MATHS -9 (CH-08 - QUADRILATERALS)

MATHS -9 (CH-08 -8.1- QUADRILATERALS)

Question 1:

The angles of quadrilateral are in the ratio 3: 5: 9: 13. Find all the angles of the quadrilateral.

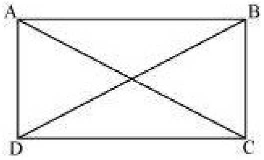
Answer 1:

Let the common ratio between the angles be x. Therefore, the angles will be $3x$, $5x$, $9x$, and $13x$ respectively.
As the sum of all interior angles of a quadrilateral is 360°,
$\therefore 3x + 5x + 9x + 13x = 360°$
$30x = 360°$
$x = 12°$
Hence, the angles are
$3x = 3 \times 12 = 36°$
$5x = 5 \times 12 = 60°$
$9x = 9 \times 12 = 108°$
$13x = 13 \times 12 = 156°$

Question 2:

If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Answer 2:



Let ABCD be a parallelogram. To show that ABCD is a rectangle, we have to prove that one of its interior angles is 90º.

In ΔABC and ΔDCB,

AB = DC (Opposite sides of a parallelogram are equal)

BC = BC (Common)

AC = DB (Given)

∴ ΔABC ≅ ΔDCB (By SSS Congruence rule)

⇒ ∠ABC = ∠DCB

It is known that the sum of the measures of angles on the same side of transversal is 180º.

∠ABC + ∠DCB = 180º (AB || CD)

⇒ ∠ABC + ∠ABC = 180º

⇒ 2∠ABC = 180º

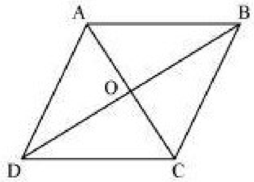
⇒ ∠ABC = 90º

Since ABCD is a parallelogram and one of its interior angles is 90º, ABCD is a rectangle.

Question 3:

Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Answer 3:



Let ABCD be a quadrilateral, whose diagonals AC and BD bisect each other at right angle i.e., OA = OC, OB = OD, and ∠AOB = ∠BOC = ∠COD = ∠AOD = 90º. To prove ABCD a rhombus, we have to prove ABCD is a parallelogram and all the sides of ABCD are equal.

In ΔAOD and ΔCOD,

OA = OC (Diagonals bisect each other)

∠AOD = ∠COD (Given)

OD = OD (Common)

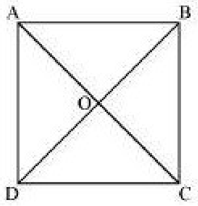
∴ ΔAOD ≅ ΔCOD (By SAS congruence rule)

∴ AD = CD (1)
Similarly, it can be proved that
AD = AB and CD = BC (2)
From equations (1) and (2),
AB = BC = CD = AD
Since opposite sides of quadrilateral ABCD are equal, it can be said that ABCD is a parallelogram. Since all sides of a parallelogram ABCD are equal, it can be said that ABCD is a rhombus.

Question 4:

Show that the diagonals of a square are equal and bisect each other at right angles.

Answer 4:



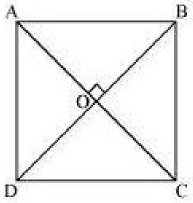
Let ABCD be a square. Let the diagonals AC and BD intersect each other at a point O. To prove that the diagonals of a square are equal and bisect each other at right angles, we have to prove AC = BD, OA = OC, OB = OD, and ∠AOB = 90º.
In ΔABC and ΔDCB,
AB = DC (Sides of a square are equal to each other)
∠ABC = ∠DCB (All interior angles are of 90∘)
BC = CB (Common side)
∴ ΔABC ≅ ΔDCB (By SAS congruency)

∴ AC = DB (By CPCT)
Hence, the diagonals of a square are equal in length.
In ΔAOB and ΔCOD,
∠AOB = ∠COD (Vertically opposite angles)
∠ABO = ∠CDO (Alternate interior angles)
AB = CD (Sides of a square are always equal)
∴ ΔAOB ≅ ΔCOD (By AAS congruence rule)
∴ AO = CO and OB = OD (By CPCT)
Hence, the diagonals of a square bisect each other.
In ΔAOB and ΔCOB,
As we had proved that diagonals bisect each other, therefore,
AO = CO
AB = CB (Sides of a square are equal)
BO = BO (Common)
∴ ΔAOB ≅ ΔCOB (By SSS congruency)
∴ ∠AOB = ∠COB (By CPCT)
However, ∠AOB + ∠COB = 180° (Linear pair)
2∠AOB = 180°
∠AOB = 90°
Hence, the diagonals of a square bisect each other at right angles.

Question 5:

Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Answer 5:



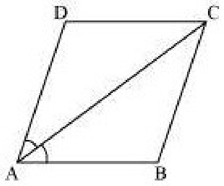
Let us consider a quadrilateral ABCD in which the diagonals AC and BD intersect each other at O. It is given that the diagonals of ABCD are equal and bisect each other at right angles. Therefore, AC = BD, OA = OC, OB = OD, and ∠AOB = ∠BOC = ∠COD = ∠AOD = 90°. To prove ABCD is a square, we have to prove that ABCD is a parallelogram, AB = BC = CD = AD, and one of its interior angles is 90°.
In ΔAOB and ΔCOD,

In ΔAOB and ΔCOD,
AO = CO (Diagonals bisect each other)
OB = OD (Diagonals bisect each other)
$\angle AOB = \angle COD$ (Vertically opposite angles)
$\therefore \Delta AOB \cong \Delta COD$ (SAS congruence rule)
$\therefore$ AB = CD (By CPCT) ... (1)
And, $\angle OAB = \angle OCD$ (By CPCT)
However, these are alternate interior angles for line AB and CD and alternate interior angles are equal to each other only when the two lines are parallel.
$\therefore$ AB || CD ... (2)
From equations (1) and (2), we obtain
ABCD is a parallelogram.
In ΔAOD and ΔCOD,
AO = CO (Diagonals bisect each other)
$\angle AOD = \angle COD$ (Given that each is 90º)
OD = OD (Common)
$\therefore \Delta AOD \cong \Delta COD$ (SAS congruence rule)

$\therefore$ AD = DC ... (3)
However, AD = BC and AB = CD (Opposite sides of parallelogram ABCD)
$\therefore$ AB = BC = CD = DA
Therefore, all the sides of quadrilateral ABCD are equal to each other.
In ΔADC and ΔBCD,
AD = BC (Already proved)
AC = BD (Given)
DC = CD (Common)
$\therefore \Delta ADC \cong \Delta BCD$ (SSS Congruence rule)
$\therefore \angle ADC = \angle BCD$ (By CPCT)
However, $\angle ADC + \angle BCD = 180°$ (Co-interior angles)
$\Rightarrow \angle ADC + \angle ADC = 180°$
$\Rightarrow 2\angle ADC = 180°$
$\Rightarrow \angle ADC = 90°$
One of the interior angles of quadrilateral ABCD is a right angle.
Thus, we have obtained that ABCD is a parallelogram, AB = BC = CD = AD and one of its interior angles is 90º. Therefore, ABCD is a square.

Question 6:

Diagonal AC of a parallelogram ABCD bisects ∠A (see the given figure). Show that
(i) It bisects ∠C also,
(ii) ABCD is a rhombus.



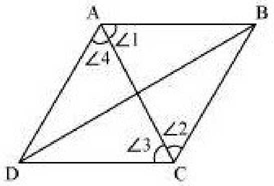
Answer 6:

(i) ABCD is a parallelogram.
∴ ∠DAC = ∠BCA (Alternate interior angles) ... (1)
And, ∠BAC = ∠DCA (Alternate interior angles) ... (2)
However, it is given that AC bisects ∠A.
∴ ∠DAC = ∠BAC ... (3)
From equations (1), (2), and (3), we obtain
∠DAC = ∠BCA = ∠BAC = ∠DCA ... (4)
⇒ ∠DCA = ∠BCA
Hence, AC bisects ∠C.
(ii)From equation (4), we obtain
∠DAC = ∠DCA
∴ DA = DC (Side opposite to equal angles are equal)
However, DA = BC and AB = CD (Opposite sides of a parallelogram)
∴ AB = BC = CD = DA
Hence, ABCD is a rhombus.

Question 7:

ABCD is a rhombus. Show that diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.

Answer 7:

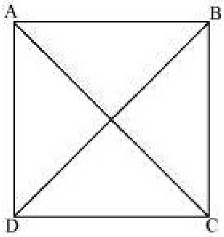


Let us join AC.
In ΔABC,
BC = AB (Sides of a rhombus are equal to each other)
∴ $\angle 1 = \angle 2$ (Angles opposite to equal sides of a triangle are equal)
However, $\angle 1 = \angle 3$ (Alternate interior angles for parallel lines AB and CD)
$\Rightarrow \angle 2 = \angle 3$
Therefore, AC bisects ∠C.
Also, $\angle 2 = \angle 4$ (Alternate interior angles for || lines BC and DA)
$\Rightarrow \angle 1 = \angle 4$
Therefore, AC bisects ∠A.
Similarly, it can be proved that BD bisects ∠B and ∠D as well.

Question 8:

ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that:
(i) ABCD is a square (ii) diagonal BD bisects ∠B as well as ∠D.

Answer 8:

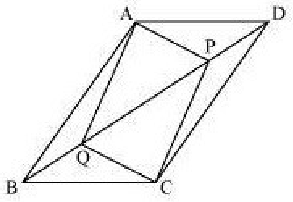


(i) It is given that ABCD is a rectangle.
$\therefore \angle A = \angle C$
$\Rightarrow \frac{1}{2}\angle A = \frac{1}{2}\angle C$
$\Rightarrow \angle DAC = \angle DCA$ (AC bisects ∠A and ∠C)

CD = DA (Sides opposite to equal angles are also equal)
However, DA = BC and AB = CD (Opposite sides of a rectangle are equal)
∴ AB = BC = CD = DA
ABCD is a rectangle and all of its sides are equal.
Hence, ABCD is a square.
(ii) Let us join BD.
In ΔBCD,
BC = CD (Sides of a square are equal to each other)
∠CDB = ∠CBD (Angles opposite to equal sides are equal)
However, ∠CDB = ∠ABD (Alternate interior angles for AB || CD)
∴ ∠CBD = ∠ABD
⇒ BD bisects ∠B.
Also, ∠CBD = ∠ADB (Alternate interior angles for BC || AD)
⇒ ∠CDB = ∠ABD
∴ BD bisects ∠D.

Question 9:

In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see the given figure).



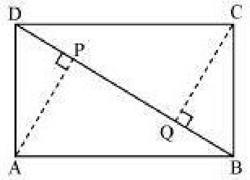
Show that:
(i) ΔAPD ≅ ΔCQB
(ii) AP = CQ
(iii) ΔAQB ≅ ΔCPD
(iv) AQ = CP
(v) APCQ is a parallelogram

Answer 9:

(i) In ΔAPD and ΔCQB,
$\angle ADP = \angle CBQ$ (Alternate interior angles for BC || AD)
AD = CB (Opposite sides of parallelogram ABCD)
DP = BQ (Given)
$\therefore \Delta APD \cong \Delta CQB$ (Using SAS congruence rule)
(ii) As we had observed that $\Delta APD \cong \Delta CQB$,
$\therefore$ AP = CQ (CPCT)
(iii) In ΔAQB and ΔCPD,
$\angle ABQ = \angle CDP$ (Alternate interior angles for AB || CD)
AB = CD (Opposite sides of parallelogram ABCD)
BQ = DP (Given)
$\therefore \Delta AQB \cong \Delta CPD$ (Using SAS congruence rule)
(iv) As we had observed that $\Delta AQB \cong \Delta CPD$,
$\therefore$ AQ = CP (CPCT)
(v) From the result obtained in (ii) and (iv),
AQ = CP and
AP = CQ
Since opposite sides in quadrilateral APCQ are equal to each other, APCQ is a parallelogram.

Question 10:

ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (See the given figure). Show that



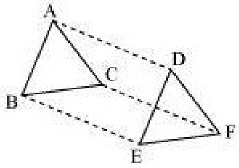
(i) $\Delta APB \cong \Delta CQD$
(ii) AP = CQ

Answer 10:

(i) In ΔAPB and ΔCQD,
$\angle APB = \angle CQD$ (Each 90°)
AB = CD (Opposite sides of parallelogram ABCD)
$\angle ABP = \angle CDQ$ (Alternate interior angles for AB || CD)
$\therefore \Delta APB \cong \Delta CQD$ (By AAS congruency)
(ii) By using the above result
$\Delta APB \cong \Delta CQD$, we obtain
AP = CQ (By CPCT)

Question 11:

In ΔABC and ΔDEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see the given figure). Show that



(i) Quadrilateral ABED is a parallelogram
(ii) Quadrilateral BEFC is a parallelogram
(iii) AD || CF and AD = CF
(iv) Quadrilateral ACFD is a parallelogram
(v) AC = DF
(vi) ΔABC ≅ ΔDEF.

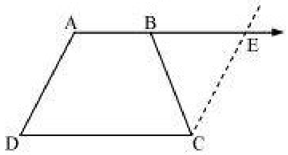
Answer 11:

(i) It is given that AB = DE and AB || DE.
If two opposite sides of a quadrilateral are equal and parallel to each other, then it will be a parallelogram.
Therefore, quadrilateral ABED is a parallelogram.
(ii) Again, BC = EF and BC || EF
Therefore, quadrilateral BCEF is a parallelogram.
(iii) As we had observed that ABED and BEFC are parallelograms, therefore
AD = BE and AD || BE
(Opposite sides of a parallelogram are equal and parallel)
And, BE = CF and BE || CF
(Opposite sides of a parallelogram are equal and parallel)
∴ AD = CF and AD || CF

(iv) As we had observed that one pair of opposite sides (AD and CF) of quadrilateral ACFD are equal and parallel to each other, therefore, it is a parallelogram.
(v) As ACFD is a parallelogram, therefore, the pair of opposite sides will be equal and parallel to each other.
∴ AC || DF and AC = DF
(vi) ΔABC and ΔDEF,
AB = DE (Given)
BC = EF (Given)
AC = DF (ACFD is a parallelogram)
∴ ΔABC ≅ ΔDEF (By SSS congruence rule)

Question 12:

ABCD is a trapezium in which AB || CD and AD = BC (see the given figure). Show that



(i) ∠A = ∠B
(ii) ∠C = ∠D
(iii) ΔABC ≅ ΔBAD
(iv) diagonal AC = diagonal BD
[*Hint*: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

Answer 12:

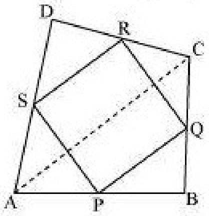
Let us extend AB. Then, draw a line through C, which is parallel to AD, intersecting AE at point E. It is clear that AECD is a parallelogram.
(i) AD = CE (Opposite sides of parallelogram AECD)
However, AD = BC (Given)
Therefore, BC = CE
∠CEB = ∠CBE (Angle opposite to equal sides are also equal)
Consider parallel lines AD and CE. AE is the transversal line for them.
∠A + ∠CEB = 180º (Angles on the same side of transversal)
∠A + ∠CBE = 180º (Using the relation∠CEB = ∠CBE) ... (1)
However, ∠B + ∠CBE = 180º (Linear pair angles) ... (2)
From equations (1) and (2), we obtain
∠A = ∠B

(ii) AB || CD
∠A + ∠D = 180º (Angles on the same side of the transversal)
Also, ∠C + ∠B = 180° (Angles on the same side of the transversal)
∴ ∠A + ∠D = ∠C + ∠B
However, ∠A = ∠B [Using the result obtained in (i)]
∴ ∠C = ∠D
(iii) In ΔABC and ΔBAD,
AB = BA (Common side)
BC = AD (Given)
∠B = ∠A (Proved before)
∴ ΔABC ≅ ΔBAD (SAS congruence rule)
(iv) We had observed that,
ΔABC ≅ ΔBAD
∴ AC = BD (By CPCT)

MATHS -9 (CH-08 -8.2- QUADRILATERALS)

Question 1:

ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see the given figure). AC is a diagonal. Show that:



(i) SR || AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Answer 1:

(i) In ΔADC, S and R are the mid-points of sides AD and CD respectively.

In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it.

∴ SR || AC and SR = $\frac{1}{2}$ AC ... (1)

(ii) In ΔABC, P and Q are mid-points of sides AB and BC respectively. Therefore, by using mid-point theorem,

PQ || AC and PQ = $\frac{1}{2}$ AC ... (2)

Using equations (1) and (2), we obtain

PQ || SR and PQ = SR ... (3)

⇒ PQ = SR

(iii) From equation (3), we obtained

PQ || SR and PQ = SR

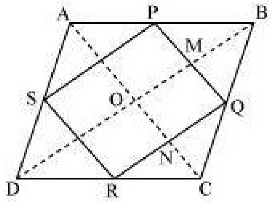
Clearly, one pair of opposite sides of quadrilateral PQRS is parallel and equal.

Hence, PQRS is a parallelogram.

Question 2:

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Answer 2:



In ΔABC, P and Q are the mid-points of sides AB and BC respectively.

∴ PQ || AC and PQ = $\frac{1}{2}$AC (Using mid-point theorem) ... (1)

In ΔADC,

R and S are the mid-points of CD and AD respectively.

∴ RS || AC and RS = $\frac{1}{2}$AC (Using mid-point theorem) ... (2)

From equations (1) and (2), we obtain

PQ || RS and PQ = RS

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram.

Let the diagonals of rhombus ABCD intersect each other at point O.

In quadrilateral OMQN,

MQ || ON (∵ PQ || AC)

QN || OM (∵ QR || BD)

Therefore, OMQN is a parallelogram.

⇒ ∠MQN = ∠NOM

⇒ ∠PQR = ∠NOM

However, ∠NOM = 90° (Diagonals of a rhombus are perpendicular to each other)

∴ ∠PQR = 90°

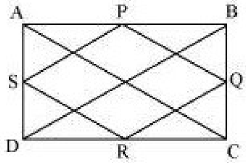
Clearly, PQRS is a parallelogram having one of its interior angles as 90°.

Hence, PQRS is a rectangle.

Question 3:

ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Answer 3:

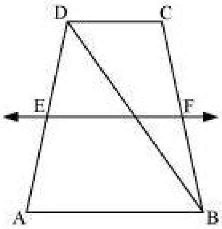


Let us join AC and BD.
In ΔABC,
P and Q are the mid-points of AB and BC respectively.
∴ PQ || AC and PQ = $\frac{1}{2}$AC (Mid-point theorem) ... (1)
Similarly in ΔADC,

SR || AC and SR = $\frac{1}{2}$ AC (Mid-point theorem) ... (2)
Clearly, PQ || SR and PQ = SR
Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram.
∴ PS || QR and PS = QR (Opposite sides of parallelogram)... (3)
In ΔBCD, Q and R are the mid-points of side BC and CD respectively.
∴ QR || BD and QR = $\frac{1}{2}$BD (Mid-point theorem) ... (4)
However, the diagonals of a rectangle are equal.
∴ AC = BD ...(5)
By using equation (1), (2), (3), (4), and (5), we obtain
PQ = QR = SR = PS
Therefore, PQRS is a rhombus.

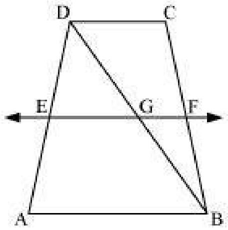
Question 4:

ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid - point of AD. A line is drawn through E parallel to AB intersecting BC at F (see the given figure). Show that F is the mid-point of BC.



Answer 4:

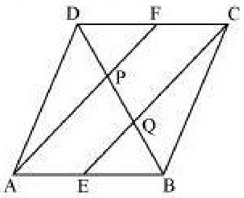
Let EF intersect DB at G.



By converse of mid-point theorem, we know that a line drawn through the mid-point of any side of a triangle and parallel to another side, bisects the third side.
In ΔABD,
EF || AB and E is the mid-point of AD.
Therefore, G will be the mid-point of DB.
As EF || AB and AB || CD,
∴ EF || CD (Two lines parallel to the same line are parallel to each other)
In ΔBCD, GF || CD and G is the mid-point of line BD. Therefore, by using converse of mid-point theorem, F is the mid-point of BC.

Question 5:

In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see the given figure). Show that the line segments AF and EC trisect the diagonal BD.



Answer 5:

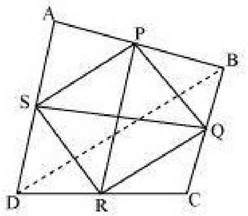
ABCD is a parallelogram.
∴AB || CD
And hence, AE || FC
Again, AB = CD (Opposite sides of parallelogram ABCD)
$\frac{1}{2}AB = \frac{1}{2}CD$
AE = FC (E and F are mid-points of side AB and CD)
In quadrilateral AECF, one pair of opposite sides (AE and CF) is parallel and equal to each other.
Therefore, AECF is a parallelogram.
⇒ AF || EC (Opposite sides of a parallelogram)
In ΔDQC, F is the mid-point of side DC and FP || CQ (as AF || EC). Therefore, by using the converse of mid-point theorem, it can be said that P is the mid-point of DQ.
⇒ DP = PQ ... (1)

Similarly, in ΔAPB, E is the mid-point of side AB and EQ || AP (as AF || EC). Therefore, by using the converse of mid-point theorem, it can be said that
Q is the mid-point of PB.
⇒ PQ = QB ... (2)
From equations (1) and (2),
DP = PQ = BQ
Hence, the line segments AF and EC trisect the diagonal BD.

Question 6:

Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Answer 6:



Let ABCD is a quadrilateral in which P, Q, R, and S are the mid-points of sides AB, BC, CD, and DA respectively. Join PQ, QR, RS, SP, and BD.
In ΔABD, S and P are the mid-points of AD and AB respectively. Therefore, by using mid-point theorem, it can be said that
SP || BD and SP = $\frac{1}{2}$ BD ... (1)
Similarly in ΔBCD,
QR || BD and QR = $\frac{1}{2}$ BD ... (2)
From equations (1) and (2), we obtain
SP || QR and SP = QR
In quadrilateral SPQR, one pair of opposite sides is equal and parallel to each other. Therefore, SPQR is a parallelogram.
We know that diagonals of a parallelogram bisect each other.
Hence, PR and QS bisect each other.

Question 7:

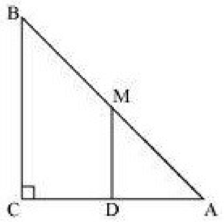
ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC

(ii) MD ⊥ AC

(iii) $CM = MA = \frac{1}{2}AB$

Answer 7:



(i) In ΔABC,

It is given that M is the mid-point of AB and MD || BC.

Therefore, D is the mid-point of AC. (Converse of mid-point theorem)

(ii) As DM || CB and AC is a transversal line for them, therefore,

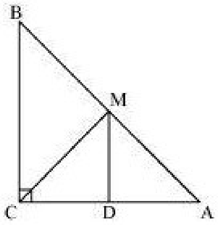
∠MDC + ∠DCB = 180º (Co-interior angles)

∠MDC + 90º = 180º

∠MDC = 90º

∴ MD ⊥ AC

(iii) Join MC.



In ΔAMD and ΔCMD,

AD = CD (D is the mid-point of side AC)

∠ADM = ∠CDM (Each 90º)

DM = DM (Common)

∴ΔAMD ≅ ΔCMD (By SAS congruence rule)

Therefore, AM = CM (By CPCT)

However, AM = $\frac{1}{2}$ AB (M is the mid-point of AB)

Therefore, it can be said that

CM = AM = $\frac{1}{2}$ AB