By. Er. Dharmendra Sir 7974073108, 9584873492

DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

BASIC NOTES (CBSE/NCERT)

TRIANGLE

PART -1 OUT OF 1

6^{th}, 7^{th} & 8^{th}

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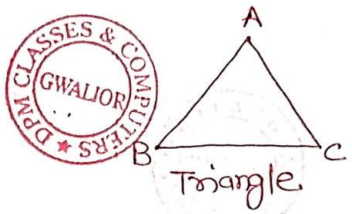
-: Triangles :-

Q. Triangle : A triangle is a simple closed curve made of three line segments. It has three vertics, three sides and three angles.



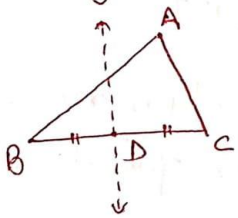
- Sides are - AB, BC and AC
- Vertics are - A, B, C
- Angles are - $\angle ABC$, $\angle BCA$, $\angle CAB$

Opposite sides of vertics A is BC, other are similar find.

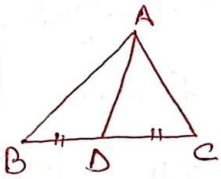
Q. classify triangles based on (i) side (ii) Angles.

- Based on sides : Scalane, Isosceles and Equilateral.
- Based on Angles : Acute-Angled, obtuse angled, Right-angled triangles.

Q. medians of a triangle :- A median connects a vertex of a triangle to the mid-point of the opposite side.



Perpendicular bisector of BC at D

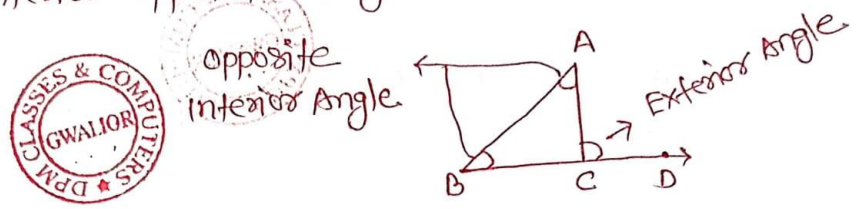


D Connect with A then AD is medium of ΔABC

Q. Altitude of a triangle :- The height of the triangle is called Altitude of a triangle. Altitude is always 90°. Angle create between vertex and opposite side.

-: Triangle :-

(1). Exterior angle of a triangle and its property :-
An Exterior angle of a triangle is equal to the sum of its interior opposites angles.



Exterior Angle = Sum of opposites interior angle

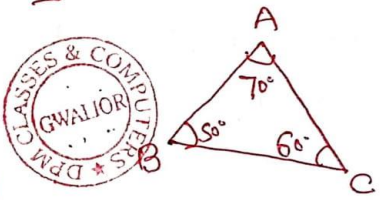
$\angle ACD = \angle BAC + \angle ABC$

(2). Angle Sum property of a triangle :- The sum of the measures of the three angles of a triangle is 180°

$\angle A + \angle B + \angle C = 180°$

$70° + 50° + 60° = 180°$

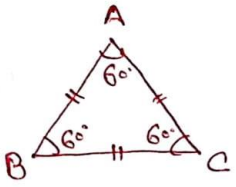
$180 = 180$ (verified)



(3). Two Special triangles : Equilateral and Isosceles :-

(a) Equilateral triangle : A triangle in which all the three sides are of equal lengths is called an equilateral triangle.

- (i) All sides have same length
- (ii) each angles has measure 60°



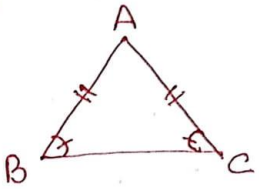
$AB = BC = CA$, $\angle A = \angle B = \angle C = 60°$

-: Triangles :-

(b) Isosceles triangle: A triangle in which two sides are of equal length is called as isosceles triangle.

(i) Two sides have same length.
(ii) Base angles opposite to the equal sides are equal.

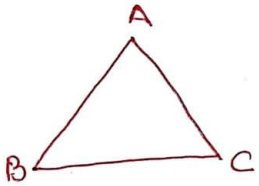
$AB = AC$
$\angle ACB = \angle ABC$



(2). Sum of the lengths of two sides of triangle:-
the sum of the lengths of any two sides of a triangle is greater than the third side.

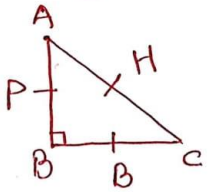
Ex- sides are
AB, BC, CA (AC)

$AB + BC > AC$
$BC + AC > AB$
$AC + AB > BC$



(3). Right-Angled triangle and Pythagoras Property :-

Pythagoras theorem applied when triangle is Right angled triangle



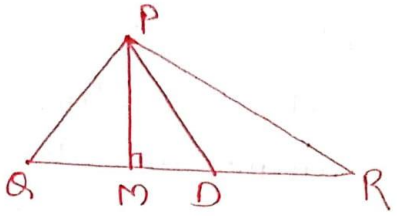
$H^2 = B^2 + P^2$

$H = \sqrt{B^2 + P^2}$

H = Hypofenuse
P = perpendicular
B = Base
$\llcorner$ = 90°

② In ΔPQR, D is the mid-point of $\overline{QR}$.

(a) $\overline{PM}$ is .....
(b) PD is .....
(c) is $QM = MR$ .....

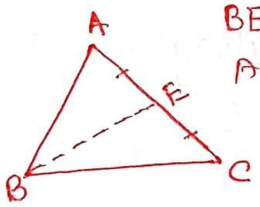


Sol. (a) Altitude
(b) median
(c) No

③ Draw rough sketches for the following:

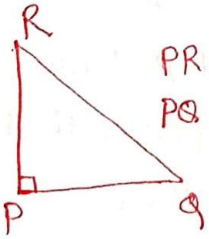
(a) In ΔABC, BE is a median.
(b) In ΔPQR, PQ and PR are altitudes of the triangle.
(c) In ΔXYZ, YL is an altitude in the exterior of the triangle.

Sol. (a)



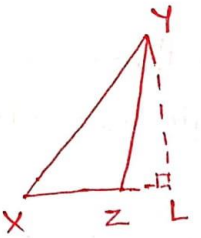
BE is a median
$AE = EC$

(b)



$PR \perp PQ$
$PQ \perp PR$

(c)

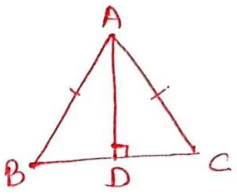


$YL \perp XZ$

Here, it can be observed that for ΔXYZ, YL is an Altitude drawn exterior to side XZ which is extended up to point L

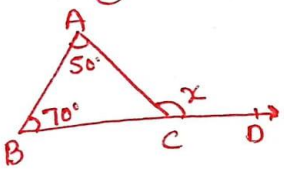
Q. Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.

Sol. Draw a line segment AD Perpendicular to BC. it is an altitude for this triangle. It can be observed that the length of BD and DC is also same, therefore, AD is also a median of this triangle.



Q. Find the value of the unknown exterior angle x in the following diagrams:

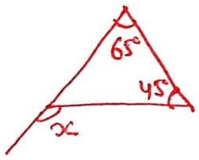
(i)



$x = 70 + 50$ (Exterior angle property)

$x = 120^\circ$

(ii)



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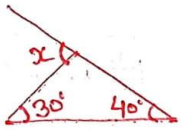

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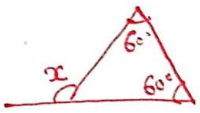
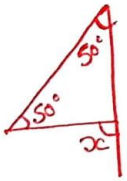
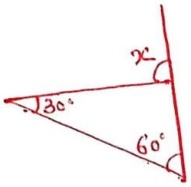


(v)



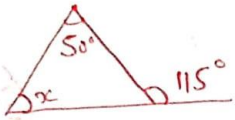
(vi)



[do it self]

Q. Find the value of the unknown interior angle x in the following figures:

(i)

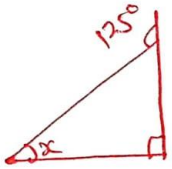


$x + 50 = 115$ (Exterior angle property)

$x = 115 - 50$

$x = 65°$

(iii)

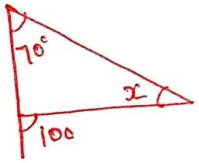


$x + 90 = 125$ (Exterior angle property)

$x = 125 - 90$

$x = 35°$

(ii)



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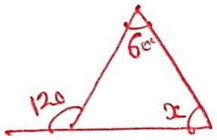
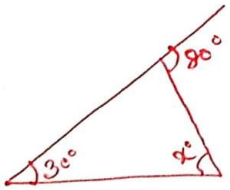


(v)



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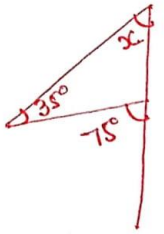
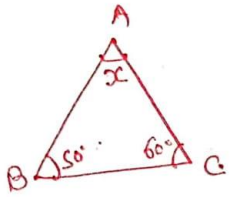


[do itself]

Q. Find the value of the unknown x in the following diagrams :-

(i)



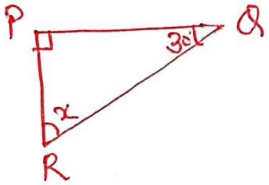
$x + 50 + 60 = 180^\circ$ (Sum of all interior angles of a triangles is 180°)

$x + 110 = 180$

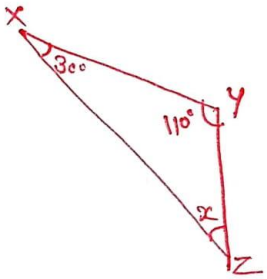
$x = 180 - 110$

$x = 70^\circ$

(ii)



(iii)



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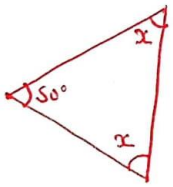

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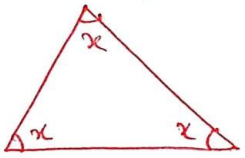

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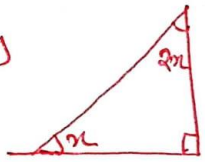
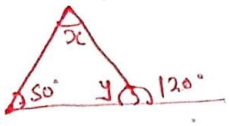


[do itself]

Q. Find the values of the unknowns x and y in the following diagrams:

(i) $x + 50 = 120$ (Exterior angles)



$x = 120 - 50$

$x = 70$

$x + y + 50 = 180°$ (Angle Sum property)

$70 + y + 50 = 180$

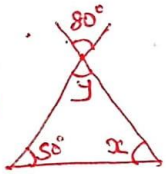
$120 + y = 180$

$y = 180 - 120$.

$y = 60°$

Therefore, $x = 70°$, $y = 60°$

(ii) $y = 80°$ (vertically opposite Angle)



$x + y + 50 = 180$ (Angle Sum property)

$x + 80 + 50 = 180$

$x + 130 = 180$

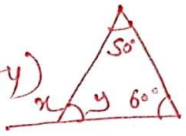
$x = 180 - 130$

$x = 50°$

Therefore, $x = 50°$, $y = 80°$

Q.. Find the values of the unknown x and y in the following diagrams:

(i)



$50 + 60 + y = 180$ (Angle Sum property)

$110 + y = 180$

$y = 180 - 110$

$y = 70°$

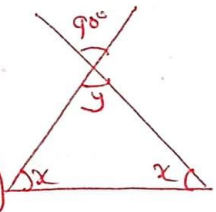
$x + y = 180°$ (Linear pair Property)

$x + 70 = 180$

$x = 180 - 70$

$x = 110°$

(ii) $y = 90°$ (vertically opposite)



$x + y + x = 180$ (Angle Sum property)

$2x + 90 = 180$

$2x = 180 - 90$

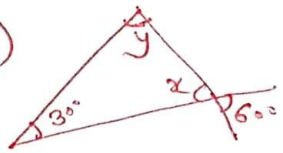
$2x = 90$

$x = \frac{90}{2}$

$x = 45°$

Therefore, $x = 45°$, $y = 90°$

(iii)



$x = 60^\circ$ (vertically opposite angles)

$x + y + 30 = 180^\circ$ (Angle Sum property)

$60 + y + 30 = 180^\circ$

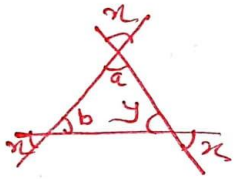
$90 + y = 180$

$y = 180 - 90$

$y = 90$

Therefore, $x = 60^\circ$, $y = 90^\circ$

(iv)



$a = x$ (vertically opposite angles)

$b = x$ (vertically opposite angles)

$y = x$ (vertically opposite angles)

$a + b + y = 180^\circ$ (Angle Sum property)

$x + x + x = 180^\circ$

$3x = 180$

$x = \frac{180}{3}$

$x = 60^\circ$

$y = x$

$y = 60^\circ$

Therefore, $x = 60^\circ$, $y = 60^\circ$

Q. Is it possible to have a triangle with the following sides ?

(i) 2 cm, 3 cm, 5 cm (ii) 3cm, 6cm, 7cm

(iii) 6cm, 3 cm, 2cm

Sol. In a triangle, the sum of the lengths of either two sides is always greater than the third side.

(i) 2 cm, 3cm, 5cm

$2+5 > 5$

$5 \not> 5$

However, $5\text{cm} = 5\text{cm}$

Hence, this triangle is not possible.

(ii) 3 cm, 6cm, 7cm

$3+6 > 7$ $\quad 6+7 > 3$ $\quad 7+3 > 6$

$9 > 7$ $\quad 13 > 3$ $\quad 10 > 6$

Hence, this triangle is possible.

(iii) 6 cm, 3 cm, 2 cm

$6+3 > 2$ $\quad 3+2 \not> 6$

$9 > 2$ $\quad 5 \not> 6$

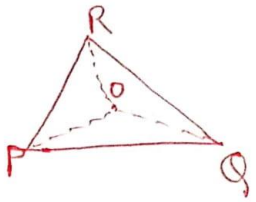
Hence, this triangle is not possible.

Q. Take any point O in the interior of a triangle PQR. is

(i) $OP + OQ > PQ$?

(ii) $OQ + OR > QR$?

(iii) $OR + OP > RP$?



Sol. If O is a point in the interior of a given triangle, then three triangles ΔOPQ, ΔOQR and ΔORP can be constructed. In a triangle, the sum of the lengths of either two sides is always greater than the third side.

(i) Yes, ΔOPQ is a triangle with sides OP, OQ and PQ.

$OP + OQ > PQ$.

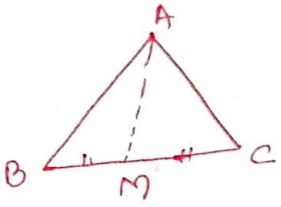
(ii) yes, ΔOQR is a triangle with sides OR, OQ and QR.

$OQ + OR > QR$

(iii) yes, ΔORP is a triangle with sides OR, OP and PR

$OR + OP > RP$

Q. AM is a median of a triangle ABC is $AB+BC+CA > 2AM$



Sol. In a triangle, the sum of the lengths of either two sides is always greater than the third side.

In $\triangle ABM$

$$AB+BM > AM \quad ---(i)$$

In $\triangle ACM$

$$AC+CM > AM \quad ---(ii)$$

Adding eq (i) and (ii) we get,

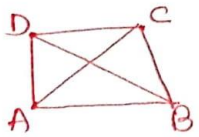
$$AB+BM+AC+CM > AM+AM$$

$$AB+BM+CM+AC > 2AM$$

$$AB+BC+CA > 2AM \qquad [BM+MC=BC]$$

Yes, the given expression is true.

Q. ABCD is a quadrilateral.
is $AB+BC+CD+DA > AC+BD$?



Sol. In a triangle, the sum of the lengths of either two sides is always greater than the third side.

Consider ΔABC

$AB+BC > CA$ ----(i)

in ΔBCD

$BC+CD > DB$ ----(ii)

in ΔCDA

$CD+DA > AC$ ----(iii)

in ΔDAB

$DA+AB > DB$ ----(iv)

Adding eq (i) (ii), (iii) and (iv)

$AB+BC+BC+CD+CD+DA+DA+AB > AC+BD+AC+BD$

$2AB+2BC+2CD+2DA > 2AC+2BD$

$2(AB+BC+CD+DA) > 2(AC+BD)$

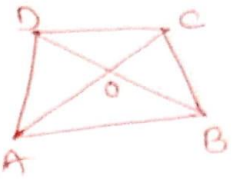
$(AB+BC+CD+DA) > (AC+BD)$

Yes, the given expression is true.

Q. ABCD is quadrilateral. Is

$AB+BC+CD+DA < 2(AC+BD)$?

Sol. In a triangle, the sum of the lengths of either two sides is always greater than the third side.



In ΔOAB

$OA+OB > AB$ ... (i)

In ΔOBC

$OB+OC > BC$ ... (ii)

In ΔOCD

$OC+OD > CD$ ... (iii)

In ΔODA

$OD+OA > DA$ ... (iv)

Add Eq (i), (ii), (iii) and (iv) We get

$OA+OB+OB+OC+OC+OD+OD+OA > AB+BC+CD+DA$

$2OA+2OB+2OC+2OD > AB+BC+CD+DA$

$2(OA+OC)+2(OB+OD) > AB+BC+CD+DA$

$2(AC)+2(BD) > AB+BC+CD+DA$

Q. the lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

Sol. In a triangle, the sum of the lengths of either two sides is always greater than the third side and also, the difference of the lengths of either two sides is always lesser than the third side.

Here, the third side will be lesser the sum of two sides and also, it will be greater than the difference of these two

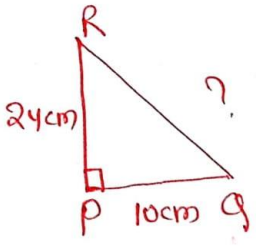
$12+15 = 27$

$15-12 = 3$

Therefore, those two measures are 27 cm and 3 cm

Q. PQR is a triangle, Right-angled at P. If PQ = 10 cm and PR = 24 cm find QR

Sol. By applying pythagoras theorem



$(H)^2 = (B)^2 + (P)^2$

$(QR)^2 = (PQ)^2 + (PR)^2$

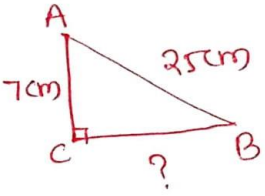
$(QR)^2 = (10)^2 + (24)^2$

$(QR)^2 = 100 + 576$

$(QR)^2 = 676$

$(QR)^2 = (26)^2 \Rightarrow QR = 26\text{cm}$

Q. ABC is a triangle, right-angled at C. If AB = 25cm and AC = 7cm find BC



Sol. By applying pythagoras theorem,

$(H)^2 = (B)^2 + (P)^2$

$(AB)^2 = (BC)^2 + (AC)^2$

$(25)^2 = (BC)^2 + (7)^2$

$625 = (BC)^2 + 49$

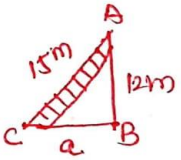
$625 - 49 = (BC)^2$

$576 = (BC)^2$

$(24)^2 = (BC)^2$

$BC = 24$ cm

Q. A 15m long ladder reached a window 12m high from the ground on placing it against a wall at a distance a. Find the distance of the feet of the ladder from the wall.



Sol. By applying pythagoras theorem,

$(H)^2 = (B)^2 + (P)^2$

$(15)^2 = (9)^2 + (12)^2$

$225 = a^2 + 144$

$225 - 144 = a^2$

$81 = a^2$

$(9)^2 = (a)^2 \Rightarrow a = 9$ m

Q. Which of the following can be sides of a right △ ?

(i) 2.5 cm, 6.5 cm, 6 cm
(ii) 2 cm, 2 cm, 5 cm
(iii) 1.5 cm, 2 cm, 2.5 cm

In case of right-angled triangles, identify the right angles.

Sol. (i) 2.5 cm, 6.5 cm, 6 cm

$(6.5)^2 = (2.5)^2 + (6)^2$

$42.25 = 6.25 + 36$

$42.25 = 42.25$

65×65 (+1)
42 | 25

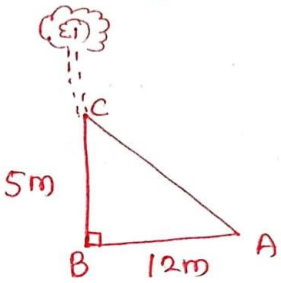
25×25 (+1)
6 | 25

The squares of the length of one side is the sum of the squares of the lengths of the remaining two sides.

Hence, these are the sides of a right-angled triangles.
Right angle will be in front of the side of 6.5 cm measure.

[do other it self]

Q. A tree is broken at a height of 5m from the ground and its top touches the ground at a distance of 12m from the base of the tree. Find the original height of the tree. (Imp)



Sol. Applying pythagoras theorem,

$(H)^2 = (B)^2 + (P)^2$

$(AC)^2 = (AB)^2 + (BC)^2$

$(AC)^2 = (12)^2 + (5)^2$

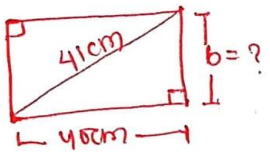
$(AC)^2 = 144 + 25$

$(AC)^2 = 169$

$(AC)^2 = (13)^2 \Rightarrow AC = 13\,m$

Thus, original height of tree = AC+BC = 13+5 = 18m

Q. Find the perimeter of the rectangle whose length is 40cm and diagonal is 41 cm.



Sol. Applying pythagoras theorem,

$(H)^2 = (B)^2 + (P)^2$

$(41)^2 = (40)^2 + (b)^2$

$1681 = 1600 + (b)^2$

$1681 - 1600 = b^2$

$81 = b^2$

$(9)^2 = (b)^2$

$b = 9$ cm

Perimeter of rectangle = $2(l+b)$

$= 2(40+9)$

$= 2 \times 49$

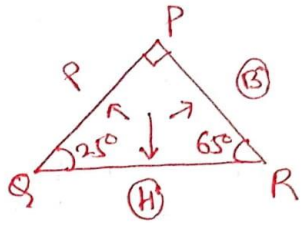
$= 98$ cm

Q. Angles Q and R of a ΔPQR are 25° and 65°. Write which of the following is true:

(i) $(PQ)^2 + (QR)^2 = (RP)^2$

(ii) $(PQ)^2 + (RP)^2 = (QR)^2$

(iii) $(RP)^2 + (QR)^2 = (PQ)^2$



Sol. $\angle P + \angle Q + \angle R = 180$ (Angle Sum property)

$\angle P + 25 + 65^\circ = 180$

$\angle P + 90 = 180$

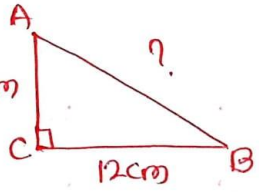
$\angle P = 180 - 90$

$\angle P = 90^\circ$

Therefore, $\angle P$ is right angled at point P

$(PQ)^2 + (RP)^2 = (QR)^2$

Q. ΔABC is right angled at C. If AC = 5 cm and BC = 12 cm find the length of AB



Sol. By applying pythagoras theorem,

$(H)^2 = (B)^2 + (P)^2$

$(AB)^2 = (BC)^2 + (AC)^2$

$(AB)^2 = (12)^2 + (5)^2$

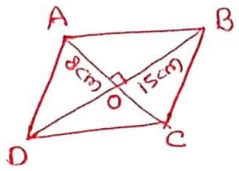
$(AB)^2 = 144 + 25$

$(AB)^2 = 169$

$(AB)^2 = (13)^2 \Rightarrow AB = 13$ cm.

Q. The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.

Sol. Let ABCD be a rhombus (all sides are of equal length) and its diagonals AC and BD are intersecting each other at point O, Diagonals in a rhombus bisect each other at 90°



$$AO = \frac{AC}{2} = \frac{16}{2} = 8\text{ cm}$$

$$BO = \frac{BD}{2} = \frac{30}{2} = 15\text{ cm}$$

By applying pythagores theorem in ΔAOB

$$OA^2 + OB^2 = AB^2$$
$$8^2 + 15^2 = AB^2$$
$$64 + 225 = AB^2$$
$$289 = AB^2$$
$$(17)^2 = AB^2$$
$$AB = 17\text{ cm}$$

Therefore, the length of the side of rhombus is 17 cm

Perimeter of rhombus = 4 × Side
= 4 × 17 = 68 cm.