2 \times 3 \times 3 = 6 \times 6 = (6)^2$$

$$= 49p^2 - 36$$

$$= (7p)^2 - (6)^2 \quad \left[\text{using } (a+b)(a-b) = a^2 - b^2 \right]$$

$$= (7p+6)(7p-6)$$

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Q. Factors of the form $(x+a)(x+b)$

$$(x+a)(x+b) = x^2 + (a+b)x + ab \text{ (imp)}$$

Ex - Factorise $x^2 + 5x + 6$

Sol. $x^2 + 5x + 6$

$$ax^2 + bx + c$$

compare identified variables and constant

$$a=1, b=5, c=6$$

$$a \times c = 1 \times 6 = 6$$

$$a+b = 5 \text{ (find ?)}$$

Let factor of 6

$$\begin{array}{r|l} 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$a \times b = a \times b \quad 2 \times 3 = 6$$

$$a+b = a+b \quad 2+3 = 5$$

Formula : —

$$x^2 + (a+b)x + ab$$

$$x^2 + (2+3)x + 2 \times 3$$

$$x^2 + 2x + 3x + 6$$

$$x(x+2) + 3(x+2) \text{ (grouping)}$$

$$(x+2)(x+3)$$

Therefore, factors of $x^2 + 5x + 6$ are $(x+2)$ and $(x+3)$

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∴ Factorisation (Algebra): -

(A). Some more example: - (for negative term)

Find the factors of $y^2 - 7y + 12$

$$\text{Sol. } y^2 - 7y + 12$$

$$ax^2 + bx + c$$

$$a \times c = 1 \times 12$$

$$= 12$$

factor of 12

Formula use: -

$$x^2 + (a+b)x + ab$$

$$x^2 + (-4-3)x + (-4) \times (-3) \rightarrow (\text{imp term})$$

$$x^2 - 4x - 3x + 12$$

$$x(x-4) - 3(x-4)$$

$$(x-4)(x-3)$$

$$4 \leftarrow \begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$3 \leftarrow$$

If a and c are positive and b is negative then both factors are in negative

$$a + b = -4 - 3 = -7$$

$$a \times b = -4 \times -3 = 12$$

Therefore, factors are $(x-4)(x-3)$

Try: -

$$\cdot z^2 - 4z - 12 \quad (a \text{ is negative and } b \text{ is positive})$$

$$\cdot 3m^2 + 9m + 6 \quad (both \text{ are positive})$$

$$\cdot (l+m)^2 - 4lm \quad (open (a+b)^2 = a^2 + 2ab + b^2)$$

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∴ Factorisation (Algebra) :-

Q. Division of algebraic expressions :-

• Division of a monomial by another monomial :-

Ex - $6x^3 \div 2x$

Sol. $6x^3 \div 2x$

$$6x^3 = 6 \times x^3$$
$$= 2 \times 3 \times x \times x \times x$$

$$2x = 2 \times x$$



We group factors of $6x^3$ to separate $2x$



$$6x^3 = 2 \times x \times 3 \times x \times x$$
$$= (2x) \times (3x^2)$$

Therefore, $6x^3 \div 2x = 3x^2$

Shortest way to depict cancellation of common factors is as we do in division of numbers :-

Ex - (Integer) $77 \div 7 = \frac{77}{7} = \frac{7 \times 11}{7} = 11$

Similarly :- $6x^3 \div 2x = \frac{6x^3}{2x}$

$$= \frac{2 \times 3 \times x \times x \times x}{2 \times x}$$

$$= 3 \times x \times x$$

$$= 3x^2$$

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∴ Factorisation (Algebra) :-

Q. Division of a polynomial by a monomial :-

Ex - $4y^3 + 5y^2 + 6y$ of the monomial $2y$

Sol. $4y^3 + 5y^2 + 6y \div 2y$ [required common $2y$]

$$\begin{aligned}
 4y^3 + 5y^2 + 6y &= 2 \times 2xy \times y \times y + 5xy \times y + 2 \times 3y \\
 &= 2 \times 2xy \times y \times y + 5 \times \frac{2}{2} xy \times y + 2 \times 3xy \\
 &= 2y (2xy \times y + \frac{5}{2}xy + 3) \\
 &= 2y (2y^2 + \frac{5}{2}y + 3)
 \end{aligned}$$

therefore, $4y^3 + 5y^2 + 6y \div 2y$ is $(2y^2 + \frac{5}{2}y + 3)$

Alternate, Cancellation method :-

$$4y^3 + 5y^2 + 6y \div 2y = \frac{4y^3 + 5y^2 + 6y}{2y}$$

$$\begin{aligned}
 &= \frac{4y^3}{2y} + \frac{5y^2}{2y} + \frac{6y}{2y} \\
 &= \frac{2 \times 2xy \times y \times y}{2xy} + \frac{5xy \times y}{2xy} + \frac{2 \times 3xy}{2xy} \\
 &= 2xy \times y + \frac{5}{2}xy + 3 \\
 &= 2y^2 + \frac{5}{2}y + 3
 \end{aligned}$$

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-! Factorisation (Algebra): -

Q. Division of algebraic Expressions continued: -
(Polynomial $\div$ polynomial)Ex - Divide $44(x^4 - 5x^3 - 24x^2)$ by $11x(x-8)$ Sol. $44(x^4 - 5x^3 - 24x^2) \div 11x(x-8)$

$$\begin{aligned}
 44(x^4 - 5x^3 - 24x^2) &= 2 \times 2 \times 11 (x \times x \times x \times x - 5x \times x \times x - 24x \times x \times x) \\
 &= 2 \times 2 \times 11 x x x x (x^2 - 5x - 24) \\
 &= 2 \times 2 \times 11 x x x x [x^2 - 8x + 3x - 24] \\
 &= 2 \times 2 \times 11 x x x x [x(x-8) + 3(x-8)] \\
 &= 2 \times 2 \times 11 x x x x [(x-8)(x+3)] \\
 &= 2 \times 2 \times 11 x x^2 (x-8)(x+3)
 \end{aligned}$$

Therefore, $44(x^4 - 5x^3 - 24x^2) \div 11x(x-8)$

$$= \frac{2 \times 2 \times 11 x x x x (x-8)(x+3)}{11 x x x (x-8)}$$

$$= 2 \times 2 \times x (x+3)$$

$$= 4x(x+3)$$



$$\begin{array}{r}
 2 \overline{) 24} \\
 \underline{2} \\
 0 \\
 2 \overline{) 12} \\
 \underline{2} \\
 0 \\
 2 \overline{) 6} \\
 \underline{2} \\
 0 \\
 3 \overline{) 3} \\
 \underline{3} \\
 0
 \end{array}$$

Solve by

$$ax^2 + bx + c$$

$$x^2 - 8x + 5x - 24 \quad a \times c = 1 \times -24 = -24$$

$$\downarrow \quad -8 + 3 = -5$$

$$\text{see above} \quad -8 \times 3 = -24$$

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∴ Factorisation (Algebra) :-

Q. Find the common factors of the given terms.

(i) $12x, 36$ (ii) $2y, 22xy$ (iii) $14p^2, 28p^2q^2$

(iv) $2x, 3x^2, 4$ (v) $6abc, 24ab^2, 12a^2b$

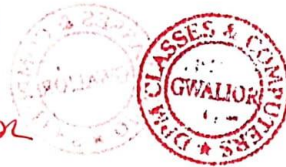
(vi) $16x^3, -4x^2, 32x$ (vii) $10p^2, 20p^2, 30xp$

(viii) $3x^2y^3, 10x^3y^3, 6x^2y^2z$

Sol. (i) $12x, 36$

$12x = 2 \times 2 \times 3 \times x$

$36 = 2 \times 2 \times 3 \times 3$



$$\begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

The common factors are 2, 2, 3

(v) $6abc, 24ab^2, 12a^2b$

$6abc = 2 \times 3 \times a \times b \times c$

$24ab^2 = 2 \times 2 \times 2 \times 3 \times a \times b \times b$

$12a^2b = 2 \times 2 \times 3 \times a \times a \times b$

$$\begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

The common factors are 2, 3, a, b

[do other itself]



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∴ Factorisation (Algebra) :-

Q. Factorise the following expressions :-

(i) $7x - 42$ (ii) $6p - 12q$ (iii) $7a^2 + 14a$

(iv) $-16z + 20z^3$ (v) $20l^2m + 30alm$ (vi) $5x^2y - 15xy^2$

(vii) $10a^2 - 15b^2 + 20c^2$ (viii) $-4a^2 + 4ab - 4ca$

(ix) $x^2yz + xy^2z + xyz^2$ (x) $ax^2y + bxy^2 + cxyz$

Sol. (i) $7x - 42$

$$7x = 7 \times x$$

$$42 = 2 \times 3 \times 7$$



The common factor is 7

$$\begin{aligned} 7x - 42 &= (7 \times x) - (2 \times 3 \times 7) \\ &= 7(x - 6) \end{aligned}$$

(ix) $x^2yz + xy^2z + xyz^2$

$$x^2yz = (x \times x \times y \times z)$$

$$xy^2z = (x \times y \times y \times z)$$

$$xyz^2 = (x \times y \times z \times z)$$



The common factors are x, y, z

$$x^2yz + xy^2z + xyz^2 = xyz(x + y + z)$$

[do other itself]

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∴ Factorisation (Algebra) :-

Q) Factorise :

(i) $x^2 + xy + 8x + 8y$

(ii) $15xy - 6x + 5y - 2$

(iii) $ax + bx - ay - by$

(iv) $15p^2 + 15 + 9p + 25p$

(v) $z - 7 + 7xy - xyz$

Sol. (i) $x^2 + xy + 8x + 8y$

$$= x^2 + xy + 8x + 8y$$

$$= x(x+y) + 8(x+y)$$

$$= (x+y)(x+8)$$



(iii) $ax + bx - ay - by$

$$= ax + bx - ay - by$$

$$= x(a+b) - y(a+b)$$

$$= (a+b)(x-y)$$

(v) $z - 7 + 7xy - xyz$

$$= z - 7 + 7xyz - xyz$$

$$= 1(z-7) + xy(7-z)$$

$$[(z-7) = -(7-z)]$$

$$= 1(z-7) - xy(z-7)$$

$$= (z-7)(1-xy)$$



[do other itself]

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∴ Factorisation (Algebra) :-

Q. Factorise the following expressions:

(i) $a^2 + 8a + 16$ (ii) $p^2 - 10p + 25$

(iii) $25m^2 + 30m + 9$ (iv) $49y^2 + 84yz + 36z^2$

(v) $4x^2 - 8x + 4$ (vi) $121b^2 - 88bc + 16c^2$

(vii) $(l+m)^2 - 4lm$ (viii) $a^4 + 2a^2b^2 + b^4$

Sol. (iii) $25m^2 + 30m + 9$

$$= 5^2m^2 + 30m + 3^2$$

$$= (5m)^2 + 30m + (3)^2$$

[Formula: $a^2 + 2ab + b^2 = (a+b)^2$]

$$= (5m)^2 + 2 \cdot 5m \cdot 3 + (3)^2 \text{ [Verified]}$$

$$= (5m+3)^2$$

$$= (5m+3)(5m+3)$$



$$\left[\begin{array}{l} 25 = 5^2 \\ 9 = 3^2 \\ [5^2m^2 = (5m)^2] \end{array} \right]$$

(vii) $(l+m)^2 - 4lm$

[Formula: $(a+b)^2 = a^2 + 2ab + b^2$]

$$= l^2 + 2lm + m^2 - 4lm$$

$$= l^2 - 2lm + m^2$$

[Formula: $a^2 - 2ab + b^2 = (a-b)^2$]

$$= l^2 - 2 \cdot l \cdot m + m^2$$

$$= (l-m)^2$$

[do other itself]

$$= (l-m)(l-m)$$

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∴ Factorisation (Algebra) :-

Q. Factorise :

(i) $4p^2 - 9q^2$ (ii) $63a^2 - 112b^2$ (iii) $49x^2 - 36$

(iv) $16x^5 - 144x^3$ (v) $(l+m)^2 - (l-m)^2$ (vi) $9x^2y^2 - 16$

(vii) $(x^2 - 2xy + y^2) - z^2$ (viii) $25a^2 - 4b^2 + 28bc - 49c^2$

Sol. (ii) $63a^2 - 112b^2$

$$= 7(9a^2 - 16b^2)$$

$$= 7[3^2a^2 - 4^2b^2]$$

$$= 7[(3a)^2 - (4b)^2] \quad [\text{Formula} = a^2 - b^2 = (a+b)(a-b)]$$

$$= 7[(3a+4b)(3a-4b)]$$

$$= 7(3a+4b)(3a-4b)$$

$$\begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 112 \\ \hline 2 & 56 \\ \hline 2 & 28 \\ \hline 2 & 14 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

(v) $(l+m)^2 - (l-m)^2$

$$[\text{Formula} : (a+b)^2 = a^2 + 2ab + b^2, (a-b)^2 = a^2 - 2ab + b^2]$$

$$= (l^2 + 2lm + m^2) - (l^2 - 2lm + m^2)$$

$$= \cancel{l^2} + 2lm + \cancel{m^2} - \cancel{l^2} + 2lm - \cancel{m^2}$$

$$= 4lm$$



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Factorisation (Algebra)

(vii) $(x^2 - 2xy + y^2) - z^2$

[Formula: $a^2 - 2ab + b^2 = (a-b)^2$]

$= (x^2 - 2xy + y^2) - z^2$

$= (x-y)^2 - z^2$

[Formula: $a^2 - b^2 = (a+b)(a-b)$]

$= (x-y+z)(x-y-z)$



(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

Rearrange according to $28bc$

$= 25a^2 - (4b^2 - 28bc + 49c^2)$

$= 5^2a^2 - [2^2b^2 - 28bc + 7^2c^2]$

$= (5a)^2 - [(2b)^2 - 28bc + (7c)^2]$

[Formula: $a^2 - 2ab + b^2 = (a-b)^2$]

$= (5a)^2 - [(2b)^2 - 2 \cdot 2b \cdot 7c + (7c)^2]$

$= (5a)^2 - (2b-7c)^2$



[Formula: $a^2 - b^2 = (a+b)(a-b)$]

$= (5a + 2b - 7c)(5a - 2b + 7c)$

[do other itself]

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~ Factorisation (Algebra) ~

Q. Factorise the expressions:

(i) $ax^2 + bx$ (ii) $7p^2 + 21q^2$ (iii) $2x^3 + 2xy^2 + 2xz^2$

(iv) $am^2 + bm^2 + bn^2 + an^2$ (v) $(lm+l) + m+1$

(vi) $y(y+z) + 9(y+z)$ (vii) $5y^2 - 20y - 8z + 2yz$

(viii) $10ab + 4a + 5b + 2$ (ix) $6xy - 4y + 6 - 9x$

Sol. (i) $ax^2 + bx$

$$= ax \times x + bx$$

$$= x(ax+b)$$



(iv) $am^2 + bm^2 + bn^2 + an^2$

$$m^2(a+b) + n^2(b+a)$$

$$m^2(a+b) + n^2(a+b)$$

$$(a+b)(m^2+n^2)$$

(v) $(lm+l) + m+1$

$$l(m+1) + (m+1)$$

$$(m+1)(l+1)$$



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∴ Factorisation (Algebra) :-

$$(vii) \quad 5y^2 - 20y - 8z + 2yz$$

$$= 5xyxy - 4x5xy - 2x4xz + 2xyxz$$

$$= 5y(y-4) + 2z(-4+y)$$

$$= 5y(y-4) + 2z(y-4)$$

$$= (y-4)(5y+2z)$$



$$(viii) \quad 10ab + 4a + 5b + 2$$

$$= 2 \times 5 \times a \times b + 2 \times 2 \times a + 5 \times b + 2$$

$$= 2a(5b+2) + (5b+2)$$

$$= (5b+2)(2a+1)$$



$$(ix) \quad 6xy - 4y + 6 - 9x$$

$$= 2 \times 3 \times x \times y - 2 \times 2 \times y + 2 \times 3 - 3 \times 3 \times x$$

$$= 2y(3x-2) + 3(2-3x) \quad [(3x-2) = -(2-3x)]$$

$$= 2y(3x-2) - 3(3x-2)$$

$$= (3x-2)(2y-3)$$



[do other it self]

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~ Factorisation (Algebra) ~

Q. Factorise:

(i) $a^4 - b^4$

(ii) $p^4 - 81$

(iii) $x^4 - (y+4)^4$

(iv) $x^4 - (x-z)^4$

(v) $a^4 - 2a^2b^2 + b^4$

Sol. (i) $a^4 - b^4$
 $= (a^2)^2 - (b^2)^2$

[Formula: $a^2 - b^2 = (a+b)(a-b)$]

$= (a^2 + b^2)(a^2 - b^2)$

$= (a^2 + b^2)(a+b)(a-b)$

(iii) $x^4 - (y+4)^4$

$= (x^2)^2 - [(y+4)^2]^2$

[Formula: $a^2 - b^2 = (a+b)(a-b)$]

$= [x^2 + (y+4)^2] [x^2 - (y+4)^2]$

$= [x^2 + (y+4)^2] (x+y+4)(x-y-4)$

(ii) $p^4 - 81$

$p^4 - 9^2$

$(p^2)^2 - (9)^2$

[Formula: $a^2 - b^2 = (a+b)(a-b)$]

$(p^2 + 9)(p^2 - 9)$

$(p^2 + 9)(p+3)(p-3)$

[$p^2 - 9 = p^2 - 3^2$]

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∴ Factorisation (Algebra) :-

$$(iv) \quad x^4 - (x-z)^4$$

$$= (x^2)^2 - [(x-z)^2]^2$$

$$[\text{Formula: } a^2 - b^2 = (a+b)(a-b)]$$

$$= [x^2 + (x-z)^2] [x^2 - (x-z)^2]$$

$$= [x^2 + x^2 - 2xz + z^2] (x + x - z) (x - x + z)$$

$$= (2x^2 - 2xz + z^2) (2x - z)(z)$$



$$(v) \quad a^4 - 2a^2b^2 + b^4$$

$$= (a^2)^2 - 2 \cdot a^2b^2 + (b^2)^2$$

$$[\text{Formula: } (a-b)^2 = a^2 - 2ab + b^2]$$

$$= [a^2 - b^2]^2$$

$$[\text{Formula: } a^2 - b^2 = (a+b)(a-b)]$$

$$= [(a+b)(a-b)]^2$$

$$= (a+b)^2 (a-b)^2$$



do other itself.

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∴ Factorisation (Algebra) :-

Q. Factorise the following expressions :- (imp)

(i) $p^2 + 6p + 8$ (ii) $q^2 - 10q + 21$ (iii) $p^2 + 6p - 16$

Sol. (i) $p^2 + 6p + 8$ (Use split of middle term)

Compare $ax^2 + bx + c$

$$= p^2 + 4p + 2p + 8$$

$$= p(p+4) + 2(p+4)$$

$$= (p+4)(p+2)$$

$$a \times c = 1 \times 8 = 8$$

$$= 4 \times 2 = 8$$

$$b = 6 = 4 + 2$$



$$\begin{array}{r|l} 2 & 8 \\ 4 \swarrow & 4 \\ \hline 2 & 2 \\ 2 \swarrow & 1 \end{array}$$

(ii) $q^2 - 10q + 21$

$$= ax^2 + bx + c$$

$$= q^2 - 3q - 7q + 21$$

$$= q(q-3) - 7(q-3)$$

$$= (q-3)(q-7)$$

$$a \times c = 1 \times 21 = 21$$

$$= -3 \times -7 = 21$$

$$b = -10 = -3 - 7$$

$$\begin{array}{r|l} 3 & 21 \\ 7 \swarrow & 7 \\ \hline 7 & 7 \\ 7 \swarrow & 1 \end{array}$$

(iii) $p^2 + 6p - 16$

$$= ax^2 + bx + c$$

$$= p^2 + 8p - 2p - 16$$

$$= p(p+8) - 2(p+8)$$

$$= (p+8)(p-2)$$



$$a \times c = 1 \times -16 = -16$$

$$= -8 \times 2 = -16$$

$$b = 6$$

$$= 8 - 2$$

$$\begin{array}{r|l} 2 & 16 \\ -2 \swarrow & 8 \\ \hline 2 & 8 \\ +8 \swarrow & 2 \\ \hline 2 & 2 \\ 2 \swarrow & 1 \end{array}$$

DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Factorisation (Algebra) :-

Q. Carry out the following divisions :-

(i) $28x^4 \div 56x$ (ii) $-36y^3 \div 9y^2$

(iii) $66p^2q^3 \div 11q^2$ (iv) $34x^3y^3z^3 \div 51xy^2z^3$

Sol.

(i) $28x^4 = 2 \times 2 \times 7 \times x \times x \times x \times x$

$56x = 2 \times 2 \times 2 \times 7 \times x$

$28x^4 \div 56x$

$$\frac{28x^4}{56x} = \frac{2 \times 2 \times 7 \times x \times x \times x \times x}{2 \times 2 \times 2 \times 7 \times x}$$

$$= \frac{x^3}{2} = \frac{1}{2}x^3$$

(iv) $34x^3y^3z^3 = 2 \times 17 \times x \times x \times x \times y \times y \times y \times z \times z \times z$

$51xy^2z^3 = 3 \times 17 \times x \times y \times y \times z \times z \times z$

$$\frac{34x^3y^3z^3}{51xy^2z^3} = \frac{2 \times 17 \times x \times x \times x \times y \times y \times y \times z \times z \times z}{3 \times 17 \times x \times y \times y \times z \times z \times z}$$



$$\frac{2x^2y}{3} = \frac{2}{3}x^2y$$

[do other it self]

DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

~ Factorisation (Algebra) ! ~

Q. Divide the given polynomial by the given monomials :

(i) $(5x^2 - 6x) \div 3x$ (ii) $(3y^8 - 4y^6 + 5y^4) \div y^4$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$

(iv) $(x^3 + 2x^2 + 3x) \div 2x$ (v) $(p^3q^6 - p^6q^3) \div p^3q^3$

Sol. (iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$

$$= 8x^2y^2z^2(x+y+z) \div 4x^2y^2z^2$$

$$= \frac{8x^2y^2z^2(x+y+z)}{4x^2y^2z^2}$$

$$= 2(x+y+z)$$



(iv) $(x^3 + 2x^2 + 3x) \div 2x$

$$= x(x^2 + 2x + 3) \div 2x$$

$$= \frac{x(x^2 + 2x + 3)}{2x}$$

$$= \frac{x^2 + 2x + 3}{2}$$

$$= \frac{1}{2}(x^2 + 2x + 3) \text{ [do other it self]}$$



DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Factorisation (Algebra) :-

Q. Work out the following divisions :-

(i) $(10x-25) \div 5$ (ii) $(10x-25) \div (2x-5)$

(iii) $10y(6y+21) \div 5(2y+7)$ (iv) $9x^2y^2(3z-24) \div 27xy(z-8)$

(v) $96abc(3a-12)(5b-30) \div 144(a-4)(b-6)$

Sol. (i) $9x^2y^2(3z-24) \div 27xy(z-8)$

$$= \frac{9x^2y^2(3z-24)}{27xy(z-8)}$$

$$= \frac{xy(3z-24)}{3(z-8)}$$

$$= \frac{xy \times 3(z-8)}{3(z-8)}$$

$$= xy$$

(ii) $(10x-25) \div (2x-5)$

$$= 5(2x-5) \div (2x-5)$$

$$= \frac{5(2x-5)}{(2x-5)}$$

$$= 5$$

[do other itself]



DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Factorisation (Algebra) :-

Q. Factorise the expressions and divide them as directed : (imp)

(i) $(y^2 + 7y + 10) \div (y + 5)$ (ii) $(m^2 - 14m - 32) \div (m + 2)$

(iii) $(5p^2 - 25p + 20) \div (p - 1)$ (iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

(v) $5p^2(p^2 - 2^2) \div 2p(p + 2)$ (vi) $12xy(x^2 - 16y^2) \div 4xy(x + 4y)$

(vii) $39y^3(50y^2 - 98) \div 26y^2(5y + 7)$

Sol.

$$\begin{aligned}
 \text{(i)} \quad & y^2 + 7y + 10 \\
 &= ax^2 + bx + c \quad (\text{compare}) \\
 &= y^2 + 5y + 2y + 10 \\
 &= y(y + 5) + 2(y + 5) \\
 &= (y + 5)(y + 2)
 \end{aligned}$$



$a \times c = 1 \times 10 = 10$

$= 5 \times 2 = 10$

$b = 5 + 2 = 7$

$$\begin{array}{r}
 2 \overline{) 10} \\
 \underline{5} \\
 5
 \end{array}$$

$= (y^2 + 7y + 10) \div (y + 5)$

$$= \frac{(y + 5)(y + 2)}{(y + 5)} = (y + 2)$$

(iii) $5p^2 - 25p + 20$

$ax^2 + bx + c$

$5p^2 - 5p - 20p + 20$

$5p(p - 1) - 20(p - 1)$

$(p - 1)(5p - 20)$

$(5p^2 - 25p + 20) \div (p - 1)$



$a \times c = 5 \times 20 = 100$

$= -5 \times -20 = 100$

$b = -25 = -5 - 20$

$$\begin{array}{l}
 5p^2 - 25p + 20 \\
 = \frac{\quad}{(p - 1)}
 \end{array}$$

$$\begin{array}{l}
 = \frac{(5p - 20)(p - 1)}{(p - 1)} \\
 = 5(p - 4)
 \end{array}$$

24.

DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Factorisation (Algebra) :-

Q. Divide as directed :-

(i) $5(2x+1)(3x+5) \div (2x+1)$

(ii) $26xy(x+5)(y-4) \div 13x(y-4)$

(iii) $52pqr(p+2)(q+r)(r+p) \div 104pr(q+r)(r+p)$

(iv) $20(y+4)(y^2+5y+3) \div 5(y+4)$

(v) $x(x+1)(x+2)(x+3) \div x(x+1)$

Sol. (ii) $26xy(x+5)(y-4) \div 13x(y-4)$

$$= \frac{26xy(x+5)(y-4)}{13x(y-4)}$$



$$= 2y(x+5)$$

(iii) $52pqr(p+2)(q+r)(r+p) \div 104pr(q+r)(r+p)$

$$= \frac{52pqr(p+2)(q+r)(r+p)}{104pr(q+r)(r+p)}$$

$$= \frac{(p+2)}{2} = \frac{1}{2}(p+2)$$

[do other as self].



DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Factorisation (Algebra) :-

$$(iv) \quad 4yz (z^2 + 6z - 16) \div 2y (z+8)$$

$$= z^2 + 6z - 16$$

$$= ax^2 + bx + c$$

$$= z^2 + 8z - 2z - 16$$

$$= z(z+8) - 2(z+8)$$

$$= (z+8)(z-2)$$

$$= 4yz (z^2 + 6z - 16) \div 2y (z+8)$$

$$= \frac{4yz (z+8)(z-2)}{2y (z+8)}$$

$$= 2z (z-2)$$

$$(v) \quad 5p^2 (p^2 - 2^2) \div 2p (p+2)$$

$$= 5p^2 (p+2)(p-2) \div 2p (p+2)$$

$$= \frac{5p^2 (p+2)(p-2)}{2p (p+2)}$$

$$= \frac{5}{2} p (p-2)$$

$$a \times c = 1 \times -16 = -16$$

$$= 8 \times -2 = -16$$

$$b = 6 = 8 - 2$$

$$\begin{array}{r|l} 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Factorisation (Algebra): -

$$(vi) \quad 12xy (9x^2 - 16y^2) \div 4xy (3x + 4y)$$

$$\begin{aligned} & 9x^2 - 16y^2 \\ &= 3^2x^2 - 4^2y^2 \\ &= (3x)^2 - (4y)^2 \quad [a^2 - b^2 = (a+b)(a-b)] \\ &= (3x+4y)(3x-4y) \end{aligned}$$

$$12xy (9x^2 - 16y^2) \div 4xy (3x + 4y)$$

$$= \frac{12xy (3x+4y)(3x-4y)}{4xy (3x+4y)}$$

$$= 3 (3x-4y)$$

$$(vii) \quad 39y^3 (50y^2 - 98) \div 26y^2 (5y + 7)$$

$$\begin{aligned} & 50y^2 - 98 \\ &= 2(25y^2 - 49) \\ &= 2[s^2y^2 - 7^2] \\ &= 2[(5y)^2 - (7)^2] \\ &= 2(5y+7)(5y-7) \end{aligned}$$

$$39y^3 (50y^2 - 98) \div 26y^2 (5y + 7)$$

$$= \frac{39y^3 \cancel{x} \cancel{x} (5y+7)(5y-7)}{\cancel{26}y^2 (5y+7)}$$

$$= 3y (5y-7)$$

DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Factorisation (Algebra) :-

Q. Find the correct the errors in the following mathematical statements.

1. $4(x-5) = 4x-5$

2. $x(3x+2) = 3x^2 + 2$

3. $2x+3y = 5xy$

4. $x+2x+3x = 5x$

5. $5y+2y+y-7y = 0$

6. $3x+2x = 5x^2$

7. $(2x)^2 + 4(2x) + 7 = 2x^2 + 8x + 7$

8. $(2x)^2 + 5x = 4x + 5x = 9x$

9. $(3x+2)^2 = 3x^2 + 6x + 4$

10. Substituting $x = -3$ in

(a) $x^2 + 5x + 4$ gives $(-3)^2 + 5(-3) + 4 = 9 - 15 + 4 = -2$

(b) $x^2 - 5x + 4$ gives $(-3)^2 - 5(-3) + 4 = 9 + 15 + 4 = 28$

(c) $x^2 + 5x$ gives $(-3)^2 + 5(-3) = 9 - 15 = -6$

11. $(y-3)^2 = y^2 - 9$

12. $(z+5)^2 = z^2 + 25$

13. $(2a+3b)(a-b) = 2a^2 - 3b^2$

14. $(a+4)(a+2) = a^2 + 8$

15. $(a-4)(a-2) = a^2 - 8$

16. $\frac{3x^2}{3x^2} = 0$

17. $\frac{3x^2+1}{3x} = 1+1 = 2$

18. $\frac{3x}{3x+2} = \frac{1}{2}$

19. $\frac{3}{4x+3} = \frac{1}{4x}$

20. $\frac{4x+5}{4x} = 5$

21. $\frac{7x+5}{5} = 7x$

DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Factorisation (Algebra) :-

$$1. \quad x(3x+2) = 3x^2 + 2x$$

$$\text{LHS} = x(3x+2)$$

$$= 3x^2 + 2x \quad \text{This is correct statement}$$

$$= \text{RHS}$$

$$8. \quad (2x)^2 + 5x = 4x + 5x = 9x$$

$$\text{LHS} = (2x)^2 + 5x$$

$$= 4x^2 + 5x$$

$$= \text{RHS} \quad \text{This is correct statement}$$



$$10. (a) \quad x^2 + 5x + 4 \text{ given } (-3)^2 + 5(-3) + 4 \neq 9 + 2 + 4 = 15$$

$$\text{LHS} = x^2 + 5x + 4$$

$$= (-3)^2 + 5(-3) + 4$$

$$= 9 - 15 + 4$$

$$= 13 - 15$$

$$= -2 \quad \text{This is correct answer}$$

$$11. \quad (y-3)^2 = y^2 - 9$$

$$\text{LHS} = (y-3)^2$$

$$= y^2 - 2 \cdot y \cdot 3 + 3^2 \quad [a^2 - 2ab + b^2 = (a-b)^2]$$

$$= y^2 - 6y + 9 \quad \text{This is correct answer}$$

[do other itself]

