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DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

BASIC NOTES (CBSE/NCERT)

FRACTIONS WITH RATIONAL NUMBERS

PART -1 OUT OF 1

6th, 7th & 8th

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∴ Fractions (Rational numbers) :-

②. Fraction = $\frac{\text{Numerator (N)}}{\text{Denominator (D)}}$ = rational numbers.

- Proper fraction = $N < D$ (Ex - $\frac{3}{7}$)
- Improper fraction = $N > D$ (Ex - $\frac{7}{3}$)
- Mix fraction = $W + N < D$ (Ex - $2\frac{1}{3}$) W = Whole



③. like fractions - when denominators are same.

Ex - $\frac{9}{5}, -\frac{2}{5}, \frac{7}{5}$ etc. These are like fractions

④. unlike fractions - when denominators are different.

Ex - $\frac{2}{3}, \frac{9}{7}, -\frac{8}{9}$ etc. These are unlike fractions.



⑤. Convert unlike fraction into like fraction (Imp.)

Ex - $\frac{2}{3}, \frac{9}{7}, \frac{2}{5} \Rightarrow \frac{70}{105}, \frac{135}{105}, \frac{42}{105}$ LCM of 3, 7 and 5

$$\frac{2}{3} = \frac{2}{3} \times \frac{35}{35} = \frac{70}{105} \quad (\text{Like fractions})$$

$$\frac{9}{7} = \frac{9}{7} \times \frac{15}{15} = \frac{135}{105}$$

$$\frac{2}{5} = \frac{2}{5} \times \frac{21}{21} = \frac{42}{105}$$

$$\begin{array}{r|rrr} 3 & 3 & 7 & 5 \\ 5 & 1 & 7 & 1 \\ 7 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 \end{array}$$

For deno. 3 - $5 \times 7 = 35$

For deno. 7 - $3 \times 5 = 15$

For deno. 5 - $3 \times 7 = 21$

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∴ Fractions (Rational numbers) :-

①. Convert improper fraction to mix fractions -

$$\text{Ex - } \frac{9}{7} = 1\frac{2}{7} = Q\frac{R}{D}$$

Division $\rightarrow 7 \overline{) 9} \begin{matrix} 1 \rightarrow \text{Quotient} \\ 2 \rightarrow \text{Remainder} \end{matrix}$

②. Convert mix fraction to improper -

$$\text{Ex - } 1\frac{2}{7} = Q\frac{R}{D}$$

$$= \frac{Q \times D + R}{D} = \frac{1 \times 7 + 2}{7} = \frac{7+2}{7} = \frac{9}{7}$$

③. Add and Subtract fractions - First Convert into like from proper, improper or mix fractions.

$$\text{EX - } \frac{9}{2} + \frac{7}{3}$$

$$= \frac{9}{2} \times \frac{3}{3} + \frac{7}{3} \times \frac{2}{2}$$

$$= \frac{27}{6} + \frac{14}{6}$$

$$= \frac{27+14}{6}$$

$$= \frac{41}{6}$$

$$= 6\frac{5}{6}$$



$$\begin{array}{r} 2 \overline{) 2,3} \\ 3 \overline{) 1,3} \\ \hline 1,1 \end{array}$$

For deno. 2 is 3

For deno 3 is 2



$$D \rightarrow 6 \overline{) 41} \begin{matrix} 6 \rightarrow Q \\ 5 \rightarrow R \end{matrix}$$

Similarly subtracted this question.

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-! Fractions (Rational numbers) :-

①. Add and Subtract mix fractions :-

Ex - $2\frac{9}{20} - 5\frac{7}{10}$

$$(2-5) + \left(\frac{9}{20} - \frac{7}{10}\right)$$

$$-3 + \left(\frac{9}{20} \times \frac{1}{1} - \frac{7}{10} \times \frac{2}{2}\right)$$

$$-3 + \left(\frac{9}{20} - \frac{14}{20}\right)$$

$$-3 + \left(\frac{9-14}{20}\right)$$

$$-3 + \left(\frac{-5}{20}\right)$$

$$-3 + \left(\frac{-1}{4}\right)$$

$$-3\frac{1}{4}$$

(minus take common)

$$[\text{Trick: } -4 + \left(-\frac{2}{3}\right) = -4\frac{2}{3}]$$

2	20, 10
2	10, 5
5	5, 5
1	1

For deno. 20 is 1

For deno 10 is 2

②. Multiplication and divide of fractions :-

Ex - $\frac{3^3}{3} \times \frac{4}{7} = \frac{3}{1} \times \frac{4}{7} = \frac{12}{7} = 1\frac{5}{7}$

$$\frac{3}{7} \div \frac{4}{7} = \frac{3}{7} \times \frac{7}{4} = \frac{3 \times 1}{1 \times 4} = \frac{3}{4}$$

③ Multiplication and divide of mix fractions :- First convert into improper fraction.

Ex - $2\frac{1}{3} \times 2\frac{2}{3} = \frac{7}{2} \times \frac{8}{3} = \frac{7 \times 2}{1 \times 3} = \frac{14}{3} = 4\frac{2}{3}$

$$6\frac{2}{5} \div 9\frac{2}{7} = \frac{32}{5} \times \frac{7}{65} = \frac{224}{325} \quad (\text{Use mind and get proper solve})$$

3.

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Q. What fraction of the a day is 8 hours ?

Sol. 1 day = 24 hours

$$\text{fraction} = \frac{8 \text{ hours}}{1 \text{ day}} = \frac{8 \text{ hours}}{24 \text{ hours}} = \frac{8}{24}$$



Q. What fraction of an hour is 40 minutes ?

Sol. do itself (hint : 1 hour = 60 minutes)

[Note: fraction find always when both Number in same unit]

Q. Arya, Abhimanyu and vivek shared lunch. Arya has bought two sandwiches, one made of vegetable and one of Jam. The other two boys forget to bring their lunch. Arya agreed to share his Sandwiches so that each person will have an equal share of each sandwich.

(a) How can Arya divide his Sandwiches so that each person has an equal share ?

(b) What part of a Sandwich will each boy receive ?

Sol. (a) Arya will divide each sandwich in three equal parts. then, he will give one part of each sandwich to each one of them (i.e. $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$)

(b) Each boy will receive $\frac{1}{3}$ part of each sandwich.



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∴ Fractions (Rational numbers) :-

Q. Kanchan dyes dresses. She had to dye 30 dresses. She has so far finished 20 dresses. What fraction of dresses has she finished?

Sol. do itself (Hint : No. of done / Total)



Q. Write the natural numbers from 2 to 12. What fraction of them are prime numbers?

Sol. Prime numbers : Which numbers divided by 1 or itself
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, ...

Prime Numbers b/w 2 to 12 are 2, 3, 5, 7, 11

No. of prime number b/w 2 to 12 = 5

* [Total no. b/w 2 to 12 are = 9 possible mistake]

Total no from 2 to 12 are = 11

$$\text{Fraction} = \frac{\text{Prime Number}}{\text{Total given Number}} = \frac{5}{11}$$



Q. Write the natural numbers from 102 to 113. What fraction of them are prime numbers?

Sol. do itself as above

Hint : 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113

Total numbers are 12

Prime numbers are 4

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∴ Fractions (Rational numbers):-

Q. Krishna received a CD player for her birthday. She bought 3 CDs and received 5 others as gifts. What fraction of her total CDs did she buy and what fraction did she receive as gifts?

Sol. Total CDs are = $3 + 5 = 8$

She bought CDs are = 3

Received in gift CDs are = 5

fraction = $\frac{3}{8}$ and $\frac{5}{8}$



Q. Draw number lines and locate the point on them

Ex - $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{4}{4}$

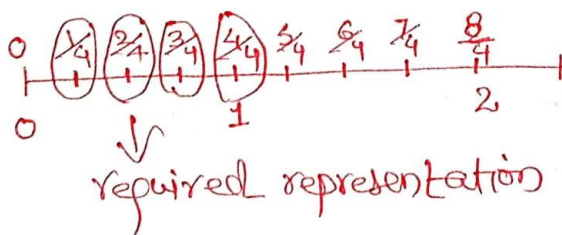
Sol. Firstly, convert into like fractions by denominators are same

$\frac{1}{2} = \frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$ and other are also in like

$\frac{2}{4}, \frac{1}{4}, \frac{3}{4}, \frac{4}{4}$ arrange in Ascending order

$\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$ [$\frac{4}{4} = 1$ that means nos are b/w 0 and 1]

denominator is 4 so divide 0 and 1 equal 4 parts



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∴ Fraction (Rational Numbers) :-

Q. Equivalent fraction : To find an equivalent fraction, we may divide both the numerator and denominator by the same number. Ex - $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}$ etc.

Ex - Find five equivalent fractions of $\frac{2}{3}$?

Sol. $\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$

$$\frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$$

$$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$$

$$\frac{2}{3} \times \frac{5}{5} = \frac{10}{15}$$

$$\frac{2}{3} \times \frac{6}{6} = \frac{12}{18}$$

Therefore equivalent fraction of $\frac{2}{3}$ are :

$$\frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}$$



EX - Find the equivalent fraction of $\frac{15}{35}$ with denominator 7.

Sol. $\frac{15}{35} = \frac{\square}{7}$

$$\frac{15 \div 5}{35 \div 5} = \frac{\boxed{3}}{7}$$



$$\begin{array}{r} 7 \overline{) 35} 5 \\ - 35 \\ \hline 0 \end{array}$$

EX - Find the equivalent fraction of $\frac{2}{9}$ with denominator 63

Sol. $\frac{2}{9} = \frac{\square}{63}$

$$\frac{2 \times 7}{9 \times 7} = \frac{\boxed{14}}{63}$$

$$\begin{array}{r} 9 \overline{) 63} 7 \\ - 63 \\ \hline 0 \end{array}$$

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∴ Fraction (Rational Number) :-

③. Simplest form of a fraction :- A fraction is said to be in the simplest form (or lowest) if its numerator and denominator have no common factor except 1.

[Note: Shortest way to find simplest form find HCF of the numerator and denominator and then divide both by them by the HCF]

EX - Convert $\frac{36}{24}$ into simplest form.

Sol.

$$\frac{36}{24} = \frac{36 \div 12}{24 \div 12} = \frac{3}{2}$$

Note: Find HCF also by division method check: $24 < 36$

$$\begin{array}{r} 24 \overline{) 36} 1 \\ \underline{24} \\ 12 \overline{) 24} 2 \\ \underline{24} \\ 0 \end{array}$$

HCF is 12



$$\begin{array}{r} 2 \overline{) 36} \\ \underline{2} \quad 18 \\ 2 \overline{) 18} \\ \underline{2} \quad 9 \\ 3 \overline{) 9} \\ \underline{3} \quad 3 \\ 3 \overline{) 3} \\ \underline{3} \quad 1 \end{array}$$

$$36 = 2 \times 2 \times 3 \times 3$$

$$24 = 2 \times 2 \times 2 \times 3$$

$$\begin{aligned} \text{HCF} &= 2 \times 2 \times 3 \\ &= 4 \times 3 = 12 \end{aligned}$$



④. Comparing like fractions :- Like fractions are fractions with the same denominator.

EX - $\frac{1}{5}, \frac{11}{5}, \frac{4}{5}, \frac{3}{5}, \frac{7}{5}$

Sol. $\frac{1}{5} < \frac{3}{5} < \frac{4}{5} < \frac{7}{5} < \frac{11}{5}$

[If not in like fraction then convert into like by taking LCM]

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-: Fraction (Rational numbers) :-

Q. Comparing unlike fractions :- If the numerator is the same in two fractions, the fraction with the smaller denominator is greater of the two.

Ex - $\frac{2}{1}, \frac{2}{13}, \frac{2}{9}, \frac{2}{5}, \frac{2}{7}$

Sol. Arrange in Ascending order - (increasing order)

$\frac{2}{13}, \frac{2}{9}, \frac{2}{7}, \frac{2}{5}, \frac{2}{1}$ (Smallest denominator is greater)

Q. Compare when Numerator and denominator are different :-

Ex - $\frac{4}{5}$ and $\frac{5}{6}$ Compare

Sol. Equivalent fractions of $\frac{4}{5}$ and $\frac{5}{6}$ are :

$\frac{4}{5} = \frac{8}{10} = \frac{12}{15} = \frac{16}{20} = \frac{20}{25} = \frac{24}{30} = \frac{28}{35} = \dots$

$\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{20}{24} = \frac{25}{30} = \frac{30}{36} = \dots$

$\frac{4}{5} = \frac{24}{30}$ and $\frac{5}{6} = \frac{25}{30}$

$\frac{24}{30} < \frac{25}{30}$ So, $\frac{5}{6} > \frac{4}{5}$

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∴ Fractions (Rational Numbers) :-

Q. Compare by common multiple :-

Ex - Compare $\frac{5}{6}$ and $\frac{13}{15}$

Sol. The fractions are unlike, then denominators are same by common multiple (Lcm) :-

$$\frac{5}{6} \times \frac{5}{5} = \frac{25}{30}$$

$$\frac{13}{15} \times \frac{2}{2} = \frac{26}{30}$$

$$\frac{25}{30} < \frac{26}{30} \text{ So, } \frac{5}{6} < \frac{13}{15}$$



$$\begin{array}{r|l} 2 & 6, 15 \\ \hline 3 & 3, 15 \\ \hline 5 & 1, 5 \\ \hline & 1, 1 \end{array}$$

$$\text{Lcm} = 2 \times 3 \times 5 = 30$$

$$30 \div 6 = 5$$

$$30 \div 15 = 2$$

Q. Ila read 25 pages of a book containing 100 pages. Lalita read $\frac{2}{5}$ of the same book. Who read less?

Sol. Number of pages read by Lalita = $\frac{2}{5} \times 100$
 $= 2 \times 20$
 $= 40 \text{ pages}$

Number of pages read by Ila = 25 pages

Hence, Ila has read less number of pages.



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-: Fraction (Rational Number) :-

Q. Rafik exercised for $\frac{3}{6}$ of an hour, while Rohit exercised for $\frac{3}{4}$ of an hour. Who exercised for a longer time?

Sol. do itself as above.



Q. In a class A of 25 students, 20 passed with 60% or more marks; in ~~another~~ another class B of 30 students, 24 passed with 60% or more marks, in which class was a greater fraction of students getting with 60% or more marks?

Sol. fraction of class A = $\frac{20}{25} = \frac{20 \div 5}{25 \div 5} = \frac{4}{5}$

fraction of class B = $\frac{24}{30} = \frac{24 \div 6}{30 \div 6} = \frac{4}{5}$



From both class, an equal fraction of students passed in first class.

Q. Shubham painted $\frac{2}{3}$ of the wall space in his room. his sister madhavi helped and painted $\frac{1}{3}$ of the wall space. How much did they paint together.

Sol. Together means (+)

They painted together = $\frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1$.
= The Complete wall.

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∴ Fraction (Rational Numbers):-

Q. Javed was given $\frac{5}{7}$ of a basket of oranges. What fraction of oranges was left in the basket?

Sol. Fraction given to Javed = $\frac{5}{7}$

$$\begin{aligned}
 \text{Fraction left in the basket} &= 1 - \frac{5}{7} \\
 &= \frac{1}{1} - \frac{5}{7} \\
 &= \frac{1 \times 7}{1} - \frac{5}{7} \\
 &= \frac{7}{7} - \frac{5}{7} \\
 &= \frac{7-5}{7} = \frac{2}{7}
 \end{aligned}$$



Q. Sarita bought $\frac{2}{5}$ metre of ribbon and Lalita $\frac{3}{4}$ metre of ribbon. What is the total length of the ribbon they bought?

Sol. do itself (total means +)



Q. Naina was given $1\frac{1}{2}$ piece of cake and Najma was given $1\frac{1}{3}$ piece of cake. Find the total amount of cake was given to both of them.

Sol. do itself (total mean +)

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∴ Fraction (Rational Numbers) :-

Q. A piece of wire $\frac{7}{8}$ metre long broke into two pieces. One piece was $\frac{1}{4}$ metre long. How long is the other piece?

Sol. do it self (How long means comparing (-))

Q. Nandini's house is $\frac{9}{10}$ km from her school. She walked some distance and then took a bus for $\frac{1}{2}$ km to reach the school. How far did she walk?

Sol. Distance ^{Walked} ~~covered~~ by Nandini =

Total distance - Distance covered by bus

$$= \frac{9}{10} - \frac{1}{2}$$

$$= \frac{9 \times 1 - 1 \times 5}{10}$$

$$= \frac{9-5}{10} = \frac{4}{10} = \frac{2}{5} \text{ km}$$

Q. Aasha and Samuel have bookshelves of the same size partly filled with books. Aasha's shelf is $\frac{5}{6}$ th full and Samuel's shelf is $\frac{2}{5}$ th full, whose bookshelf is more full? What fraction?

Sol. Solve it self (-)

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∴ Fraction (Rational) Numbers :-

Q. Jaidev takes $2\frac{1}{5}$ minutes to walk across the school ground. Rahul takes $\frac{7}{4}$ minutes to do the same. Who takes less time and by what fraction?

Sol. Time taken by Jaidev = $2\frac{1}{5} = \frac{11}{5}$ min

Time taken by Rahul = $\frac{7}{4}$ min

Converting these into like fractions.

$$\frac{11}{5} = \frac{11}{5} \times \frac{4}{4} = \frac{44}{20}$$

$$\frac{7}{4} = \frac{7}{4} \times \frac{5}{5} = \frac{35}{20}$$

$$\frac{44}{20} > \frac{35}{20}$$

Hence, Rahul takes lesser time

$$\text{Difference} = \frac{44}{20} - \frac{35}{20}$$

$$= \frac{44-35}{20}$$

$$= \frac{9}{20} \text{ min}$$

$$\begin{array}{r} 2 \overline{) 514} \\ \underline{2} \\ 31 \\ \underline{20} \\ 11 \\ \underline{10} \\ 10 \\ \underline{10} \\ 0 \end{array}$$

$$\text{Lcm} = 20$$

$$20 \div 5 = 4$$

$$20 \div 4 = 5$$

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∴ Fractions (Rational Numbers) :-

①. Rational Numbers :- A rational number is defined as a number that can be expressed in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

Thus, rational numbers include integers and fractions.

Ex - $\frac{-5}{2}$, $\frac{3}{8}$, $\frac{0}{3}$ etc.



②. Equivalent rational numbers :- By multiplying the numerator and denominator of a rational number by the same non-zero integer, we obtain another rational number equivalent to the given rational number. This is exactly like obtaining equivalent fractions.

③. Positive and Negative Rational Numbers :-

Ex - Positive integer $\frac{-2}{7}$, $\frac{3}{7}$ etc.

Negative integer $-\frac{4}{7}$, $-\frac{2}{7}$ etc.



④. Rational Numbers in Standard form :- A standard form of rational number is if its denominator is positive integer and the numerator and denominator have no common factor other than 1.

Ex - $-\frac{3}{2}$, $-\frac{4}{5}$ etc.

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∴ Fraction (Rational Number) :-

Q. Rational Numbers between two rational numbers :-
We can find unlimited number of rational numbers between any two rational numbers.

EX- find five rational number b/w $-\frac{3}{5}$ and $-\frac{1}{3}$

$$-\frac{3}{5} = -\frac{3}{5} \times \frac{3}{3} = -\frac{9}{15} \times \frac{2}{2} = -\frac{18}{30}$$

$$-\frac{1}{3} = -\frac{1}{3} \times \frac{5}{5} = -\frac{5}{15} \times \frac{2}{2} = -\frac{10}{30}$$



$$\begin{array}{r} 3 \overline{) 5, 3} \\ \underline{5} \\ 1 \end{array}$$

$$\text{LCM} = 3 \times 5$$

$$= 15$$

$$15 \div 5 = 3$$

$$15 \div 3 = 5$$

$$\frac{-17}{30}, \frac{-16}{30}, \frac{-15}{30}, \frac{-14}{30}, \frac{-13}{30}$$

Q.



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∴ Fraction (Rational Number) :-

Q. Properties of rational numbers :-

- 1: closure property :- Rational numbers are closed under addition, that is, for any two rational numbers a and b , $a+b$ is also a rational number.

$$\text{Ex - } \frac{3}{8} + \frac{(-5)}{7} = \frac{21 + (-40)}{56} = \frac{-19}{56} \quad (a+b)$$

- Rational numbers are closed under subtraction, that is, for any rational numbers a and b , $a-b$ is also a rational number.

$$\text{Ex - } \frac{-5}{7} - \frac{2}{3} = \frac{-5 \times 3 - 2 \times 7}{21} = \frac{-29}{21} \quad (a-b)$$

- Rational numbers are closed under multiplication, that is, for any two rational numbers a and b , $a \times b$ is also a rational number.

$$\text{Ex - } \frac{-2}{8} \times \frac{4}{5} = \frac{-2 \times 4}{3 \times 5} = \frac{-8}{15} \quad (a \times b)$$

- Rational numbers are not closed under division, if exclude zero then the collection of, all other rational numbers is closed under division.

$$\text{Ex - } \frac{-5}{3} \div \frac{2}{5} = \frac{-5}{3} \times \frac{5}{2} = \frac{-5 \times 5}{3 \times 2} = \frac{-25}{6} \quad (a \div b)$$

[Note: $a \div 0$ is not defined]

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∴ Fraction (Rational Numbers) :-

2. Commutativity :-

- Addition :- Two rational numbers can be added in any order. We say that addition is commutative for rational numbers. That is, for any two rational numbers a and b , $a+b = b+a$.

$$\text{Ex - } -\frac{2}{3} + \frac{5}{7} = \frac{-14+15}{21} = \frac{1}{21} \quad (a+b)$$

$$\frac{5}{7} + \left(-\frac{2}{3}\right) = \frac{15+(-14)}{21} = \frac{1}{21} \quad (b+a)$$

$a+b = b+a$ verified.

- Subtraction :- Note that subtraction is not commutative for integers and integers are also rational numbers, so, subtraction will not be commutative for rational numbers too.

$$\text{Ex - } \frac{2}{3} - \frac{5}{4} = \frac{2 \times 4 - 5 \times 3}{12} = \frac{8-15}{12} = -\frac{7}{12} \quad (a-b)$$

$$\frac{5}{4} - \frac{2}{3} = \frac{5 \times 3 - 2 \times 4}{12} = \frac{15-8}{12} = \frac{7}{12} \quad (b-a)$$

$a-b \neq b-a$ Not verified.

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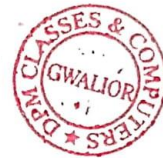
∴ Fraction (Rational Numbers) :-

- Multiplication :- multiplication is commutative for rational numbers. In general $a \times b = b \times a$ for any two rational numbers a and b .

$$\text{EX - } -\frac{7}{3} \times \frac{6}{5} = \frac{-7 \times 6}{3 \times 5} = \frac{-42}{15} \quad (a \times b)$$

$$\frac{6}{5} \times -\frac{7}{3} = \frac{6 \times -7}{5 \times 3} = \frac{-42}{15} \quad (b \times a)$$

$$\boxed{a \times b = b \times a} \quad \text{verified}$$



- Division :- Division is not commutative for rational numbers. $a \div b \neq b \div a$

$$\text{EX - } -\frac{5}{4} \div \frac{3}{7} = -\frac{5}{4} \times \frac{7}{3} = \frac{-5 \times 7}{4 \times 3} = \frac{-35}{12} \quad (a \div b)$$

$$\frac{3}{7} \div -\frac{5}{4} = \frac{3}{7} \times \frac{4}{-5} = \frac{3 \times 4}{7 \times -5} = \frac{12}{-35} \quad (b \div a)$$

$$\boxed{a \div b \neq b \div a} \quad \text{Not verified}$$



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-; Fraction (Rational Numbers) :-

3. Associativity :-

- Addition :- Addition is associative for rational numbers, that is, for any three rational numbers a , b and c , $a + (b + c) = (a + b) + c$

EX -
$$\begin{aligned} -\frac{2}{3} + \left[\frac{3}{5} + \left(-\frac{5}{6} \right) \right] &= -\frac{2}{3} + \left[\frac{3 \times 6 + (-5) \times 5}{30} \right] \\ &= -\frac{2}{3} + \left[\frac{18 - 25}{30} \right] \\ &= -\frac{2}{3} - \frac{7}{30} \\ &= \frac{-2 \times 10 - 7 \times 1}{30} \\ &= \frac{-20 - 7}{30} \\ &= \frac{-27}{30} \\ &= -\frac{9}{10} \quad [a + (b + c)] \end{aligned}$$

Verified.

$$a + (b + c) = (a + b) + c$$

$$\begin{aligned} \left(-\frac{2}{3} + \frac{3}{5} \right) + \frac{-5}{6} &= \left[\frac{-2 \times 5 + 3 \times 3}{15} \right] + \frac{-5}{6} \\ &= \left[\frac{-10 + 9}{15} \right] + \frac{-5}{6} \\ &= \frac{-1}{15} + \frac{-5}{6} \\ &= \frac{-1 \times 2 + (-5) \times 5}{30} = \frac{-2 - 25}{30} \\ &= \frac{-27}{30} = -\frac{9}{10} \quad [(a + b) + c] \end{aligned}$$

DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Fraction (Rational Numbers) :-

- Subtraction :- Subtraction is not associative for integers, so, subtraction is not associative for rational numbers.

$$a - (b - c) \neq (a - b) - c$$

- Multiplication :- multiplication is associative for rational numbers, that is for any three rational numbers a, b and c , $a \times (b \times c) = (a \times b) \times c$

Ex - $-\frac{7}{3} \times \left(\frac{5}{4} \times \frac{2}{9}\right) = -\frac{7}{3} \times \left[\frac{5 \times 2}{4 \times 9}\right]$

$= -\frac{7}{3} \times \frac{10}{36}$

$= \frac{-70}{108} = \frac{-35}{54} \quad [a \times (b \times c)]$

$\left(-\frac{7}{3} \times \frac{5}{4}\right) \times \frac{2}{9} = \left(\frac{-7 \times 5}{3 \times 4}\right) \times \frac{2}{9}$

$= \left(\frac{-35}{12}\right) \times \frac{2}{9}$

$= \frac{-70}{108} = \frac{-35}{54} \quad [(a \times b) \times c]$

$$[(a \times b) \times c = a \times (b \times c)] \text{ verified.}$$

- Division :- Division is not associative for integers, so Division is not associative for rational numbers.

$$(a \div b) \div c \neq a \div (b \div c)$$

DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Fraction (Rational Numbers) :-

4. Distributivity of multiplication over addition for rational numbers :- $a(b+c) = ab+ac$

Ex - $-\frac{3}{4} \times \left[\frac{2}{3} + \frac{-5}{6} \right] = -\frac{3}{4} \times \left[\frac{2 \times 2 + (-5) \times 1}{6} \right]$

$$= -\frac{3}{4} \times \left[\frac{4-5}{6} \right]$$

$$= -\frac{3}{4} \times \frac{-1}{6}$$

$$= \frac{3 \times 1}{4 \times 6} \quad [- \times - = +]$$

$$= \frac{1}{8} [a(b+c)]$$

$$-\frac{3}{4} \times \frac{2}{3} + \left(-\frac{3}{4} \right) \times \left(\frac{-5}{6} \right) = \frac{-3 \times 2}{4 \times 3} + \frac{-3 \times -5}{4 \times 6}$$

$$= \frac{-6}{12} + \frac{15}{24}$$

$$= \frac{-6 \times 2 + 15 \times 1}{24}$$

$$= \frac{-12 + 15}{24} = \frac{3}{24} = \frac{1}{8}$$

[ab+ac]

$$\boxed{a \times (b+c) = a \times b + a \times c} \text{ verified}$$

Similarly :- Distributivity of multiplication over subtraction for rational numbers

$$\boxed{a \times (b-c) = a \times b - a \times c}$$

DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Fraction (Rational Numbers):-

③. The Role of zero (0) :- zero is called the identity for the addition of rational numbers; it is the additive identity for integers and whole numbers as well.

$$a + 0 = a = 0 + a$$

④. The Role of 1 :- 1 is the multiplicative identity for rational numbers

$$a \times 1 = a = 1 \times a$$



⑤. Negative of a number or Additive inverse :-

EX - Additive inverse of $\frac{2}{3}$ is $-\frac{2}{3}$

Additive inverse of $-\frac{2}{3}$ is $\frac{2}{3}$

$$a + (-a) = (-a) + a = 0$$

⑥. Reciprocal or multiplicative inverse :-

EX - Reciprocal of $\frac{8}{7}$ is $\frac{7}{8}$

Reciprocal of $-\frac{8}{9}$ is $-\frac{9}{8}$



Note: zero (0) has no reciprocal.

$$\frac{a}{b} \times \frac{c}{d} = 1$$

DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Fractions (Rational Numbers):-

Q. A rectangle sheet of paper is $12\frac{1}{2}$ cm long and $10\frac{2}{3}$ cm wide. Find its perimeter.

Sol.

$$\text{length} = 12\frac{1}{2} \text{ cm} = \frac{25}{2} \text{ cm} = \frac{25}{2} \times \frac{3}{3} = \frac{75}{6} \text{ cm}$$

$$\text{breadth} = 10\frac{2}{3} \text{ cm} = \frac{32}{3} \text{ cm} = \frac{32}{3} \times \frac{2}{2} = \frac{64}{6} \text{ cm}$$

Perimeter of rectangle sheet = 2 (length + breadth)

$$= 2 \left(\frac{75}{6} + \frac{64}{6} \right)$$

$$= 2 \left(\frac{75+64}{6} \right)$$

$$= 2 \times \frac{139}{6}$$

$$= \frac{139}{3}$$

$$= 46\frac{1}{3} \text{ cm}$$

Q. Salil wants to put a picture in the frame. The picture is $7\frac{3}{5}$ cm wide. To fit in the frame the picture cannot be more than $7\frac{3}{10}$ cm wide. How much should the picture be trimmed? (-)

Sol.

$$\text{picture width} = 7\frac{3}{5} = \frac{38}{5} \times \frac{2}{2} = \frac{76}{10} \text{ cm}$$

$$\text{Frame width} = 7\frac{3}{10} = \frac{73}{10} \times \frac{1}{1} = \frac{73}{10} \text{ cm}$$

$$\text{Picture will be trimmed} = \frac{76}{10} - \frac{73}{10} = \frac{3}{10} \text{ cm}$$

DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Fractions (Rational Numbers) :-

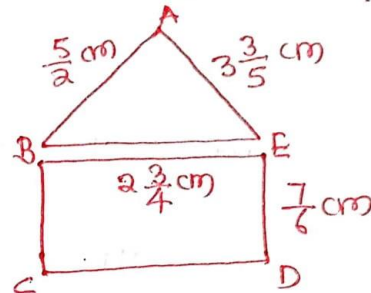
Q. Find the perimeters of (i) $\triangle ABE$ (ii) the rectangle BCDE in this figure. Whose perimeter is greater?

Sol. $AB = \frac{5}{2} \text{ cm}$

$AE = 3\frac{3}{5} = \frac{18}{5} \text{ cm}$

$BE = 2\frac{3}{4} = \frac{11}{4} \text{ cm}$

$ED = \frac{7}{6} \text{ cm}$



(i) Perimeter of $\triangle ABE$ = Sum of all sides

= $AB + BE + EA$

= $\frac{5}{2} + \frac{18}{5} + \frac{11}{4}$

= $\frac{5 \times 10 + 18 \times 4 + 11 \times 5}{20}$

= $\frac{50 + 72 + 55}{20}$

= $\frac{177}{20} = 8\frac{17}{20} \text{ cm}$

$$\begin{array}{r} 2 \overline{) 254} \\ 2 \overline{) 152} \\ 5 \overline{) 151} \\ \hline 111 \end{array}$$

Len = 20

$20 \div 2 = 10$

$20 \div 5 = 4$

$20 \div 4 = 5$

$177 \div 20 = 8\frac{17}{20}$

changing in like fraction

$$\begin{array}{r} 20 \overline{) 177} \\ 20 \overline{) 177} \\ \hline 177 \\ 20 \overline{) 177} \\ \hline 177 \\ 20 \overline{) 177} \\ \hline 177 \end{array}$$

(ii) Perimeter of rectangle BCDE = $2(\text{length} + \text{breadth})$

= $2(BE + ED)$

= $2\left(\frac{11}{4} + \frac{7}{6}\right)$

= $2\left(\frac{11 \times 3 + 7 \times 2}{12}\right)$

= $2\left(\frac{33 + 14}{12}\right) = 2 \times \frac{47}{12} = \frac{47}{6} \text{ cm}$
25.

DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Fractions (Rational Numbers):

Q. Ritu ate $\frac{3}{5}$ part of an apple and the remaining apple was eaten by her brother Somu. How much part of the apple did Somu eat? Who had the larger share? By how much?

Sol. Part of apple eaten by Ritu = $\frac{3}{5}$

Part of apple eaten by Somu = $1 - \frac{3}{5}$

$$= \frac{1}{1} \times \frac{5}{5} - \frac{3}{5} \times \frac{1}{1}$$

$$= \frac{5}{5} - \frac{3}{5}$$

$$= \frac{5-3}{5}$$

$$= \frac{2}{5}$$

Therefore, Somu ate $\frac{2}{5}$ part of the apple.

Since $\frac{3}{5} > \frac{2}{5}$, Ritu had the larger share.

$$\text{Difference} = \frac{3}{5} - \frac{2}{5} = \frac{1}{5}$$

Therefore, Ritu's share is larger than the share of Somu by $\frac{1}{5}$.



DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

-∴ Fractions (Rational Numbers):-

Q. Michael finished colouring a picture in $\frac{7}{12}$ hours.
 Vaibhav finished colouring the same picture in $\frac{3}{4}$ hours.
 Who worked longer? By what fraction was it longer?

Sol. Time taken by Michael = $\frac{7}{12}$ hours

Time taken by Vaibhav = $\frac{3}{4}$ hours

Converting into like fractions

$$\frac{7}{12} = \frac{7}{12} \times \frac{1}{1} = \frac{7}{12}$$

$$\frac{3}{4} = \frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$$



$$\begin{array}{r} 2 \overline{) 412} \\ \underline{2} 6 \\ 2 6 \\ \underline{3} 3 \\ 11 \\ 11 \end{array}$$

Lcm = 12

$$12 \div 12 = 1$$

$$12 \div 4 = 3$$

Since, $\frac{9}{12} > \frac{7}{12}$

Vaibhav worked longer.

$$\text{Difference} = \frac{9}{12} - \frac{7}{12} = \frac{2}{12} = \frac{1}{6} \text{ hours.}$$



[Note: in fractions 'of' represents multiplication]

Q. Vidhya and Pratap went for a picnic. Their mother gave them a water bottle that contained 5 litres of water. Vidhya consumed $\frac{2}{5}$ of the water. Pratap consumed remaining water.

(i) How much water did Vidhya drink?

(ii) What fraction of the total quantity of water did Pratap drink?

27.

DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Fractions (Rational Numbers) :-

Sol. (i) Water consumed by Vidya = $\frac{2}{5}$ of 5 litres

$$= \frac{2}{5} \times 5$$

$$= 2 \text{ litres}$$

(ii) Water consumed by Prateep = $1 - \frac{2}{5}$

$$= \frac{1}{1} \times \frac{5}{5} - \frac{2}{5} \times \frac{1}{1}$$

$$= \frac{5}{5} - \frac{2}{5}$$

$$= \frac{5-2}{5}$$

$$= \frac{3}{5} \text{ of the total water}$$

Q. Find: $\frac{1}{2}$ of (i) 24 (ii) 46

Sol. (i) $\frac{1}{2}$ of 46 = $\frac{1}{2} \times 46 = \frac{46}{2} = 23$

(ii) $\frac{1}{2}$ of 24 = $\frac{1}{2} \times 24 = \frac{24}{2} = 12$

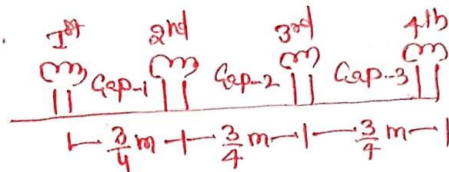
DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Fractions (Rational Numbers) :-

Q. Saili plants 4 saplings (पौधे), in a row, in her garden. The distance b/w two adjacent saplings is $\frac{3}{4}$ m. Find the distance b/w the first and the last saplings.

Sol.



$$\text{length of 1 gap} = \frac{3}{4} \text{ m}$$

$$\begin{aligned} \text{Therefore, Distance b/w I and IV Sapling} &= 3 \times \frac{3}{4} \\ &= \frac{9}{4} \\ &= 2\frac{1}{4} \text{ m} \end{aligned}$$

Q. Lipika reads a book for $1\frac{3}{4}$ hours everyday. She reads the entire book in 6 days. How many hours in all were required by her to read the book?

Sol.

$$\text{Number of hours reads in 1 day} = 1\frac{3}{4} = \frac{7}{4} \text{ hours}$$

$$\text{Number of days} = 6$$

$$\begin{aligned} \text{Therefore, Number of hours reads in 6 days} &= \frac{7}{4} \times 6^3 \\ &= \frac{7 \times 3}{2} \\ &= \frac{21}{2} \\ &= 10\frac{1}{2} \text{ hours.} \end{aligned}$$

DPM CLASSES

6th to 10th (Math's & Science), 11th & 12th (Physics, Chemistry, Math's)

∴ Fractions (Rational Numbers) :-

Q. A car runs 16 km using 1 litre of petrol. How much distance will it cover using $2\frac{3}{4}$ litres of petrol.

Sol.

$$\text{Car runs in 1 litre of petrol} = 16 \text{ km}$$

$$\begin{aligned} \text{Car runs in } 2\frac{3}{4} \text{ litres of petrol} &= 16 \times 2\frac{3}{4} \\ &= 16 \times \frac{11}{4} \\ &= 4 \times 11 \\ &= 44 \text{ km} \end{aligned}$$

Q. Sushant reads $\frac{1}{3}$ part of a book in 1 hour. How much part of the book will he read in $2\frac{1}{5}$ hours?

Sol. $\text{book read in 1 hour} = \frac{1}{3} \text{ part}$

$$\text{book read in } 2\frac{1}{5} \text{ hour} = \frac{1}{3} \times 2\frac{1}{5}$$

$$= \frac{1}{3} \times \frac{11}{5}$$

$$= \frac{11}{15}$$