MATHS -8 (CH-09 – ALGERAIC EXPRESSIONS & IDENTITIES)

MATHS -8 (CH-09 -9.1- ALGERAIC EXPRESSIONS & IDENTITIES)

Question 1:

Identify the terms, their coefficients for each of the following expressions.

(i) $5xyz^2 - 3zy$

(ii) $1 + x + x^2$

(iii) $4x^2y^2 - 4x^2y^2z^2 + z^2$

(iv) $3 - pq + qr - rp$

(v) $\frac{x}{2}+\frac{y}{2}-xy$

(vi) $0.3a - 0.6ab + 0.5b$

Answer 1:

The terms and the respective coefficients of the given expressions are as follows.

-	Terms	Coefficients
(i)	$5xyz^2$ $-3zy$	5 −3
(ii)	1 x x^2	1 1 1
(iii)	$4x^2y^2$ $-4x^2y^2z^2$ z^2	4 −4 1
(iv)	3 $-pq$ qr $-rp$	3 −1 1 −1
(v)	$\frac{x}{2}$ $\frac{y}{2}$ $-xy$	$\frac{1}{2}$ $\frac{1}{2}$ −1
(vi)	$0.3a$ $-0.6ab$ $0.5b$	0.3 −0.6 0.5

Question 2:

Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories?

$x + y$, 1000, $x + x^2 + x^3 + x^4$, $7 + y + 5x$, $2y - 3y^2$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$, $4z - 15z^2$, $ab + bc + cd + da$, pqr, $p^2q + pq^2$, $2p + 2q$

Answer 2:

The given expressions are classified as

Monomials: 1000, pqr

Binomials: $x + y$, $2y - 3y^2$, $4z - 15z^2$, $p^2q + pq^2$, $2p + 2q$

Trinomials: $7 + y + 5x$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$

Polynomials that do not fit in any of these categories are

$x + x^2 + x^3 + x^4$, $ab + bc + cd + da$

Question 3:

Add the following.

(i) $ab - bc$, $bc - ca$, $ca - ab$

(ii) $a - b + ab$, $b - c + bc$, $c - a + ac$

(iii) $2p^2q^2 - 3pq + 4$, $5 + 7pq - 3p^2q^2$

(iv) $l^2 + m^2$, $m^2 + n^2$, $n^2 + l^2$, $2lm + 2mn + 2nl$

Answer 3:

The given expressions written in separate rows, with like terms one below the other and then the addition of these expressions are as follows.

(i)

$$\begin{array}{rrr} & ab - bc & \\ + & bc - ca & \\ + & -ab \quad & +ca \\ \hline & 0 & \end{array}$$

Thus, the sum of the given expressions is 0.

(ii)

$$\begin{array}{rrrr} & a - b + ab & & \\ + & b & -c + bc & \\ + & -a & c & +ac \\ \hline & ab & +bc + ac & \end{array}$$

Thus, the sum of the given expressions is $ab + bc + ac$.

(iii)

$$\begin{array}{r} 2p^2q^2 - 3pq + 4 \\ +\quad -3p^2q^2 + 7pq + 5 \\ \hline -\ p^2q^2 + 4pq + 9 \end{array}$$

Thus, the sum of the given expressions is $-p^2q^2 + 4pq + 9$.

(iv)

$$\begin{array}{r} l^2 + m^2 \\ +\quad\ m^2 + n^2 \\ +\quad l^2 \quad\ + n^2 \\ +\quad\ \ 2lm + 2mn + 2nl \\ \hline 2l^2 + 2m^2 + 2n^2 + 2lm + 2mn + 2nl \end{array}$$

Thus, the sum of the given expressions is $2(l^2 + m^2 + n^2 + lm + mn + nl)$.

Question 4:

(a) Subtract $4a - 7ab + 3b + 12$ from $12a - 9ab + 5b - 3$

(b) Subtract $3xy + 5yz - 7zx$ from $5xy - 2yz - 2zx + 10xyz$

(c) Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$

Answer 4:

The given expressions in separate rows, with like terms one below the other and then the subtraction of these expressions is as follows.

(a)

$$\begin{array}{r} 12a - 9ab + 5b - 3 \\ 4a - 7ab + 3b + 12 \\ \hline (-)\quad (+)\quad (-)\quad (-) \\ \hline 8a - 2ab + 2b - 15 \end{array}$$

(b)

$$\begin{array}{r} 5xy - 2yz - 2zx + 10xyz \\ 3xy + 5yz - 7zx \\ \hline (-)\quad (-)\quad (+) \\ \hline 2xy - 7yz + 5zx + 10xyz \end{array}$$

(c)

$$\begin{array}{r} 18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q \\ -10 - 8p + 7q - 3pq + 5pq^2 + 4p^2q \\ \hline (+)\ (+)\ (-)\ (+)\ (-)\ (-) \\ \hline 28 + 5p - 18q + 8pq - 7pq^2 + p^2q \end{array}$$

MATHS -8 (CH-09 -9.2- ALGERAIC EXPRESSIONS & IDENTITIES)

Question 1:

Find the product of the following pairs of monomials.
(i) 4, $7p$ (ii) $-4p$, $7p$ (iii) $-4p$, $7pq$
(iv) $4p^3$, $-3p$ (v) $4p$, 0

Answer 1:

The product will be as follows.
(i) $4 \times 7p = 4 \times 7 \times p = 28p$
(ii) $-4p \times 7p = -4 \times p \times 7 \times p = (-4 \times 7) \times (p \times p) = -28\,p^2$
(iii) $-4p \times 7pq = -4 \times p \times 7 \times p \times q = (-4 \times 7) \times (p \times p \times q) = -28p^2q$
(iv) $4p^3 \times -3p = 4 \times (-3) \times p \times p \times p \times p = -12\,p^4$
(v) $4p \times 0 = 4 \times p \times 0 = 0$

Question 2:

Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively.
(p, q); ($10m$, $5n$); ($20x^2$, $5y^2$); ($4x$, $3x^2$); ($3mn$, $4np$)

Answer 2:

We know that,
Area of rectangle = Length × Breadth
Area of 1st rectangle = $p \times q = pq$
Area of 2nd rectangle = $10m \times 5n = 10 \times 5 \times m \times n = 50\,mn$
Area of 3rd rectangle = $20x^2 \times 5y^2 = 20 \times 5 \times x^2 \times y^2 = 100\,x^2y^2$
Area of 4th rectangle = $4x \times 3x^2 = 4 \times 3 \times x \times x^2 = 12x^3$
Area of 5th rectangle = $3mn \times 4np = 3 \times 4 \times m \times n \times n \times p = 12mn^2p$

Question 3:

Complete the table of products.

First monomial → / Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	...	...	...	...	...
$-5y$	...	...	$-15x^2y$	...	...	...
$3x^2$	...	...	...	...	...	...
$-4xy$	...	...	...	...	...	...
$7x^2y$	...	...	...	...	...	...
$-9x^2y^2$	...	...	...	...	...	...

Answer 3:

The table can be completed as follows.

First monomial → / Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

Question 4:

Obtain the volume of rectangular boxes with the following length, breadth and height respectively.
(i) $5a, 3a^2, 7a^4$ (ii) $2p, 4q, 8r$ (iii) $xy, 2x^2y, 2xy^2$
(iv) $a, 2b, 3c$

Answer 4:

We know that,
Volume = Length × Breadth × Height
(i) Volume = $5a \times 3a^2 \times 7a^4 = 5 \times 3 \times 7 \times a \times a^2 \times a^4 = 105\,a^7$
(ii) Volume = $2p \times 4q \times 8r = 2 \times 4 \times 8 \times p \times q \times r = 64pqr$
(iii) Volume = $xy \times 2x^2y \times 2xy^2 = 2 \times 2 \times xy \times x^2y \times xy^2 = 4x^4y^4$

(iv) Volume = $a \times 2b \times 3c = 2 \times 3 \times a \times b \times c = 6abc$

Question 5:

Obtain the product of
(i) xy, yz, zx (ii) $a, -a^2, a^3$ (iii) $2, 4y, 8y^2, 16y^3$
(iv) $a, 2b, 3c, 6abc$ (v) $m, -mn, mnp$

Answer 5:

(i) $xy \times yz \times zx = x^2y^2z^2$
(ii) $a \times (-a^2) \times a^3 = -a^6$
(iii) $2 \times 4y \times 8y^2 \times 16y^3 = 2 \times 4 \times 8 \times 16 \times y \times y^2 \times y^3 = 1024\,y^6$
(iv) $a \times 2b \times 3c \times 6abc = 2 \times 3 \times 6 \times a \times b \times c \times abc = 36a^2b^2c^2$
(v) $m \times (-mn) \times mnp = -m^3n^2p$

MATHS -8 (CH-09 -9.3- ALGERAIC EXPRESSIONS & IDENTITIES)

Question 1:

Carry out the multiplication of the expressions in each of the following pairs.
(i) $4p, q + r$ (ii) $ab, a - b$ (iii) $a + b, 7a^2b^2$
(iv) $a^2 - 9, 4a$ (v) $pq + qr + rp, 0$

Answer 1:

(i) $(4p) \times (q + r) = (4p \times q) + (4p \times r) = 4pq + 4pr$
(ii) $(ab) \times (a - b) = (ab \times a) + [ab \times (-b)] = a^2b - ab^2$
(iii) $(a + b) \times (7a^2 b^2) = (a \times 7a^2b^2) + (b \times 7a^2b^2) = 7a^3b^2 + 7a^2b^3$
(iv) $(a^2 - 9) \times (4a) = (a^2 \times 4a) + (-9) \times (4a) = 4a^3 - 36a$
(v) $(pq + qr + rp) \times 0 = (pq \times 0) + (qr \times 0) + (rp \times 0) = 0$

Question 2:

Complete the table

---	First expression	Second Expression	Product
(i)	a	$b + c + d$	-
(ii)	$x + y - 5$	$5xy$	-
(iii)	p	$6p^2 - 7p + 5$	-
(iv)	$4p^2q^2$	$p^2 - q^2$	-
(v)	$a + b + c$	abc	-

Answer 2:

The table can be completed as follows.

-	First expression	Second Expression	Product
(i)	a	$b + c + d$	$ab + ac + ad$
(ii)	$x + y - 5$	$5xy$	$5x^2y + 5xy^2 - 25xy$
(iii)	p	$6p^2 - 7p + 5$	$6p^3 - 7p^2 + 5p$
(iv)	$4p^2q^2$	$p^2 - q^2$	$4p^4q^2 - 4p^2q^4$
(v)	$a + b + c$	abc	$a^2bc + ab^2c + abc^2$

Question 3:

Find the product.
(i) $(a^2) \times (2a^{22}) \times (4a^{26})$
(ii) $\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right)$
(iii) $\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right)$
(iv) $x \times x^2 \times x^3 \times x^4$

Answer 3:

(i) $(a^2) \times (2a^{22}) \times (4a^{26}) = 2 \times 4 \times a^2 \times a^{22} \times a^{26} = 8a^{50}$

(ii) $\left(\frac{2}{3}xy\right)\times\left(\frac{-9}{10}x^2y^2\right)=\left(\frac{2}{3}\right)\times\left(\frac{-9}{10}\right)\times x\times y\times x^2\times y^2=\frac{-3}{5}x^3y^3$

(iii) $\left(\frac{-10}{3}pq^3\right)\times\left(\frac{6}{5}p^3q\right)=\left(\frac{-10}{3}\right)\times\left(\frac{6}{5}\right)\times pq^3\times p^3q=-4p^4q^4$

(iv) $x \times x^2 \times x^3 \times x^4 = x^{10}$

Question 4:

(a) Simplify $3x(4x-5)+3$ and find its values for (i) $x = 3$, (ii) $x=\frac{1}{2}$.

(b) $a(a^2 + a + 1) + 5$ and find its values for (i) $a = 0$, (ii) $a = 1$, (iii) $a = -1$.

Answer 4:

(a) $3x(4x - 5) + 3 = 12x^2 - 15x + 3$

(i) For $x = 3$, $12x^2 - 15x + 3 = 12(3)^2 - 15(3) + 3$

$= 108 - 45 + 3$

$= 66$

(ii) For $x=\frac{1}{2}, 12x^2-15x+3=12\left(\frac{1}{2}\right)^2-15\left(\frac{1}{2}\right)+3$

$=12\times\frac{1}{4}-\frac{15}{2}+3$

$=3-\frac{15}{2}+3=6-\frac{15}{2}$

$=\frac{12-15}{2}=\frac{-3}{2}$

(b) $a(a^2 + a + 1) + 5 = a^3 + a^2 + a + 5$

(i) For $a = 0$, $a^3 + a^2 + a + 5 = 0 + 0 + 0 + 5 = 5$

(ii) For $a = 1$, $a^3 + a^2 + a + 5 = (1)^3 + (1)^2 + 1 + 5$

$= 1 + 1 + 1 + 5 = 8$

(iii) For $a = -1$, $a^3 + a^2 + a + 5 = (-1)^3 + (-1)^2 + (-1) + 5$

$= -1 + 1 - 1 + 5 = 4$

Question 5:

(a) Add: $p(p - q)$, $q(q - r)$ and $r(r - p)$

(b) Add: $2x(z - x - y)$ and $2y(z - y - x)$

(c) Subtract: $3l(l - 4m + 5n)$ from $4l(10n - 3m + 2l)$

(d) Subtract: $3a(a + b + c) - 2b(a - b + c)$ from $4c(-a + b + c)$

Answer 5:

(a) First expression = $p(p - q) = p^2 - pq$
Second expression = $q(q - r) = q^2 - qr$
Third expression = $r(r - p) = r^2 - pr$
Adding the three expressions, we obtain

$$\begin{array}{l} p^2 - pq \\ + \quad q^2 - qr \\ + \quad r^2 - pq \\ \hline p^2 - pq + q^2 - qr + r^2 - pq \end{array}$$

Therefore, the sum of the given expressions is $p^2 + q^2 + r^2 - pq - qr - rp$.

(b) First expression = $2x(z - x - y) = 2xz - 2x^2 - 2xy$
Second expression = $2y(z - y - x) = 2yz - 2y^2 - 2yx$
Adding the two expressions, we obtain

$$\begin{array}{l} 2xz - 2x^2 - 2xy \\ + \quad -2yx + 2yz - 2y^2 \\ \hline 2xz - 2x^2 - 4xy + 2yz - 2y^2 \end{array}$$

Therefore, the sum of the given expressions is $-2x^2 - 2y^2 - 4xy + 2yz + 2zx$.

(c) $3l(l - 4m + 5n) = 3l^2 - 12lm + 15ln$
$4l(10n - 3m + 2l) = 40ln - 12lm + 8l^2$
Subtracting these expressions, we obtain

$$\begin{array}{l} 40ln - 12lm + 8l^2 \\ 15ln - 12lm + 3l^2 \\ \hline (-) \quad (+) \quad (-) \\ \hline +25ln \qquad +5l^2 \end{array}$$

Therefore, the result is $5l^2 + 25ln$.

(d) $3a(a + b + c) - 2b(a - b + c) = 3a^2 + 3ab + 3ac - 2ba + 2b^2 - 2bc$
$= 3a^2 + 2b^2 + ab + 3ac - 2bc$
$4c(-a + b + c) = -4ac + 4bc + 4c^2$
Subtracting these expressions, we obtain

$$\begin{array}{l} -4ac + 4bc + 4c^2 \\ 3ac - 2bc \qquad +3a^2 + 2b^2 + ab \\ \hline (-) \quad (+) \qquad (-) \quad (-) \quad (-) \\ \hline -7ac + 6bc + 4c^2 - 3a^2 - 2b^2 - ab \end{array}$$

Therefore, the result is $-3a^2 - 2b^2 + 4c^2 - ab + 6bc - 7ac$.

MATHS -8 (CH-09 -9.4- ALGERAIC EXPRESSIONS & IDENTITIES)

Question 1:

Multiply the binomials.
(i) $(2x + 5)$ and $(4x - 3)$ (ii) $(y - 8)$ and $(3y - 4)$
(iii) $(2.5l - 0.5m)$ and $(2.5l + 0.5m)$ (iv) $(a + 3b)$ and $(x + 5)$
(v) $(2pq + 3q^2)$ and $(3pq - 2q^2)$
(vi) $\left(\frac{3}{4}a^2 + 3b^2\right)$ and $4\left(a^2 - \frac{2}{3}b^2\right)$

Answer 1:

(i) $(2x + 5) \times (4x - 3) = 2x \times (4x - 3) + 5 \times (4x - 3)$
$= 8x^2 - 6x + 20x - 15$
$= 8x^2 + 14x - 15$ (By adding like terms)
(ii) $(y - 8) \times (3y - 4) = y \times (3y - 4) - 8 \times (3y - 4)$
$= 3y^2 - 4y - 24y + 32$
$= 3y^2 - 28y + 32$ (By adding like terms)
(iii) $(2.5l - 0.5m) \times (2.5l + 0.5m) = 2.5l \times (2.5l + 0.5m) - 0.5m\ (2.5l + 0.5m)$
$= 6.25l^2 + 1.25lm - 1.25lm - 0.25m^2$
$= 6.25l^2 - 0.25m^2$

(iv) $(a + 3b) \times (x + 5) = a \times (x + 5) + 3b \times (x + 5)$
$= ax + 5a + 3bx + 15b$
(v) $(2pq + 3q^2) \times (3pq - 2q^2) = 2pq \times (3pq - 2q^2) + 3q^2 \times (3pq - 2q^2)$
$= 6p^2q^2 - 4pq^3 + 9pq^3 - 6q^4$
$= 6p^2q^2 + 5pq^3 - 6q^4$
(vi) $\left(\frac{3}{4}a^2 + 3b^2\right) \times \left[4\left(a^2 - \frac{2}{3}b^2\right)\right] = \left(\frac{3}{4}a^2 + 3b^2\right) \times \left(4a^2 - \frac{8}{3}b^2\right)$
$= \frac{3}{4}a^2 \times \left(4a^2 - \frac{8}{3}b^2\right) + 3b^2 \times \left(4a^2 - \frac{8}{3}b^2\right)$
$= 3a^4 - 2a^2b^2 + 12b^2a^2 - 8b^4$
$= 3a^4 + 10a^2b^2 - 8b^4$

Question 2:

Find the product.
(i) $(5 - 2x)\ (3 + x)$ (ii) $(x + 7y)\ (7x - y)$
(iii) $(a^2 + b)\ (a + b^2)$ (iv) $(p^2 - q^2)\ (2p + q)$

Answer 2:

(i) $(5 - 2x)(3 + x) = 5(3 + x) - 2x(3 + x)$
$= 15 + 5x - 6x - 2x^2$
$= 15 - x - 2x^2$
(ii) $(x + 7y)(7x - y) = x(7x - y) + 7y(7x - y)$
$= 7x^2 - xy + 49xy - 7y^2$
$= 7x^2 + 48xy - 7y^2$
(iii) $(a^2 + b)(a + b^2) = a^2(a + b^2) + b(a + b^2)$
$= a^3 + a^2b^2 + ab + b^3$
(iv) $(p^2 - q^2)(2p + q) = p^2(2p + q) - q^2(2p + q)$
$= 2p^3 + p^2q - 2pq^2 - q^3$

Question 3:

Simplify.
(i) $(x^2 - 5)(x + 5) + 25$
(ii) $(a^2 + 5)(b^3 + 3) + 5$
(iii) $(t + s^2)(t^2 - s)$
(iv) $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$
(v) $(x + y)(2x + y) + (x + 2y)(x - y)$
(vi) $(x + y)(x^2 - xy + y^2)$
(vii) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$
(viii) $(a + b + c)(a + b - c)$

Answer 3:

(i) $(x^2 - 5)(x + 5) + 25$
$= x^2(x + 5) - 5(x + 5) + 25$
$= x^3 + 5x^2 - 5x - 25 + 25$
$= x^3 + 5x^2 - 5x$
(ii) $(a^2 + 5)(b^3 + 3) + 5$
$= a^2(b^3 + 3) + 5(b^3 + 3) + 5$
$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5$
$= a^2b^3 + 3a^2 + 5b^3 + 20$
(iii) $(t + s^2)(t^2 - s)$
$= t(t^2 - s) + s^2(t^2 - s)$
$= t^3 - st + s^2t^2 - s^3$

(iv) $(a+b)(c-d)+(a-b)(c+d)+2(ac+bd)$
$= a(c-d)+b(c-d)+a(c+d)-b(c+d)+2(ac+bd)$
$= ac-ad+bc-bd+ac+ad-bc-bd+2ac+2bd$
$= (ac+ac+2ac)+(ad-ad)+(bc-bc)+(2bd-bd-bd)$
$= 4ac$

(v) $(x+y)(2x+y)+(x+2y)(x-y)$
$= x(2x+y)+y(2x+y)+x(x-y)+2y(x-y)$
$= 2x^2+xy+2xy+y^2+x^2-xy+2xy-2y^2$
$= (2x^2+x^2)+(y^2-2y^2)+(xy+2xy-xy+2xy)$
$= 3x^2-y^2+4xy$

(vi) $(x+y)(x^2-xy+y^2)$
$= x(x^2-xy+y^2)+y(x^2-xy+y^2)$
$= x^3-x^2y+xy^2+x^2y-xy^2+y^3$
$= x^3+y^3+(xy^2-xy^2)+(x^2y-x^2y)$
$= x^3+y^3$

(vii) $(1.5x-4y)(1.5x+4y+3)-4.5x+12y$
$= 1.5x(1.5x+4y+3)-4y(1.5x+4y+3)-4.5x+12y$
$= 2.25x^2+6xy+4.5x-6xy-16y^2-12y-4.5x+12y$
$= 2.25x^2+(6xy-6xy)+(4.5x-4.5x)-16y^2+(12y-12y)$
$= 2.25x^2-16y^2$

(viii) $(a+b+c)(a+b-c)$
$= a(a+b-c)+b(a+b-c)+c(a+b-c)$
$= a^2+ab-ac+ab+b^2-bc+ca+bc-c^2$
$= a^2+b^2-c^2+(ab+ab)+(bc-bc)+(ca-ca)$
$= a^2+b^2-c^2+2ab$

MATHS -8 (CH-09 -9.5- ALGERAIC EXPRESSIONS & IDENTITIES)

Question 1:

Use a suitable identity to get each of the following products.

(i) $(x+3)(x+3)$ (ii) $(2y+5)(2y+5)$

(iii) $(2a-7)(2a-7)$ (iv) $\left(3a-\frac{1}{2}\right)\left(3a-\frac{1}{2}\right)$

(v) $(1.1m-0.4)(1.1m+0.4)$ (vi) $(a^2+b^2)(-a^2+b^2)$

(vii) $(6x-7)(6x+7)$ (viii) $(-a+c)(-a+c)$

(ix) $\left(\frac{x}{2}+\frac{3y}{4}\right)\left(\frac{x}{2}+\frac{3y}{4}\right)$ (x) $(7a-9b)(7a-9b)$

Answer 1:

The products will be as follows.

(i) $(x+3)(x+3) = (x+3)^2$

$= (x)^2 + 2(x)(3) + (3)^2$ $[(a+b)^2 = a^2 + 2ab + b^2]$

$= x^2 + 6x + 9$

(ii) $(2y+5)(2y+5) = (2y+5)^2$

$= (2y)^2 + 2(2y)(5) + (5)^2$ $[(a+b)^2 = a^2 + 2ab + b^2]$

$= 4y^2 + 20y + 25$

(iii) $(2a-7)(2a-7) = (2a-7)^2$

$= (2a)^2 - 2(2a)(7) + (7)^2$ $[(a-b)^2 = a^2 - 2ab + b^2]$

$= 4a^2 - 28a + 49$

(iv) $\left(3a-\frac{1}{2}\right)\left(3a-\frac{1}{2}\right) = \left(3a-\frac{1}{2}\right)^2$

$= (3a)^2 - 2(3a)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2$ $[(a-b)^2 = a^2 - 2ab + b^2]$

$= 9a^2 - 3a + \frac{1}{4}$

(v) $(1.1m - 0.4)(1.1m + 0.4)$

$= (1.1m)^2 - (0.4)^2$ $[(a+b)(a-b) = a^2 - b^2]$

$= 1.21m^2 - 0.16$

(vi) $(a^2 + b^2)(-a^2 + b^2) = (b^2 + a^2)(b^2 - a^2)$

$= (b^2)^2 - (a^2)^2$ $[(a+b)(a-b) = a^2 - b^2]$

$= b^4 - a^4$

(vii) $(6x-7)(6x+7) = (6x)^2 - (7)^2$ $[(a+b)(a-b) = a^2 - b^2]$

$= 36x^2 - 49$

(viii) $(-a+c)(-a+c) = (-a+c)^2$

$= (-a)^2 + 2(-a)(c) + (c)^2$ $[(a+b)^2 = a^2 + 2ab + b^2]$

$= a^2 - 2ac + c^2$

(ix) $\left(\frac{x}{2}+\frac{3y}{4}\right)\left(\frac{x}{2}+\frac{3y}{4}\right) = \left(\frac{x}{2}+\frac{3y}{4}\right)^2$

$= \left(\frac{x}{2}\right)^2 + 2\left(\frac{x}{2}\right)\left(\frac{3y}{4}\right) + \left(\frac{3y}{4}\right)^2$ $[(a+b)^2 = a^2 + 2ab + b^2]$

$= \frac{x^2}{4} + \frac{3xy}{4} + \frac{9y^2}{16}$

(x) $(7a-9b)(7a-9b) = (7a-9b)^2$

$= (7a)^2 - 2(7a)(9b) + (9b)^2$ $[(a-b)^2 = a^2 - 2ab + b^2]$

$= 49a^2 - 126ab + 81b^2$

Question 2:

Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following products.
(i) $(x + 3)(x + 7)$ (ii) $(4x + 5)(4x + 1)$
(iii) $(4x - 5)(4x - 1)$ (iv) $(4x + 5)(4x - 1)$
(v) $(2x + 5y)(2x + 3y)$ (vi) $(2a^2 + 9)(2a^2 + 5)$
(vii) $(xyz - 4)(xyz - 2)$

Answer 2:

The products will be as follows.
(i) $(x + 3)(x + 7) = x^2 + (3 + 7)x + (3)(7)$
$= x^2 + 10x + 21$
(ii) $(4x + 5)(4x + 1) = (4x)^2 + (5 + 1)(4x) + (5)(1)$
$= 16x^2 + 24x + 5$
(iii) $(4x-5)(4x-1) = (4x)^2 + [(-5) + (-1)](4x) + (-5)(-1)$
$= 16x^2 - 24x + 5$

(iv) $(4x+5)(4x-1) = (4x)^2 + [(5) + (-1)](4x) + (5)(-1)$
$= 16x^2 + 16x - 5$
(v) $(2x + 5y)(2x + 3y) = (2x)^2 + (5y + 3y)(2x) + (5y)(3y)$
$= 4x^2 + 16xy + 15y^2$
(vi) $(2a^2 + 9)(2a^2 + 5) = (2a^2)^2 + (9 + 5)(2a^2) + (9)(5)$
$= 4a^4 + 28a^2 + 45$
(vii) $(xyz - 4)(xyz - 2)$
$= (xyz)^2 + [(-4) + (-2)](xyz) + (-4)(-2)$
$= x^2y^2z^2 - 6xyz + 8$

Question 3:

Find the following squares by suing the identities.
(i) $(b - 7)^2$ (ii) $(xy + 3z)^2$ (iii) $(6x^2 - 5y)^2$
(iv) $\left(\frac{2}{3}m + \frac{3}{2}n\right)^2$ (v) $(0.4p - 0.5q)^2$ (vi) $(2xy + 5y)^2$

Answer 3:

(i) $(b - 7)^2 = (b)^2 - 2(b)(7) + (7)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$
$= b^2 - 14b + 49$
(ii) $(xy + 3z)^2 = (xy)^2 + 2(xy)(3z) + (3z)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$
$= x^2y^2 + 6xyz + 9z^2$
(iii) $(6x^2 - 5y)^2 = (6x^2)^2 - 2(6x^2)(5y) + (5y)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$
$= 36x^4 - 60x^2y + 25y^2$

(iv) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2=\left(\frac{2}{3}m\right)^2+2\left(\frac{2}{3}m\right)\left(\frac{3}{2}n\right)+\left(\frac{3}{2}n\right)^2$ $[(a+b)^2=a^2+2ab+b^2]$

$=\frac{4}{9}m^2+2mn+\frac{9}{4}n^2$

(v) $(0.4p-0.5q)^2=(0.4p)^2-2(0.4p)(0.5q)+(0.5q)^2$

$[(a-b)^2=a^2-2ab+b^2]$

$=0.16p^2-0.4pq+0.25q^2$

(vi) $(2xy+5y)^2=(2xy)^2+2(2xy)(5y)+(5y)^2$

$[(a+b)^2=a^2+2ab+b^2]$

$=4x^2y^2+20xy^2+25y^2$

Question 4:

Simplify.

(i) $(a^2-b^2)^2$ (ii) $(2x+5)^2-(2x-5)^2$

(iii) $(7m-8n)^2+(7m+8n)^2$ (iv) $(4m+5n)^2+(5m+4n)^2$

(v) $(2.5p-1.5q)^2-(1.5p-2.5q)^2$

(vi) $(ab+bc)^2-2ab^2c$ (vii) $(m^2-n^2m)^2+2m^3n^2$

Answer 4:

(i) $(a^2-b^2)^2=(a^2)^2-2(a^2)(b^2)+(b^2)^2$ $[(a-b)^2=a^2-2ab+b^2]$

$=a^4-2a^2b^2+b^4$

(ii) $(2x+5)^2-(2x-5)^2=(2x)^2+2(2x)(5)+(5)^2-[(2x)^2-2(2x)(5)+(5)^2]$

$[(a-b)^2=a^2-2ab+b^2]$

$[(a+b)^2=a^2+2ab+b^2]$

$=4x^2+20x+25-[4x^2-20x+25]$

$=4x^2+20x+25-4x^2+20x-25=40x$

(iii) $(7m-8n)^2+(7m+8n)^2$

$=(7m)^2-2(7m)(8n)+(8n)^2+(7m)^2+2(7m)(8n)+(8n)^2$

$[(a-b)^2=a^2-2ab+b^2$ and $(a+b)^2=a^2+2ab+b^2]$

$=49m^2-112mn+64n^2+49m^2+112mn+64n^2$

$=98m^2+128n^2$

(iv) $(4m + 5n)^2 + (5m + 4n)^2$
$= (4m)^2 + 2(4m)(5n) + (5n)^2 + (5m)^2 + 2(5m)(4n) + (4n)^2$
$[(a + b)^2 = a^2 + 2ab + b^2]$
$= 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2$
$= 41m^2 + 80mn + 41n^2$
(v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$
$= (2.5p)^2 - 2(2.5p)(1.5q) + (1.5q)^2 - [(1.5p)^2 - 2(1.5p)(2.5q) + (2.5q)^2]$
$[(a - b)^2 = a^2 - 2ab + b^2]$
$= 6.25p^2 - 7.5pq + 2.25q^2 - [2.25p^2 - 7.5pq + 6.25q^2]$
$= 6.25p^2 - 7.5pq + 2.25q^2 - 2.25p^2 + 7.5pq - 6.25q^2]$
$= 4p^2 - 4q^2$

(vi) $(ab + bc)^2 - 2ab^2c$
$= (ab)^2 + 2(ab)(bc) + (bc)^2 - 2ab^2c$ $[(a + b)^2 = a^2 + 2ab + b^2]$
$= a^2b^2 + 2ab^2c + b^2c^2 - 2ab^2c$
$= a^2b^2 + b^2c^2$
(vii) $(m^2 - n^2m)^2 + 2m^3n^2$
$= (m^2)^2 - 2(m^2)(n^2m) + (n^2m)^2 + 2m^3n^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$
$= m^4 - 2m^3n^2 + n^4m^2 + 2m^3n^2$
$= m^4 + n^4m^2$

Question 5:

Show that
(i) $(3x + 7)^2 - 84x = (3x - 7)^2$ (ii) $(9p - 5q)^2 + 180pq = (9p + 5q)^2$
(iii) $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$
(iv) $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$
(v) $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

Answer 5:

(i) L.H.S $= (3x + 7)^2 - 84x$
$= (3x)^2 + 2(3x)(7) + (7)^2 - 84x$
$= 9x^2 + 42x + 49 - 84x$
$= 9x^2 - 42x + 49$
R.H.S $= (3x - 7)^2 = (3x)^2 - 2(3x)(7) + (7)^2$
$= 9x^2 - 42x + 49$
L.H.S = R.H.S

(ii) L.H.S = $(9p - 5q)^2 + 180pq$
$= (9p)^2 - 2(9p)(5q) + (5q)^2 - 180pq$
$= 81p^2 - 90pq + 25q^2 + 180pq$
$= 81p^2 + 90pq + 25q^2$
R.H.S = $(9p + 5q)^2$
$= (9p)^2 + 2(9p)(5q) + (5q)^2$
$= 81p^2 + 90pq + 25q^2$
L.H.S = R.H.S

(iii) L.H.S = $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn$

$$= \left(\frac{4}{3}m\right)^2 - 2\left(\frac{4}{3}m\right)\left(\frac{3}{4}n\right) + \left(\frac{3}{4}n\right)^2 + 2mn$$
$$= \frac{16}{9}m^2 - 2mn + \frac{9}{16}n^2 + 2mn$$
$$= \frac{16}{9}m^2 + \frac{9}{16}n^2 = \text{R.H.S.}$$

(iv) L.H.S = $(4pq + 3q)^2 - (4pq - 3q)^2$
$= (4pq)^2 + 2(4pq)(3q) + (3q)^2 - [(4pq)^2 - 2(4pq)(3q) + (3q)^2]$
$= 16p^2q^2 + 24pq^2 + 9q^2 - [16p^2q^2 - 24pq^2 + 9q^2]$
$= 16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 24pq^2 - 9q^2$
$= 48pq^2$ = R.H.S
(v) L.H.S = $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a)$
$= (a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0$ = R.H.S.

Question 6:

Using identities, evaluate.
(i) 71^2 (ii) 99^2 (iii) 102^2 (iv) 998^2
(v) $(5.2)^2$ (vi) 297×303 (vii) 78×82
(viii) 8.9^2 (ix) 1.05×9.5

Answer 6:

(i) $71^2 = (70 + 1)^2$
$= (70)^2 + 2(70)(1) + (1)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$
$= 4900 + 140 + 1 = 5041$
(ii) $99^2 = (100 - 1)^2$
$= (100)^2 - 2(100)(1) + (1)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$
$= 10000 - 200 + 1 = 9801$
(iii) $102^2 = (100 + 2)^2$
$= (100)^2 + 2(100)(2) + (2)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$
$= 10000 + 400 + 4 = 10404$

(iv) $998^2 = (1000 - 2)^2$
$= (1000)^2 - 2(1000)(2) + (2)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$
$= 1000000 - 4000 + 4 = 996004$
(v) $(5.2)^2 = (5.0 + 0.2)^2$
$= (5.0)^2 + 2(5.0)(0.2) + (0.2)^2$ $[(a + b)^2 = a^2 + 2ab + b^2]$
$= 25 + 2 + 0.04 = 27.04$
(vi) $297 \times 303 = (300 - 3) \times (300 + 3)$
$= (300)^2 - (3)^2$ $[(a + b)(a - b) = a^2 - b^2]$
$= 90000 - 9 = 89991$

(vii) $78 \times 82 = (80 - 2)(80 + 2)$
$= (80)^2 - (2)^2$ $[(a + b)(a - b) = a^2 - b^2]$
$= 6400 - 4 = 6396$
(viii) $8.9^2 = (9.0 - 0.1)^2$
$= (9.0)^2 - 2(9.0)(0.1) + (0.1)^2$ $[(a - b)^2 = a^2 - 2ab + b^2]$
$= 81 - 1.8 + 0.01 = 79.21$

(ix) $1.05 \times 9.5 = 1.05 \times 0.95 \times 10$
$= (1 + 0.05)(1 - 0.05) \times 10$
$= [(1)^2 - (0.05)^2] \times 10$
$= [1 - 0.0025] \times 10$ $[(a + b)(a - b) = a^2 - b^2]$
$= 0.9975 \times 10 = 9.975$

Question 7:

Using $a^2 - b^2 = (a + b)(a - b)$, find
(i) $51^2 - 49^2$ (ii) $(1.02)^2 - (0.98)^2$ (iii) $153^2 - 147^2$
(iv) $12.1^2 - 7.9^2$

Answer 7:

(i) $51^2 - 49^2 = (51 + 49)(51 - 49)$
$= (100)(2) = 200$
(ii) $(1.02)^2 - (0.98)^2 = (1.02 + 0.98)(1.02 - 0.98)$
$= (2)(0.04) = 0.08$

(iii) $153^2 - 147^2 = (153 + 147)(153 - 147)$
$= (300)(6) = 1800$
(iv) $12.1^2 - 7.9^2 = (12.1 + 7.9)(12.1 - 7.9)$
$= (20.0)(4.2) = 84$

Question 8:

Using $(x + a)(x + b) = x^2 + (a + b)x + ab$, find
(i) 103 × 104 (ii) 5.1 × 5.2 (iii) 103 × 98 (iv) 9.7 × 9.8

Answer 8:

(i) $103 \times 104 = (100 + 3)(100 + 4)$
$= (100)^2 + (3 + 4)(100) + (3)(4)$
$= 10000 + 700 + 12 = 10712$
(ii) $5.1 \times 5.2 = (5 + 0.1)(5 + 0.2)$
$= (5)^2 + (0.1 + 0.2)(5) + (0.1)(0.2)$
$= 25 + 1.5 + 0.02 = 26.52$

(iii) $103 \times 98 = (100 + 3)(100 - 2)$
$= (100)^2 + [3 + (-2)](100) + (3)(-2)$
$= 10000 + 100 - 6$
$= 10094$
(iv) $9.7 \times 9.8 = (10 - 0.3)(10 - 0.2)$
$= (10)^2 + [(-0.3) + (-0.2)](10) + (-0.3)(-0.2)$
$= 100 + (-0.5)10 + 0.06 = 100.06 - 5 = 95.06$