Sample Solutions

4.

I. RegExps.

1. first and last symbols are diff.

$$a(a+b)^*b + b(a+b)^*a$$

3. Not having 2 conseq 0's.

$$(1+01)^*(0+\epsilon) + (0+\epsilon)(1+10)^*$$

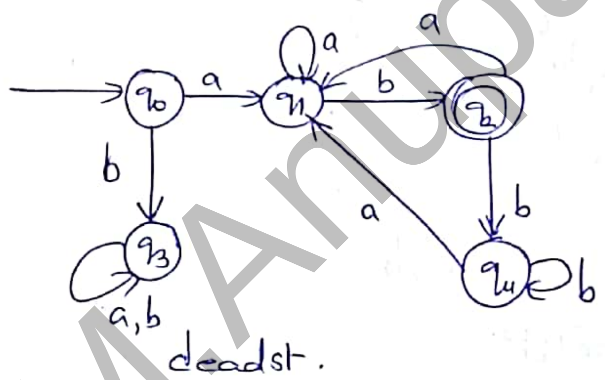
5. $L = \{a^{2n}b^{2m} \mid n,m \geq 0\}$ (even a's followed by even b's)

$$(aa)^*(bb)^*$$

9. Do not end with 01.

$$(0+1)^*(10+00+11) + (\epsilon+0+1)$$

II DFA 1.



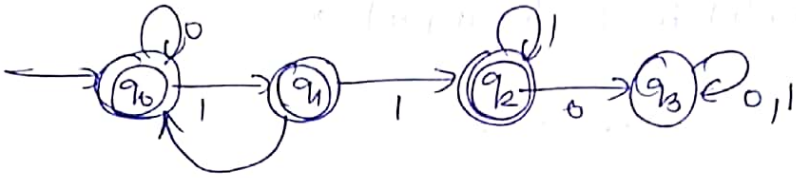
$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{a, b\}, \delta, q_0, \{q_2\})$ where

δ is

	a	b
$\rightarrow q_0$	q_1	q_3
q_1	q_1	q_2
q_2	q_1	q_4
q_3	q_3	q_3
q_4	q_1	q_4

2. Not containing 110

First construct DFA for containing 110 and interchange final and non final states. Final DFA is

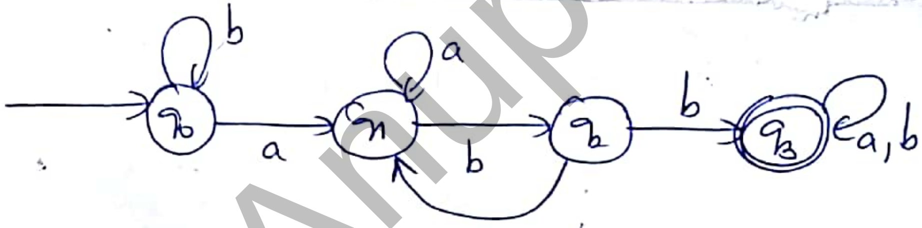


Required DFA $M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \delta, q_0, \{q_0, q_1, q_2\})$

where δ is

	0	1
$\rightarrow q_0$	q_0	q_1
q_1	q_0	q_2
q_2	q_3	q_2
q_3	q_3	q_3

4. At least one a followed by exactly 2 b's



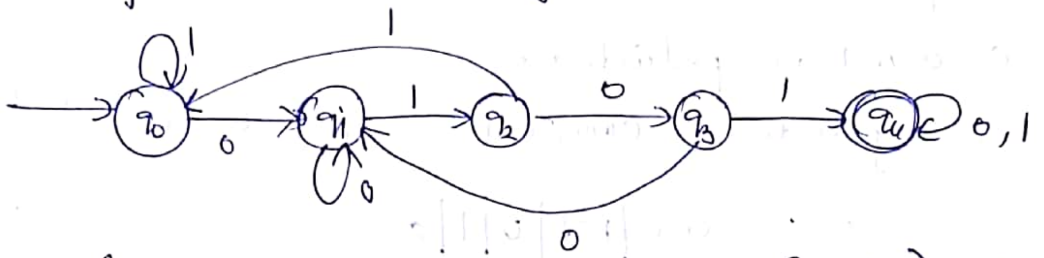
Assuming the string contains atleast one a followed by exactly 2 b's

$M = (\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_3\})$

Where δ is

	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_3	q_3

Containing 0101 as substring



$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0,1\}, \delta, q_0, \{q_4\})$ where δ is given as

	0	1
$\rightarrow q_0$	q_1	q_0
q_1	q_1	q_2
q_2	q_3	q_0
q_3	q_1	q_4
q_4	q_4	q_4

III NFA:

3. Any no. of a's followed by any no. of b's followed by any no. of c's.

$M = (\{q_0, q_1, q_2\}, \{a,b\}, \delta, [q_0], \{[q_2]\})$ where

	a	b	c	ε
$\rightarrow q_0$	$\{q_0\}$	–	–	$\{q_1\}$
q_1	–	$\{q_1\}$	–	$\{q_2\}$
q_2	–	–	$\{q_2\}$	–

4. A pair of 1's followed by a pair of 0's.

$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0,1\}, \delta, [q_0], \{[q_4]\})$

where δ is

	0	1
$\rightarrow q_0$	$\{q_0\}$	$\{q_0, q_1\}$
q_1	—	$\{q_2\}$
q_2	$\{q_3\}$	—
q_3	$\{q_4\}$	—
q_4	$\{q_4\}$	$\{q_4\}$

IV CFG:

2. Set of palindromes over $\{0,1\}$

ϵ, 0 & 1 are palindromes

If w is a palindrome 0w0, 1w1 are palindromes, so CFG is

$$S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \epsilon$$

This grammar generates both odd & even palindromes.

Hence, $G = \{V, T, P, S\}$ where $V = \{S\}$, $T = \{0,1\}$, P is given above and S is the start symbol.

3. Equal no. of a's & b's.

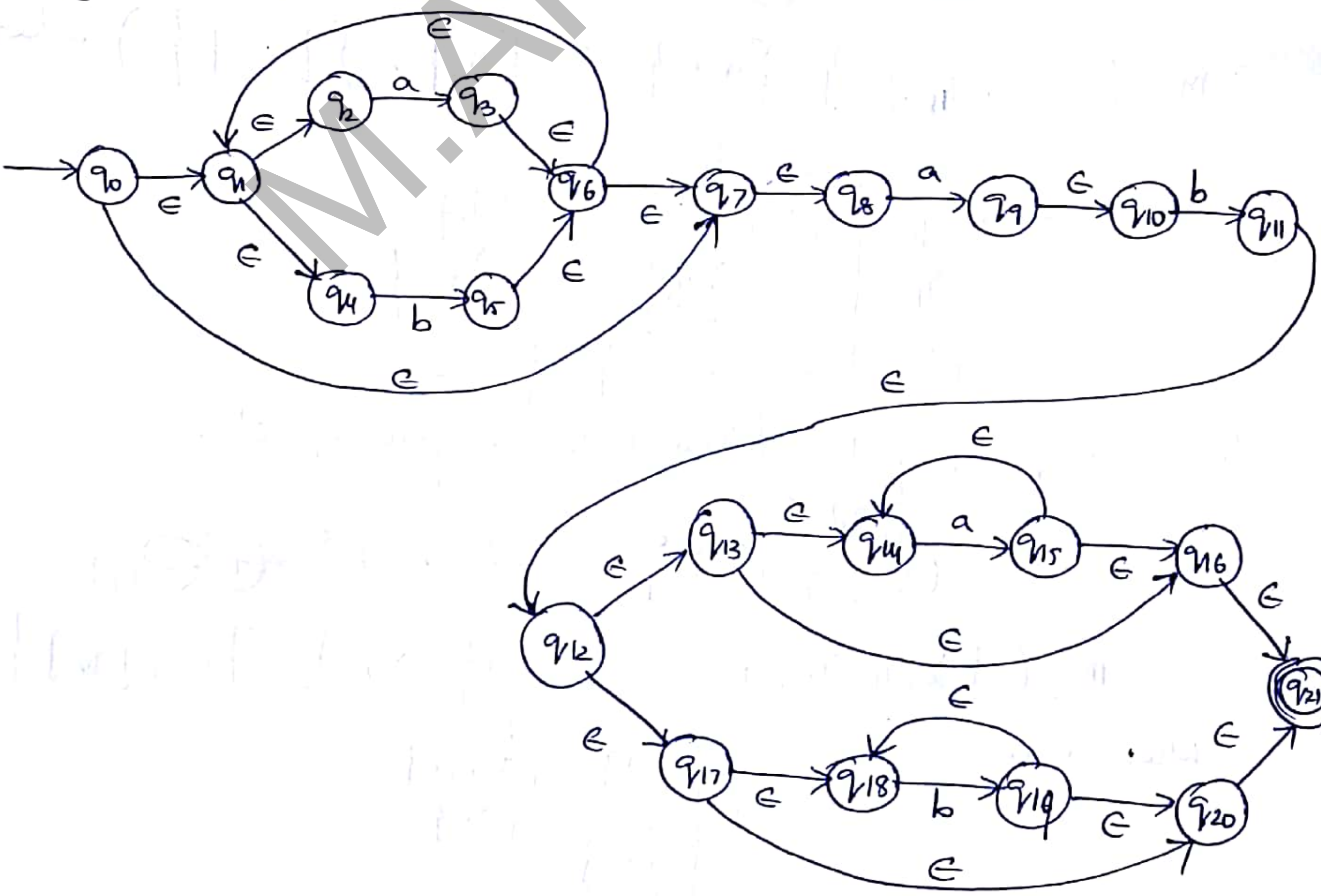
$S \rightarrow aSbS \mid bSaS \mid \epsilon$ gives equal no. of a's & b's.

$G = (V, T, P, S)$ where $V = \{S\}$, $T = \{a, b\}$, P is given above and S is the start symbol.

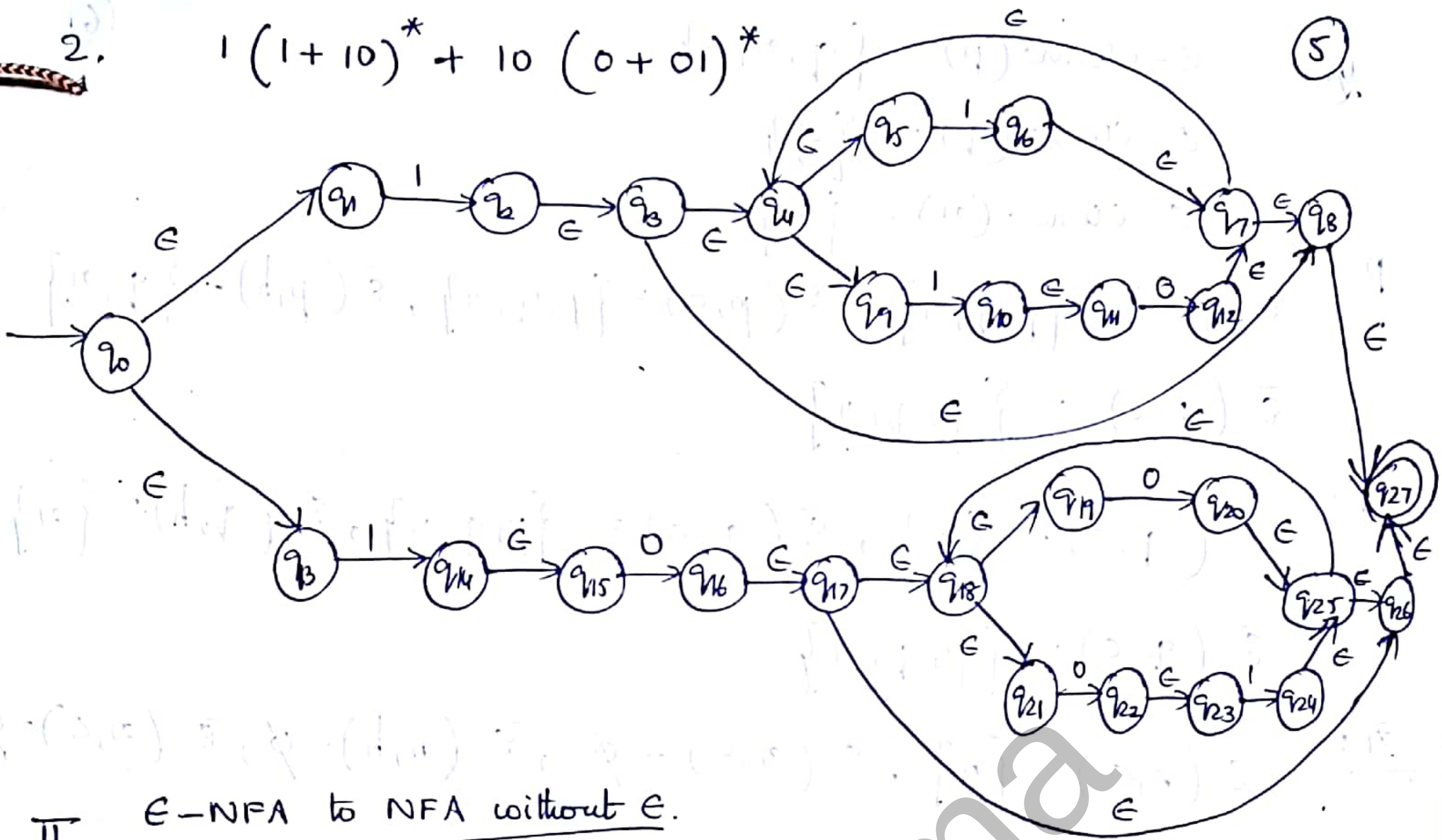
5. Convert the following:

Q 1. RE to ϵ-NFA:

$(a+b)^*ab(a^*+b^*)$



2. $1(1+10)^* + 10(0+01)^*$ (5)



II ϵ–NFA to NFA without ϵ.

1. ϵ-Closure$(p) = \{p\}$
   ϵ-Closure$(q) = \{p, q\}$
   ϵ-Closure$(r) = \{r, q, p\}$

p: $\bar{\delta}(p, \epsilon) = \{p\}$, $\bar{\delta}(p, a) = \{p\}$, $\bar{\delta}(p, b) = \{p, q\}$

$\bar{\delta}(p, c) = \{r, q, p\}$

q: $\bar{\delta}(q, \epsilon) = \{p, q\}$, $\bar{\delta}(q, a) = \{p, q\}$, $\bar{\delta}(q, b) = \{q, r, p\}$

$\bar{\delta}(q, c) = \{p, q, r\}$

r: $\bar{\delta}(r, \epsilon) = \{p, q, r\}$, $\bar{\delta}(r, a) = \{p, q, r\}$, $\bar{\delta}(r, b) = \{p, q, r\}$

$\bar{\delta}(r, c) = \{p, r, q\}$

$F' = \{r\}$

Required NFA is $M' = (Q, \Sigma, \bar{\delta}, p, F')$ where

$Q = \{p, q, r\}$, $\Sigma = \{a, b, c\}$, $\bar{\delta}$ is given above, $p = p$ and $F' = \{r\}$

2. ϵ-Closure$(p) = \{q, r, p\}$

ϵ-Closure$(q) = \{q\}$

ϵ-Closure$(r) = \{r\}$

<u>p:</u> $\bar{\delta}(p,\epsilon) = \{p,q,r\}$, $\bar{\delta}(p,a) = \{p,q,r\}$, $\bar{\delta}(p,b) = \{q,r\}$

$\bar{\delta}(p,c) = \{p,q,r\}$

<u>q:</u> $\bar{\delta}(q,\epsilon) = \{q\}$, $\bar{\delta}(q,a) = \{p,q,r\}$, $\bar{\delta}(q,b) = \{r\}$

$\bar{\delta}(q,c) = \{p,q,r\}$

<u>r:</u> $\bar{\delta}(r,\epsilon) = \{r\}$, $\bar{\delta}(r,a) = \phi$, $\bar{\delta}(r,b) = \phi$, $\bar{\delta}(r,c) = \phi$

$F' = \{p, r\}$

$M' = (\{p,q,r\}, \{a,b,c\}, \bar{\delta}$ (given above), p, $\{p,r\})$

<u>III</u> <u>NFA without ϵ to DFA:</u>

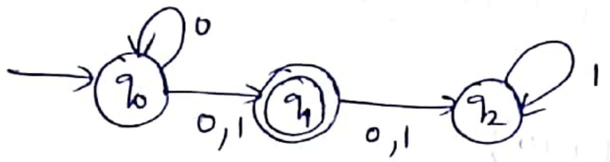
1. $M' = (Q', \Sigma, \delta', [p], F')$ where $Q' = \{[p], [p,q], [p,q,r], [p,r], [p,q,r,s], [p,q,s], [p,r,s], [p,s]\}$

δ'	0	1
$\rightarrow [p]$	$[p,q]$	$[p]$
$[p,q]$	$[p,q,r]$	$[p,r]$
$[p,q,r]$	$[p,q,r,s]$	$[p,r]$
$[p,r]$	$[p,q,s]$	$[p]$
$[p,q,r,s]$	$[p,q,r,s]$	$[p,r,s]$
$[p,q,s]$	$[p,q,r,s]$	$[p,r,s]$
$[p,r,s]$	$[p,q,s]$	$[p,s]$
$[p,s]$	$[p,q,s]$	$[p,s]$

$\Sigma = \{0, 1\}$

δ' given, $p = [p]$

and $F' = \{[p,q,r,s], [p,q,s], [p,r,s], [p,s]\}$

2. 

$M' = (Q', \Sigma, \delta', [q_0], F')$ where $Q' = \{[q_0], [q_0, q_1], [q_1], [q_0, q_1 q_2], [q_1 q_2], [q_2]\}$

δ'	0	1
$\rightarrow [q_0]$	$[q_0\ q_1]$	$[q_1]$
$[q_0\ q_1]$	$[q_0\ q_1\ q_2]$	$[q_1\ q_2]$
$[q_1]$	$[q_2]$	$[q_2]$
$[q_0\ q_1\ q_2]$	$[q_0\ q_1\ q_2]$	$[q_1\ q_2]$
$[q_1\ q_2]$	$[q_2]$	$[q_2]$
$[q_2]$	—	$[q_2]$

$\Sigma = \{0, 1\}$

δ' is as given

$q_0 = [q_0]$

$F' = \{[q_0, q_1], [q_1], [q_0, q_1, q_2], [q_1, q_2]\}$

3. $M' = (Q', \Sigma, \delta', [q_0], F')$ where

δ'	0	1
$\rightarrow [q_0]$	$[q_0\ q_1]$	$[q_0]$
$[q_0\ q_1]$	$[q_0\ q_1]$	$[q_0\ q_2]$
$[q_0\ q_2]$	$[q_0\ q_1]$	$[q_0\ q_3]$
$[q_0\ q_3]$	$[q_0\ q_1\ q_3]$	$[q_0\ q_3]$
$[q_0 q_1 q_3]$	$[q_0\ q_1\ q_3]$	$[q_0\ q_2\ q_3]$
$[q_0\ q_2\ q_3]$	$[q_0\ q_1\ q_3]$	$[q_0\ q_3]$

$Q' = \{[q_0], [q_0 q_1], [q_0 q_2], [q_0 q_3], [q_0\ q_1\ q_3], [q_0, q_2, q_3]\}$

$\Sigma = \{0, 1\}$

δ' is as given

$q_0 = [q_0]$

$F' = \{[q_0\ q_3], [q_0\ q_1\ q_3], [q_0\ q_2\ q_3]\}$

IV. NFA / DFA to REs

1. $q_0 = q_1 1 + q_2 0$ —— (1)

$q_1 = q_0 0$ —— (2)

$q_2 = q_0 1$ —— (3)

$q_3 = q_3(0+1) + q_1 0 + q_2 1$ —— (4)

substituting (2) and (3) in Eq (1)

Solving we get

$$q_0 = q_0(01+10) \qquad R = Q + RP$$

$$q_0 = \epsilon + q_0(01+10) \qquad R \simeq QP^*$$

Using Arden's theorm we get

$$\therefore \; q_0 = (01+10)^*$$ is the required RE.

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2.

$$q_0 = q_0 0 \quad \text{--- (1)}$$

$$q_1 = q_0\epsilon + q_1 1 \quad \text{--- (2)}$$

$$q_2 = q_2 2 + q_1 \epsilon \quad \text{--- (3)}$$

From (1) using Arden's theorm $q_0 = 0^*$ substituting in (2) we get

$$q_1 = 0^*\epsilon + q_1 1 \qquad \text{Using Arden's theorm.}$$

$$\therefore q_1 = 0^* 1^*$$

Substituting in (3) we get

$$q_2 = q_2 2 + 0^* 1^* \epsilon$$

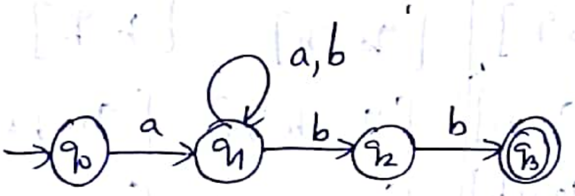
$$q_2 = 0^* 1^* + q_2 2 \qquad \text{Using Arden's theorm we get,}$$

$$\therefore q_2 = 0^* 1^* 2^*$$

Hence Required RE is $0^* 1^* 2^*$.

3. FA for $a(a+b)^*bb$

1. We first get an NFA



This is an NFA without ε transitions. ~~We should convert this to an NFA without ε transitions and then~~ We convert it into a DFA.

Given M'

	a	b
$\rightarrow q_0$	$\{q_1\}$	—
q_1	$\{q_1\}$	$\{q_1 q_2\}$
q_2	—	$\{q_3\}$
$* q_3$	—	—

We construct DFA

$$M = (Q', \Sigma, \delta', [q_0], F')$$

where δ' is given as follows.

δ'	a	b
$\rightarrow [q_0]$	$[q_1]$	
$[q_1]$	$[q_1]$	$[q_1 q_2]$
$[q_1 q_2]$	$[q_1]$	$[q_1 q_2 q_3]$
$[q_1 q_2 q_3]$	$[q_1]$	$[q_1 q_2 q_3]$

$\Sigma = \{a, b\}$, $Q' = \{[q_0], [q_1], [q_1 q_2], [q_1, q_2, q_3]\}$

$q_0 = [q_0]$ and $F' = \{[q_1, q_2, q_3]\}$

6. 1. $L = \{ww^R \mid w \in \{0,1\}^*\}$

Take $w = 001$ and w^R is 100

$\therefore$ $z = 001100$

case(i) let $x = 00$, $y = 11$, $z = 00$

let $k = 2$ $\quad xy^r z = 00(11)^r 00 \Rightarrow 0011 1100$ is a palindrome.

Case (ii)

$x = \epsilon$, $y = 001$ & $z = 100$

for $k = 2$ $\quad xy^k z \Rightarrow \epsilon(001)^r 100 \Rightarrow 001001100 \notin L$

Hence given language is not regular.

2. $C = \{w \mid w$ has equal no. of 0's & 1's$\}$

let $w = 010011$

Case(i)

$x = 01$, $y = 00$, $z = 11$, $xy^r z = 01(00)^r 11 \notin C$

case(ii) $x = \epsilon$, $y = 010$, $z = 011$, $xy^r z = (010)^r 011 \notin C$

Hence language is not regular.

7. Minimize given FA

2. $\Pi_3 = \Pi_2 = \{ \{C, D, F, G\}, \{A\}, \{H, I\}, \{B, E\} \}$

$\therefore$ $Q = \{ \Pi_3$, $\Sigma = \{a, b\}$, $F = \{(C, D, F, G)\}$.

$q_0 = \{A\}$ and δ is given as

	a	b
$\rightarrow$ [A]	[B, E]	[B, E]
[B, E]	[C, D, F, G]	[C, D, F, G]
[C, D, F, G]	[H, I]	[H, I]
[H, I]	[H, I]	[H, I]

8. b) $L = \{ a^n b a^n \mid n \geq 1 \}$

9. a) LMD: $S \rightarrow iCtS \rightarrow ibtS \rightarrow ibtiCtSeS$

$\rightarrow ibtibtSeS \rightarrow ibtibtaeS \rightarrow ibtibtaea$

RMD: $S \rightarrow iCtSeS \rightarrow iCtSea \rightarrow iCtiCtSea$

$\rightarrow iCtiCtaea \rightarrow iCtibtaea \rightarrow ibtibtaea$.

LMD: Assuming string to be "a+a"

b) $E \rightarrow E+T \rightarrow T+T \rightarrow F+T \rightarrow a+T \rightarrow a+F \rightarrow a+a$

RMD:

$E \rightarrow E+T \rightarrow E+F \rightarrow E+a \rightarrow T+a \rightarrow F+a \rightarrow a+a$

10. Ambiguous Grammars.

1. Consider the string "ab".

LMD 1: $S \to AB \to AaB \to aB \to abB \to ab$

LMD 2: $S \to AB \to B \to ab$

Since there is more than 1 LMD for the same string the grammar is ambiguous.

XII a) PDA for given CFG.

1. $V = \{S, B\}$, $T = \{0, 1\}$

$\delta(q, \epsilon, S) = \{(q, 0BB)\}$

$\delta(q, \epsilon, B) = \{(q, 0S), (q, 1S), (q, 0)\}$

$\delta(q, 0, 0) = \{(q, \epsilon)\}$

$\delta(q, 1, 1) = \{(q, \epsilon)\}$

b) CFG for given PDA.

$V = \{S, [q_i, A, q_i], [q_i, Z, q_i] \ \forall q_i \in Q\}$

$T = \{a, b\}$

S is the start symbol

1. $P_1: S \to [q_0 Z q_0]$

$P_2: S \to [q_0 Z q_1]$

$P_3: S \to [q_0 Z q_2]$

$P_4: [q_0, Z, q_0] \to a[q_0 A q_0][q_0 Z q_0]$

$P_5: \to a[q_0 A q_1][q_1 Z q_0]$

$P_6: \to a[q_0 A q_2][q_2 Z q_0]$

$P_7: [q_0 Z q_1] \to a[q_0 A q_0][q_0 Z q_1]$

$P_8: \to a[q_0 A q_1][q_1 Z q_1]$

$P_9: \to a[q_0 A q_2][q_2 Z q_1]$

$P_{10}: [q_0 Z q_2] \to a[q_0 A q_0][q_0 Z q_2]$

$P_{11}: \to a[q_0 A q_1][q_1 Z q_2]$

$P_{12}: \to a[q_0 A q_2][q_2 Z q_2]$

P_{13} : $[q_0\ A\ q_0] \rightarrow a\,[q_0\ A\ q_0]$

P_{14} : $[q_0\ A\ q_1] \rightarrow a\,[q_0\ A\ q_1]$

P_{15} : $[q_0\ A\ q_2] \rightarrow a\,[q_0\ A\ q_2]$

P_{16} : $[q_0\ A\ q_1] \rightarrow b$

P_{17} : $[q_1, Z, q_2] \rightarrow e$

$P_{16}, P_{17}, P_{14}, P_{11}, P_3$ are the prods to be retained.

XV b) Mealy to Moore :

1. Given $M_1 = (Q_1, \Sigma, \Delta, \delta, \lambda, q_0)$ where $Q_1 = \{q_0, q_1\}$, $\Sigma = \{a, b\}$, $\Delta = \{0, 1\}$ and δ and λ are

δ	a	b
q_0	q_0	q_1
q_1	q_0	q_1

and

λ	a	b
q_0	0	1
q_1	1	0

The corresponding Moore Machine is given by $M_2 = (Q_2, \Sigma, \Delta, \delta', \lambda', [q_0, 0])$. The transition function δ' is given by $\delta'([q, b], a) = [\delta(q, a), \lambda(q, a)]$ and $\lambda'[q, b] = b$

δ'	a	b
$[q_0, 0]$	$[q_0, 0]$	$[q_1, 1]$
$[q_0, 1]$	$[q_0, 0]$	$[q_1, 1]$
$[q_1, 0]$	$[q_0, 1]$	$[q_1, 0]$
$[q_1, 1]$	$[q_0, 1]$	$[q_1, 0]$

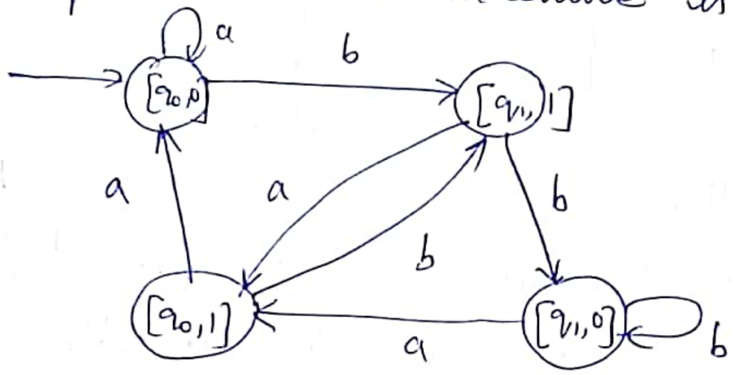
$\lambda'([q_0, 0]) = 0$

$\lambda'([q_0, 1]) = 1$

$\lambda'([q_1, 0]) = 0$

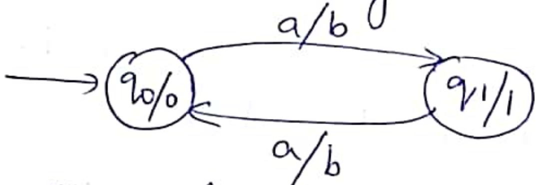
$\lambda'([q_1, 1]) = 1$

The equivalent Moore machine is given as



a) Moore to Mealy machine.

1.



Given $M_1 = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$ where

$Q = \{q_0, q_1\}$, $\Sigma = \{a, b\}$, $\Delta = \{0, 1\}$

δ	a	b
q_0	q_1	q_1
q_1	q_0	q_0

and $\lambda(q_0) = 0$

$\lambda(q_1) = 1$

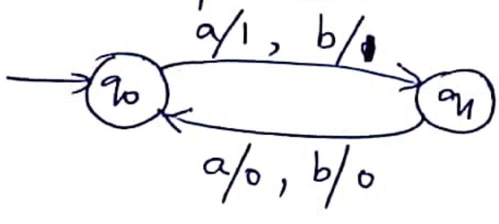
The corresponding Mealy machine for the above is given by M_2 defined as $M_2 = (Q, \Sigma, \Delta, \delta, \lambda', q_0)$ where $\lambda'(q, a)$ is $\lambda(\delta(q, a))$ for all states q and for all input symbols a.

λ'	a	b
q_0	$\lambda(q_1)$	$\lambda(q_1)$
q_1	$\lambda(q_0)$	$\lambda(q_0)$

$\Longrightarrow$

λ'	a	b
q_0	1	1
q_1	0	0

This gives the equivalent Mealy machine as follows:



XI. b) PDA for :

1. $L = \{0^n 1^n \mid n \geq 0\}$

Eg: string $a^3 b^3$.

logic: We push all a's onto stack till we see a b. Once a b is seen we start popping. If string is of the form $a^n b^n$ then an empty stack is left in the end which is the accepting condition. Assume starting state is q_0, Z is initially on top of stack and A & B represent a and b on the stack.

$\delta(q_0, a, Z) = (q_0, AZ)$ Push A

$\delta(q_0, a, A) = (q_0, AA)$ Push A (All a's are seen)

$\delta(q_0, b, A) = (q_0, \epsilon)$ Pop A

$\delta(q_0, \epsilon, Z) = (q_0, \epsilon)$ Acceptance by Empty stack.

PDA is

$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ where $M = (\{q_0\}, \{a, b\}, \{A, B, Z\}, \delta, q_0, Z_0, \phi)$